

WENTWORTH-
SMITH
ARITHMETICS

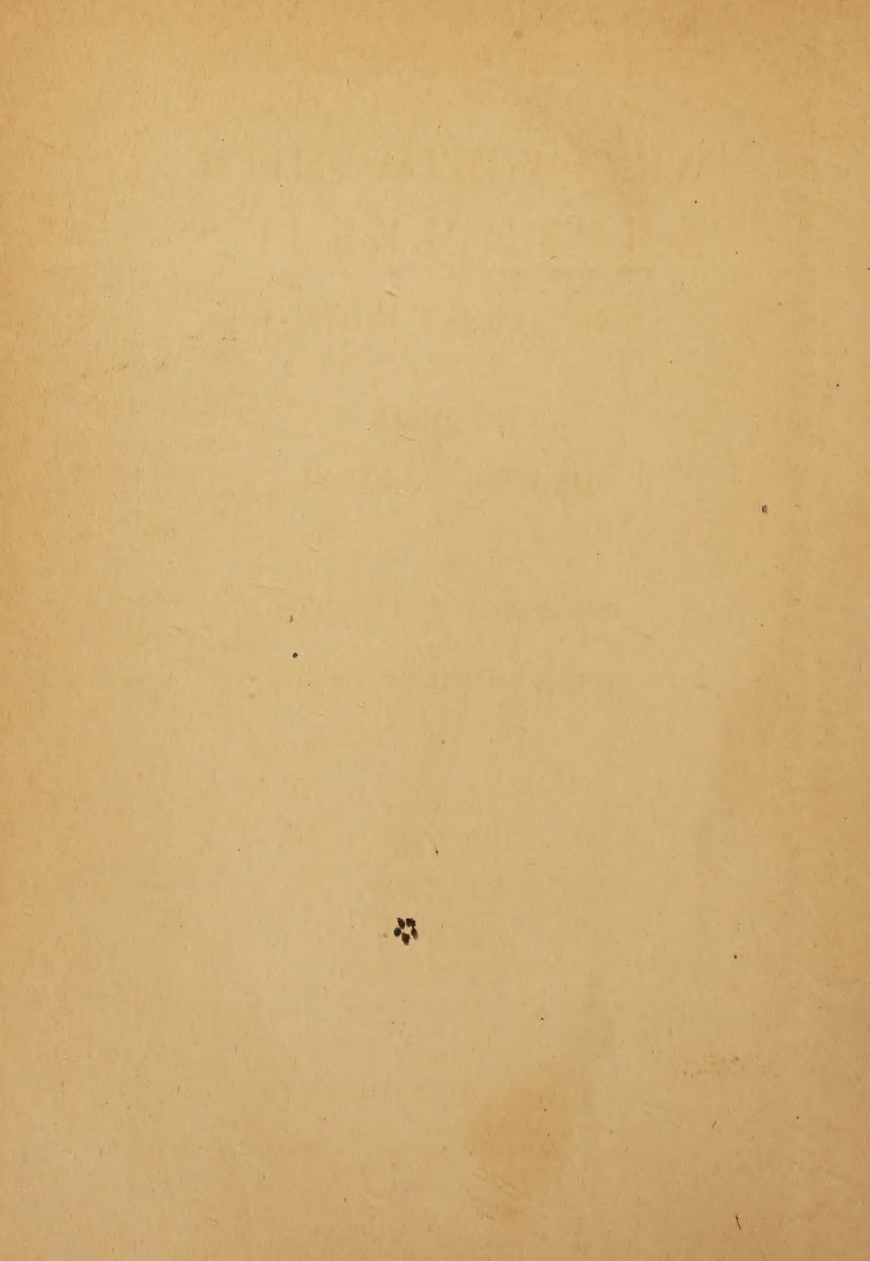
BOOK II



NEW JERSEY EDITION

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WENTWORTH-SMITH MATHEMATICAL SERIES

WENTWORTH-SMITH ARITHMETICS

NEW JERSEY EDITION

BOOK II

GRADES V AND VI

BY

GEORGE WENTWORTH

AND

DAVID EUGENE SMITH



GINN AND COMPANY

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PREFACE

In preparing this series of arithmetics for the schools of New Jersey, and of other parts of the country which may wish to follow the course of study adopted in that state, the authors have followed carefully both the spirit and the letter of the course as shown in the report of the Commissioner of Education. They believe fully in the essential principles upon which the course is based, and they feel that it has been worked out with such skill as to assure excellent results at the hands of all teachers who will heartily join in trying to make it a success. It has two predominating features: in the first place it stands for serious, well-arranged mathematics, and not for a scrappy presentation which always fails to give to the pupil that feeling of mastery of the subject to which he is entitled; and in the second place it appeals to the pupil's human interests by relating the subject to his personal needs and to the life in which he finds himself. Had the course failed to stand for these two principles, had it made an appeal merely for easy problems and for a limited number of applications with little or no drill work, had it offered such an ill-considered sequence as too often appears in contemporary textbooks, and had it failed to advocate the real in mathematics instead of dull formalism, the authors would not have been interested to undertake this series.

There can be no question that the time has passed for mere theory in elementary arithmetic. The good that is in the subject can all be secured from problems representing actual conditions of to-day, together with that drill which is essential to the fixing of habits of accurate and reasonably rapid computation. Therefore the authors have sought to establish true motives for study, to place the pupil

face to face with as real issues as possible, to furnish an abundance of drill work, to suggest to the teacher the best plan of carrying out the spirit of the official course, and to follow this course absolutely in the presentation of the topics.

The work in Grades V and VI has as its large features a thorough study of common and decimal fractions and an introduction to percentage. Along with this is carried a systematic review of the fundamental operations with whole numbers and a continued application of arithmetic to the affairs of daily life. Particular attention is called to the new feature of the Review and Drill sections found at the end of the work of each half grade. Each of these sections consists of a cumulative review drill covering all the operations with whole numbers and fractions, of a set of problems without numbers, and of certain industrial applications in which the pupil finds himself in a real situation making a personal demand upon his knowledge of arithmetic.

The authors wish to acknowledge the kindness of many friends in the state of New Jersey in furnishing problems of local and state interest, thus aiding in the effort to make arithmetic appeal to the needs and tastes of the children. They wish also to acknowledge their indebtedness to the official course of study, from which they have quoted freely throughout the work and in the Suggestions to Teachers at the end.

GEORGE WENTWORTH
DAVID EUGENE SMITH

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ARITHMETIC. BOOK II

GRADE V. FIRST HALF

I. REVIEW OF GRADE IV

1. What you have Learned of Arithmetic. You have already learned much about arithmetic. You have learned to count and to write numbers, at least to a million; how to add, subtract, multiply, and divide whole numbers, or integers; how to solve many problems; how to keep simple accounts; and how to do some easy work with fractions.

You have also learned how to write United States money, how to measure things, and how to use the ordinary tables needed in making these measurements. Besides all this, you have seen that you have need for arithmetic at the store, in your games, in reading, and in many other ways.

2. What there is still to Learn. There is, however, much more to learn about the uses of arithmetic, and more to learn about how to work with numbers. Every one of us is interested in what the world is doing and in our country and its products. We all like to know about our farms, our mines, our forests, our great industries, the accounts that we must sometimes keep, and the customs of business life, and we find much of interest about these matters in arithmetic. We shall now learn how to work with larger numbers, and how to solve problems about our individual interests and about the great industries of the country.

3. Notation. The writing of numbers by means of symbols is called *notation*, and the symbols are called *numerals*.

Sometimes the naming or reading of numbers is called *numeration*.

Our common notation started in India about two thousand years ago. The Arabs took it to Europe. Therefore our common numerals are also called *Arabic numerals* or *Hindu-Arabic numerals*. You know that there are nine of these numerals besides the *zero*, which is also called *naught* or *cipher*.

4. Periods. When more than four figures are used in an integer they are usually separated into groups of three so as to be read more easily; for example, 1,397,352, sometimes printed as 1 397 352. The groups are called *periods*.

5. Place Value. The value of a period or of a figure depends upon the place it occupies. Therefore each has a *place value*. For example, in the number 222, the 2 at the right is in the ones' (or units') place and means 2 ones (or units); the middle 2 is in the tens' place and means 2 tens; the left-hand 2 is in the hundreds' place and means 2 hundreds.

Definitions like those given above should not be memorized, but the teacher should be sure that the terms are understood and can be used properly.

6. Reading Numbers. In a number like 123,209,209 we sometimes speak of the three places in each period as *orders*.

Periods:	Millions	Thousands	Units
Orders:	Hundreds Tens Units	Hundreds Tens Units	Hundreds Tens Units
	123	,209	,209

Names of numbers beyond millions are too rarely used to be taught. A thousand million is called a *billion*, but the word does not always have this meaning, for in England it means a million million.

ORAL EXERCISE

1. What name do you give to 3 tens? to 10 tens?
2. What name do you give to 1000 thousands?
3. Looking at your fingers, do you see why people came to count by tens? Why was it?

4. Read these numbers:

296	10,000	354,200	4,246,384
1296	21,236	400,576	5,027,050
5280	34,007	148,392	15,125,000
9999	90,009	900,009	75,248,309

5. What advantage has our method of writing nine (9) over the method the Romans used, as shown on a clock face (IX)?

6. In the number 234, what place is occupied by the 4? by the 3? by the 2?

7. In the number 123,456,789, what name is given to the period 789? to the period 456? to the period 123? to the whole number?

WRITTEN EXERCISE

Write in words:

- | | | |
|-------------|-------------|-----------------|
| 1. 15,756. | 4. 230,230. | 7. 2,342,756. |
| 2. 24,908. | 5. 500,005. | 8. 31,427,642. |
| 3. 137,005. | 6. 999,999. | 9. 141,141,141. |

Write in figures:

10. Four million, twenty-five thousand, five.
11. Seventy-seven thousand, five hundred eighty-six.
12. Ninety-eight thousand, seven hundred fifty-four.
13. One hundred six thousand, seven hundred twenty.
14. Two hundred million, fourteen thousand, sixty-eight.

7. Roman Numerals. The Roman notation is used chiefly for numbering chapters, for dates, and on clocks. It employs seven capital letters, as follows:

<i>Letters,</i>	I	V	X	L	C	D	M
<i>Values,</i>	1	5	10	50	100	500	1000

The first nine numbers are written thus:

I, II, III, IV or IIII, V, VI, VII, VIII, IX.

The tens are written thus:

X, XX, XXX, XL, L, LX, LXX, LXXX, XC.

The numbers from eleven to nineteen are written thus:

XI, XII, XIII, XIV, XV, XVI, XVII, XVIII, XIX.

The following are examples of other numbers:

XXIII = 23	XLVIII = 48	LXXVI = 76
XXXVI = 36	XCVIII = 98	LXXXII = 82

The New Jersey course requires a knowledge of these numerals to M, and the writing of numbers through C. The pupils may be interested, however, to know that the hundreds are written thus: C, CC, CCC, CD, D, DC, DCC, DCCC, and DCCCC or CM. The year 1917 is written MDCCCXVII or MCMXVII, the former being the usual Roman method.

WRITTEN EXERCISE

Write in common (Arabic) numerals:

1. XI.	5. LXXII.	9. IX.	13. III.
2. LXIV.	6. XCVII.	10. XIX.	14. XIII.
3. XXVI.	7. LXXV.	11. IV.	15. XXIII.
4. LXVI.	8. XXXIV.	12. XIV.	16. XLIII.

Write in Roman numerals:

17. 31.	18. 42.	19. 25.	20. 37.	21. 99.
---------	---------	---------	---------	---------

WRITTEN EXERCISE

1. Write in the common (Arabic) system: LXIX, XLIV.
2. Write in the Roman system: 49, 79, 94, 96, 98, 46.
3. Write in the Roman system the number of this page; of the preceding page; of the page following this one.
4. Write in the common (Arabic) system the number twenty-one million, one thousand, four hundred seventy-five.
5. Write in common figures the number one hundred seven million, one thousand, one.
6. Write in common figures the number of Ex. 5 decreased by 42,675; by 12,245; by 295,001.

Write in words:

- | | | |
|-------------|----------------|------------------|
| 7. 127,007. | 10. 2,176,144. | 13. 12,123,123. |
| 8. 236,036. | 11. 3,981,426. | 14. 42,307,400. |
| 9. 417,000. | 12. 5,237,826. | 15. 100,000,000. |

Write in common numerals:

16. Six million, six; seven million, twelve.
17. Seventy-seven million, seventy-seven.
18. Five hundred fifty-five thousand, five.
19. Seven hundred seventeen thousand, one hundred one.
20. Six hundred sixty-six thousand, eighteen.
21. One hundred million, five hundred seventy-five.

Write in Roman numerals:

- | | | | | |
|---------|---------|---------|---------|---------|
| 22. 24. | 23. 39. | 24. 12. | 25. 93. | 26. 88. |
|---------|---------|---------|---------|---------|

Write in common numerals:

- | | | | | |
|--------|--------|----------|----------|----------|
| 27. C. | 28. D. | 29. XLI. | 30. LXI. | 31. XCI. |
|--------|--------|----------|----------|----------|

8. Addition. The process of finding a number that equals two or more other numbers taken together is called *addition*.

The result of adding two or more numbers is called the *sum* of the numbers.

9. Like Numbers. Numbers applied to the same unit are called *like numbers*.

Thus \$7 and \$2 have the same unit, \$1; 7 in. and 2 in. have the same unit, 1 in.; 9 and 16 have the same unit, 1.

10. Addends. The numbers added are called *addends*.

Only numbers considered as like numbers can be added, and the sum is like the addends.

That is, we cannot add \$4 and 9 ft., or 9 lb. and 6 horses. If we add \$14 and \$20 the sum must be like the addends; that is, it must be \$34, not 34 in. or 34 qt. or 34 boys. We can put 9 books with 6 erasers, but we cannot add them so as to get 15 books or 15 erasers; by calling them *things*, however, we may consider them as like numbers, and say that we have 15 *things*.

Teachers may, if they wish, be guided by the note on abstract and concrete numbers given on page 15, where the subject first assumes some importance to the pupil.

11. How to Add. In adding a column of figures, try to read a column like a word. Try especially to see the 10's, the 5's, or the 15's.

In this column, for example, you should see the two 10's and the 15, and write the answer, 35. However, it is not wise to skip numbers in order to group by 10's, because you may forget the numbers that you skipped and thus make a serious error.

$$\begin{array}{r}
 4 \\
 2 \\
 9 \\
 8 \\
 2 \\
 7 \\
 3 \\
 \hline
 35
 \end{array}
 \begin{array}{l}
 \nearrow \\
 \nearrow \\
 \nearrow \\
 \nearrow \\
 \nearrow \\
 \nearrow \\
 \nearrow
 \end{array}
 \begin{array}{l}
 15 \\
 10 \\
 10
 \end{array}$$

12. Check. To *check*, or *prove*, the work, add the column from the top down, if you first added it upwards.

WRITTEN SPEED TEST

Copy the numbers in lines 1 to 8 inclusive, and add; then add the columns numbered 9 to 16 inclusive:

	9.	10.	11.	12.	13.	14.	15.	16.
1.	3	4	6	4	3	3	9	2
2.	4	4	1	9	8	1	4	3
3.	2	9	5	3	4	8	9	5
4.	4	7	4	9	4	8	2	8
5.	8	3	3	4	1	3	7	6
6.	8	2	2	1	9	7	3	6
7.	9	5	4	2	8	1	6	3
8.	<u>6</u>	<u>4</u>	<u>8</u>	<u>4</u>	<u>3</u>	<u>9</u>	<u>9</u>	<u>2</u>

Proceed in the same way for lines numbered 17 to 24 inclusive, and for columns numbered 25 to 32 inclusive:

	25.	26.	27.	28.	29.	30.	31.	32.
17.	5	9	8	7	3	2	6	8
18.	2	5	9	2	8	5	7	6
19.	9	7	6	5	4	6	9	7
20.	2	7	4	1	7	6	4	9
21.	7	7	5	8	8	3	9	4
22.	1	3	6	6	2	8	4	7
23.	2	4	6	3	9	5	8	7
24.	<u>4</u>	<u>2</u>	<u>3</u>	<u>8</u>	<u>5</u>	<u>7</u>	<u>9</u>	<u>6</u>

Such tests are of great value, especially if the pupils keep records of their speed and accuracy and review the exercise from time to time to see how the records improve.

13. Addition of Larger Numbers. You have already learned, in primary arithmetic, how to add numbers of two or more figures, so that we need consider only one simple example. To add 67, 86, and 34, we write them so that units come under units, and so on.

Then $4 + 6 + 7 = 17$, or 1 ten + 7 units. We therefore write the 7 in the units' column and add the 1 to the tens.

67
86
34
187 sum

Then 1 ten + 3 tens + 8 tens + 6 tens = 18 tens, or 1 hundred + 8 tens. We therefore write the 8 in the tens' column and the 1 in the hundreds' column.

The sum is 187.

Arrange units of the same order in the same column.

Add each column by itself, beginning at the right, and write the sum, if it is less than 10, below the column added.

If the sum of any column is 10 or more, write the unit figure below the column added, and add the tens with the next column.

WRITTEN SPEED TEST

Copy the numbers in lines 1 to 6 inclusive, and add; then add the columns numbered 7 to 14 inclusive, and time yourself.

	7.	8.	9.	10.	11.	12.	13.	14.
1.	12	20	23	14	86	77	80	78
2.	6	10	15	17	83	85	90	94
3.	7	9	12	23	87	88	91	93
4.	13	15	14	36	42	23	31	46
5.	21	31	21	41	28	17	16	20
6.	<u>42</u>	<u>24</u>	<u>33</u>	<u>23</u>	<u>14</u>	<u>23</u>	<u>41</u>	<u>26</u>

ORAL EXERCISE

Look at each example and state only the sum:

1.	2.	3.	4.	5.	6.	7.
17	27	37	67	97	29	42
<u>2</u>	<u>2</u>	<u>22</u>	<u>72</u>	<u>32</u>	<u>10</u>	<u>30</u>
8.	9.	10.	11.	12.	13.	14.
15	21	32	10	21	11	12
<u>13</u>	<u>12</u>	<u>14</u>	<u>6</u>	<u>10</u>	<u>11</u>	<u>10</u>
15.	16.	17.	18.	19.	20.	21.
20	33	42	81	16	15	17
<u>25</u>	<u>12</u>	<u>63</u>	<u>40</u>	<u>10</u>	<u>10</u>	<u>20</u>
22.	23.	24.	25.	26.	27.	28.
21	32	41	10	21	31	40
<u>35</u>	<u>70</u>	<u>88</u>	<u>73</u>	<u>20</u>	<u>31</u>	<u>50</u>
29.	30.	31.	32.	33.	34.	35.
75	31	41	21	32	50	51
<u>32</u>	<u>92</u>	<u>21</u>	<u>36</u>	<u>40</u>	<u>60</u>	<u>62</u>
36.	37.	38.	39.	40.	41.	42.
41	23	20	21	20	30	32
62	40	42	42	96	40	43
<u>10</u>	<u>10</u>	<u>20</u>	<u>20</u>	<u>10</u>	<u>50</u>	<u>54</u>

This is a type of the rapid oral work that should be given daily in arithmetic. From three to five minutes of such rapid work in each recitation will do much to remove the charge that pupils are usually slow and inaccurate in the fundamental operations. It should always be remembered that the most important part of arithmetic is found in the four fundamental operations. Particular attention should be given to oral drill in addition by endings.

WRITTEN EXERCISE

See how long it takes to copy and add each of these columns, and to check each result. Write the time with the results.

1.	2.	3.	4.
\$147.50	\$748.06	\$375.42	\$298.04
35.70	126.53	104.41	127.63
61.80	39.40	630.67	701.96
<u>130.50</u>	<u>86.51</u>	<u>49.60</u>	<u>872.37</u>
5.	6.	7.	8.
\$359.68	\$496.48	\$396.04	\$982.69
298.76	127.92	72.63	604.05
400.92	600.00	129.87	81.97
640.32	503.52	871.64	631.83
<u>701.24</u>	<u>872.08</u>	<u>280.93</u>	<u>400.00</u>
9.	10.	11.	12.
\$1482.63	\$4761.42	\$3147.29	\$4268.75
689.74	3917.63	2873.41	3942.35
2908.37	5238.58	4192.64	2065.08
8517.37	6082.37	128.46	5095.02
9310.26	4098.07	3029.00	2711.40
<u>7091.63</u>	<u>5901.93</u>	<u>7081.00</u>	<u>1029.60</u>
13.	14.	15.	16.
\$3246.48	\$2763.48	\$2983.96	\$1430.00
4139.72	298.43	4198.73	280.00
863.41	4200.75	2299.80	7557.75
96.37	8342.66	4763.81	263.48
2800.42	528.93	989.95	2986.75
<u>437.00</u>	<u>2743.66</u>	<u>2263.48</u>	<u>3945.50</u>

17. The sales of a department store for the four weeks of December amounted to \$32,368.42, \$38,426.54, \$41,286.35, and \$53,342.60. What was the total amount?

18. The sales of a dry-goods merchant for the twelve months in a year were as follows: \$3772.60, \$4237.40, \$3816.75, \$4627.42, \$3278.75, \$3846.50, \$3121.27, \$3016.50, \$3827.52, \$4126.35, \$4335.50, \$4925.75. What was the total amount of the sales for the year?

19. The area of the United States is 3,602,990 sq. mi.; of the Philippine Islands, 143,000 sq. mi.; of Porto Rico, 3600 sq. mi.; of Hawaii, 6740 sq. mi.; and of other islands, 554 sq. mi. What is the total area of the United States and its possessions?

20. The leading manufacturing cities in New Jersey in a recent year, with the value of their products, were as follows: Newark, \$202,512,000; Jersey City, \$128,775,000; Bayonne, \$73,641,000; Paterson, \$69,584,000; Camden, \$49,138,000; Perth Amboy, \$73,093,000; Trenton, \$49,009,000; Elizabeth, \$29,147,000; Passaic, \$41,729,000; Hoboken, \$20,413,000. Find the total value of the manufactures of these cities for that year.

21. The principal grain crops produced in New Jersey in a recent year were as follows: corn, 10,374,000 bu.; buckwheat, 264,000 bu.; wheat, 1,462,000 bu.; oats, 1,849,000 bu.; and rye, 1,260,000 bu. How many bushels were there in all?

Such statistics are, of course, only approximately correct.

22. The three most important manufactures in New Jersey in a recent year were as follows: copper refining and smelting products, \$125,651,000; silk and silk goods, \$65,430,000; foundry and machine-shop products, \$65,398,000. Find the total of these manufactures in that year.

14. Subtraction. The operation of finding the number which, when added to the smaller of two numbers, makes the larger is called *subtraction*. The number thus obtained is the *difference* between the two given numbers. That is, the difference between 75 and 48 is 27, because $48 + 27 = 75$.

15. Subtrahend. In subtraction, the number to which the difference is added, or the number which is subtracted, is called the *subtrahend*.

563 minuend
127 subtrahend
<u>436</u> difference

16. Minuend. The number from which we subtract is called the *minuend*. The minuend is the sum of the difference and the subtrahend.

Subtraction may also be defined as the operation of taking one number from another, and the result may be defined as the remainder or difference. But since a great deal of subtraction is "making change," where the work is based on addition, the definition given above is coming to be preferred.

The teacher is advised to give but little time to asking for such definitions as those given above, for the terms are rarely used in business. The business man says, "Deduct \$10 and pay me the *balance*," instead of "Subtract \$10 and pay me the *difference*." The difference is sometimes called the *remainder*.

ORAL EXERCISE

Find the number which, added to the smaller of these numbers, makes the larger :

1.	$\begin{array}{r} 7 \\ 3 \\ \hline \end{array}$	$\begin{array}{r} 17 \\ 3 \\ \hline \end{array}$	$\begin{array}{r} 6 \\ 2 \\ \hline \end{array}$	$\begin{array}{r} 26 \\ 2 \\ \hline \end{array}$	$\begin{array}{r} 3 \\ 1 \\ \hline \end{array}$	$\begin{array}{r} 43 \\ 1 \\ \hline \end{array}$	$\begin{array}{r} 5 \\ 2 \\ \hline \end{array}$	$\begin{array}{r} 75 \\ 2 \\ \hline \end{array}$
----	---	--	---	--	---	--	---	--

2.	$\begin{array}{r} 27 \\ 10 \\ \hline \end{array}$	$\begin{array}{r} 27 \\ 13 \\ \hline \end{array}$	$\begin{array}{r} 36 \\ 20 \\ \hline \end{array}$	$\begin{array}{r} 36 \\ 22 \\ \hline \end{array}$	$\begin{array}{r} 43 \\ 30 \\ \hline \end{array}$	$\begin{array}{r} 43 \\ 31 \\ \hline \end{array}$	$\begin{array}{r} 85 \\ 40 \\ \hline \end{array}$	$\begin{array}{r} 85 \\ 42 \\ \hline \end{array}$
----	---	---	---	---	---	---	---	---

3.	$\begin{array}{r} 67 \\ 21 \\ \hline \end{array}$	$\begin{array}{r} 48 \\ 23 \\ \hline \end{array}$	$\begin{array}{r} 96 \\ 35 \\ \hline \end{array}$	$\begin{array}{r} 78 \\ 46 \\ \hline \end{array}$	$\begin{array}{r} 57 \\ 27 \\ \hline \end{array}$	$\begin{array}{r} 39 \\ 18 \\ \hline \end{array}$	$\begin{array}{r} 75 \\ 66 \\ \hline \end{array}$	$\begin{array}{r} 87 \\ 78 \\ \hline \end{array}$
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17. How to Subtract. To subtract 756 from 1632, we proceed as follows:

There is no number which, added to 6, makes 2; that is, we cannot subtract 6 from 2, but we can subtract 6 from 12, for $12 - 6 = 6$. We have now used 10 of the 30, leaving 20. We cannot subtract 50 from 20, but $120 - 50 = 70$. We have now used 100 of the 1600, and $1500 - 700 = 800$. Therefore the result is 876.

1632
756
876

Check. Since $876 + 756 = 1632$, the work is evidently correct.

WRITTEN EXERCISE

Subtract the following numbers:

1.	2.	3.	4.	5.
796	796	796	746	746
<u>384</u>	<u>394</u>	<u>398</u>	<u>399</u>	<u>587</u>
6.	7.	8.	9.	10.
3234	3234	3234	3234	3234
<u>872</u>	<u>815</u>	<u>845</u>	<u>1856</u>	<u>2978</u>
11.	12.	13.	14.	15.
4368	5721	2933	4203	4307
<u>299</u>	<u>4186</u>	<u>1768</u>	<u>2192</u>	<u>2358</u>
16.	17.	18.	19.	20.
3628	4023	5127	5962	6014
<u>969</u>	<u>3648</u>	<u>3675</u>	<u>3183</u>	<u>4235</u>
21.	22.	23.	24.	25.
6271	8123	8207	8190	9235
<u>4632</u>	<u>4637</u>	<u>5620</u>	<u>6391</u>	<u>7687</u>

WRITTEN EXERCISE

See how long it takes to copy these numbers and to subtract and check. Write the time with the last result.

1. 42,350 <u>21,175</u>	2. 14,762 <u>9,987</u>	3. 36,431 <u>21,596</u>	4. 50,000 <u>29,872</u>
5. 63,527 <u>41,962</u>	6. 27,998 <u>19,999</u>	7. 60,000 <u>48,762</u>	8. 70,900 <u>56,907</u>
9. \$175.62 <u>81.98</u>	10. \$129.87 <u>67.98</u>	11. \$208.09 <u>98.78</u>	12. \$400.12 <u>69.84</u>
13. \$170.08 <u>69.99</u>	14. \$370.42 <u>91.64</u>	15. \$209.18 <u>87.69</u>	16. \$302.78 <u>62.95</u>
17. \$802.65 <u>69.81</u>	18. \$200.00 <u>93.74</u>	19. \$608.75 <u>35.82</u>	20. \$672.93 <u>98.94</u>
21. \$409.98 <u>23.49</u>	22. \$682.70 <u>93.81</u>	23. \$200.00 <u>187.60</u>	24. \$325.34 <u>128.92</u>
25. \$429.81 <u>149.92</u>	26. \$308.72 <u>129.85</u>	27. \$298.05 <u>109.72</u>	28. \$408.00 <u>129.60</u>

Teachers will do well to return to these efficiency tests occasionally, comparing the records of the pupils with their earlier records.

18. Multiplication. The process of taking one number as many times as there are units in another is called *multiplication*.

This means multiplication by a whole number. Thus, to multiply \$3 by 2 means that \$3 is taken 2 times.

The number multiplied is called the *multiplicand*.

The number by which the multiplicand is multiplied is called the *multiplier*.

The result of multiplying is called the *product*.

$\$325$ multiplicand $\quad 3$ multiplier $\$975$ product

19. Abstract and Concrete Numbers. It is common to speak of a number that does not refer to any particular kind of object or measure as an *abstract number*. It is also customary to speak of a number that refers to some particular kind of object or measure as a *concrete number*.

Thus 4 is commonly called an abstract number, and 4 feet a concrete number.

This distinction is not observed so much as it formerly was, for all *number* is essentially abstract. We may specify the units (feet, yards, apples, etc.), but the *number* is still abstract. The custom of specifying the units in solving a problem is also tending to disappear, the pupil keeping the units in mind just as the business man does when he solves his problems.

For example, consider the following cases:

If one book costs 20¢ and another book costs 30¢, both books cost *as many cents* as $20 + 30$, or 50¢.

If one book costs 20¢ and another book costs 30¢, the second book costs *as many cents* more than the first as $30 - 20$, or 10¢.

If 1 orange costs 5¢, 4 oranges will cost *as many cents* as 4×5 , or 20¢.

If 1 orange costs 5¢, one can buy *as many oranges* for 20¢ as $20 \div 5$, or 4 oranges.

If 4 oranges cost 20¢, each orange costs *as many cents* as $20 \div 4$, or 5¢.

If desired, the above method may be used throughout the entire book.

In school, however, it adds to the clearness of the solution if the more important numbers are labeled with their proper units, and therefore this is done in the solutions in this book. For example, in $3 \times \$325 = \975 , the multiplicand and product are properly given the same label.

20. Nature of the Numbers. It is therefore seen that

The multiplier must be thought of as abstract, and the product is like the multiplicand.

That is, if we have 3 times 17 ft., the 3 is abstract, and the product is feet, like the multiplicand.

21. Multiple. The product of two abstract integers is called a *multiple* of either. For example, 35 is a multiple of 7 and of 5.

22. Factors. The numbers which multiplied together make another number are called its *factors*. For example, the factors of 30 are 2, 3, and 5.

23. Power. The result of taking a number any number of times as a factor is called a *power* of the number.

For example, $2 \times 2 = 4$, and 4 is called the *second* power, or the *square*, of 2. 2×2 is written 2^2 . So $2 \times 2 \times 2$, or 2^3 , is the *third* power of 2. It is also called the *cube* of 2.

ORAL EXERCISE

1. State four multiples of 7; of 3; of 9; of 11; of 15.
2. State the factors of 35; of 77; of 21; of 49; of 121.
3. State the squares of 7, 6, 9, 8, 10, 11, 20, 100, 1000.
4. What is the square of 5? the cube of 3? the fourth power of 2? the cube of 5? the cube of 2? the cube of 10?
5. What is the square of 12? the cube of 4? the fourth power of 1? the fifth power of 2?
6. Multiply, in turn, by 2, 3, 4, and so on to 9:

7	4	6	3	8	5	9	2
11	20	21	30	50	70	40	60

24. How to Multiply. You have already learned how to multiply one number by another. For example, to multiply \$635.50 by 215, 3247 by 1305, and 3247 by 1350 we have:

\$635.50	3247	3247
215	1305	1350
<u>3177 50</u>	<u>16235</u>	<u>162350</u>
6355 0	9741	9741
127100	3247	3247
<u>\$136632.50</u>	<u>4237335</u>	<u>4383450</u>

In multiplying \$635.50 by 215 we multiply as if the problem were $215 \times 63,550\phi$. Then $5 \times 63,550 = 317,750$. We next have to multiply 63,550 by 1 ten, and the result is 63,550 tens, which we write so that the 0 is in tens' place. We next have to multiply 63,550 by 2 hundreds, and the result is 127,100 hundreds, which we write so that the right-hand figure is in hundreds' place. Adding, we have 13,663,250 cents, or \$136,632.50.

WRITTEN EXERCISE

Multiply each number by each of the numbers below it in the column; thus, 376×243 , 405×243 , and so on:

I	II	III	IV	V	VI
243	307	442	581	226	822
376	476	208	622	642	344
405	921	179	389	379	296
221	330	643	427	496	473
392	407	827	462	577	892
470	920	906	330	640	487
689	821	432	448	807	745
573	139	507	229	488	307

This will give 168 examples, from which the teacher should select only as many as necessary. Upon this whole question of efficiency tests it would be well at this time to read the suggestions on page 83.

WRITTEN EXERCISE

Multiply the following :

- | | | |
|------------------------|--------------------------|-----------------------------|
| 1. 24×325 . | 25. 83×1765 . | 49. $43 \times 21,761$. |
| 2. 32×526 . | 26. 59×2763 . | 50. $56 \times 29,376$. |
| 3. 41×578 . | 27. 23×4126 . | 51. $58 \times 42,807$. |
| 4. 53×482 . | 28. 58×3279 . | 52. $64 \times 80,009$. |
| 5. 67×573 . | 29. 63×4081 . | 53. 123×1407 . |
| 6. 78×409 . | 30. 49×7756 . | 54. 236×3143 . |
| 7. 89×277 . | 31. 82×3793 . | 55. 823×4062 . |
| 8. 96×483 . | 32. 41×8872 . | 56. 347×8096 . |
| 9. 523×832 . | 33. 64×8709 . | 57. 823×7743 . |
| 10. 624×421 . | 34. 33×4981 . | 58. 629×3648 . |
| 11. 532×409 . | 35. 25×4444 . | 59. 426×3140 . |
| 12. 677×505 . | 36. 62×3436 . | 60. 408×3776 . |
| 13. 752×327 . | 37. 83×7070 . | 61. 567×9873 . |
| 14. 581×826 . | 38. 49×6236 . | 62. 228×4362 . |
| 15. 823×983 . | 39. 83×8192 . | 63. 503×8742 . |
| 16. 435×481 . | 40. 54×7762 . | 64. 621×8263 . |
| 17. 632×408 . | 41. 36×8087 . | 65. 987×7890 . |
| 18. 563×481 . | 42. 53×8129 . | 66. 602×3348 . |
| 19. 408×662 . | 43. 66×6666 . | 67. 672×4815 . |
| 20. 507×836 . | 44. 91×3473 . | 68. 360×4572 . |
| 21. 802×906 . | 45. 17×4646 . | 69. 666×7883 . |
| 22. 532×848 . | 46. 33×8308 . | 70. 565×8432 . |
| 23. 783×927 . | 47. $46 \times 12,481$. | 71. $821 \times \$248.75$. |
| 24. 860×342 . | 48. $34 \times 36,408$. | 72. $633 \times \$336.90$. |

ORAL EXERCISE

1. Multiply by 10: 10¢, 50¢, \$1.20, \$3.50.
2. Multiply by 100: 3, 42, 565, \$1.50, \$21.75.
3. State a short way of multiplying a whole number (integer) by 10; by 100; by 1000; by 10,000.
4. State a short way of multiplying a number representing United States money by 10; by 1000.

25. Multiplying by Powers of 10. *To multiply an integer by 10, annex a zero; to multiply by 100, annex two zeros; to multiply by 1000, annex three zeros.*

To multiply a number representing United States money by 10, move the decimal point one place to the right; to multiply by 100, move the decimal point two places to the right; to multiply by 1000, move the decimal point three places to the right.

Hence to multiply 27 by 40, multiply by 4 and annex 0.

To multiply \$25.16 by 300, multiply by 3 and move the decimal point two places to the right.

WRITTEN EXERCISE

Multiply the following:

- | | |
|---------------------------------|---------------------------------|
| 1. $210 \times \$346.$ | 6. $320 \times \$4215.$ |
| 2. $800 \times 932 \text{ ft.}$ | 7. $700 \times 864 \text{ rd.}$ |
| 3. $9600 \times \$4300.$ | 8. $4300 \times \$8250.$ |
| 4. $7000 \times \$15.42.$ | 9. $9000 \times \$21.75.$ |
| 5. $6520 \times \$12.50.$ | 10. $2750 \times \$31.02.$ |

11. If a railway train goes 48 mi. an hour, how far will it go in 20 hr., at the same rate?

12. If it costs \$14.75 a year to educate you, how much does it cost to educate 300 pupils, at the same rate?

ORAL EXERCISE

1. On the blackboard, multiply 68 by 5. Divide 680 by 2.
2. On the blackboard, multiply \$12.25 by 5. Also divide \$122.50 by 2. How do the results compare? Tell a short way of multiplying by 5.
3. Because $5 = 10 \div 2$, to multiply an integer by 5 is the same as to annex 0 and divide by what number?
4. On the blackboard, multiply \$42.74 by 50. Also divide \$4274 by 2. Compare the results.
5. Because $50 = 100 \div 2$, to multiply by 50 we may first multiply by what number, and then divide by what other number?
6. On the blackboard, multiply \$32.56 by 25. Also divide \$3256 by 4. Compare the results.

Multiply the following by 5, 50, and 25, in turn :

7. 4	12	16	28	40	80	120
8. 8	24	32	48	88	96	160

26. Aliquot Part. A number that will exactly divide another is called an *aliquot part* of that number.

For example, 2 and 5 are aliquot parts of 10, and 2, 5, 10, 20, 25, and 50 are aliquot parts of 100.

Therefore the aliquot parts of \$1 are 2¢, 5¢, 10¢, 20¢, 25¢, and 50¢. To these may be added $33\frac{1}{3}$ ¢, $16\frac{2}{3}$ ¢, and certain other aliquot parts which will be considered later.

27. Multiplying by Aliquot Parts. From the above exercise we have the following:

To multiply by 5, multiply by 10 and divide by 2.

To multiply by 50, multiply by 100 and divide by 2.

To multiply by 25, multiply by 100 and divide by 4.

ORAL EXERCISE

1. Multiply 20 by 5; divide 200 by 2.
2. Multiply 40 by 5; divide 400 by 2.
3. Multiply 4 by 25; divide 400 by 4.
4. At 24¢ each, what will 5 pineapples cost?
5. At 28¢ a yard, what will 5 yd. of ribbon cost?
6. At \$80 each, what will 50 ponies cost?
7. State a short way of multiplying by 5; by 50; by 500.
8. How much is 25×4 ? 25×8 ?
9. State a short way of multiplying by 25.

WRITTEN EXERCISE

1. Multiply by 50: 237, 425 ft., \$68.50, \$987.40.
2. Multiply by 25: 134, 802 ft., \$49.60, \$384.40.
3. Multiply by 500: 16, 64, 726, \$42.80, \$124.40.

Multiply by 5:

4. 48	74	88	82	448	1264	3642
5. 64	38	78	96	826	2488	8177

Multiply by 50:

6. 66	38	96	76	224	2172	4242
7. 88	46	98	42	338	3108	6543

Multiply by 25:

8. 32	36	64	56	224	4844	4036
-------	----	----	----	-----	------	------

Multiply, in turn, 50¢, 25¢, 20¢, and 10¢ by:

9. 40	200	160	240	360	480	2800
-------	-----	-----	-----	-----	-----	------

WRITTEN EXERCISE

1. At \$72 an acre, how much must a man pay for 25 acres of land? for 250 acres?

2. At \$55 each, how much must a farmer pay for 25 cows? for 50 cows? for 5 cows?

3. At \$1425 each, how much will a manufacturer receive for 25 automobiles? for 50 automobiles?

4. If 2 M (2 thousand) feet of pine cost \$96.50, what will 8 M cost?

Pupils should appreciate the importance of recognizing the *relations of numbers*. In this example they should see that 8 M is 4×2 M, and should simply multiply \$96.50 by 4.

5. If 6 thousand square feet of building paper cost \$13.68, what will 3 thousand square feet cost?

6. If 5 lb. of butter cost \$1.50, what will 15 lb. cost? what will $2\frac{1}{2}$ lb. cost? what will 50 lb. cost?

7. If a man grinds 35 bu. of corn, each bushel weighing 56 lb., how many pounds does he grind?

8. A farmer adds to his property a field of 65 acres, paying \$75 an acre for it. How much does the field cost?

9. A dealer sells 15 automobiles for \$13,875. How much does he receive for 45 automobiles at this rate?

10. If 38 cows average 408 lb. of milk apiece a month, how many pounds of milk do the cows produce in a month?

11. If a dealer sells 8 building lots for \$13,875, how much does he receive for 2 lots at this rate?

Teachers should accustom the pupils always to estimate an answer in advance and to write it down. Then if they find a great difference between their result and their estimate they have either shown poor judgment or made an error in their work. Many absurd answers can be avoided in this way.

28. Nature of Division. In multiplication two factors are given to find the product. In division the product and one of two factors are given to find the other factor.

That is, in the case of $2 \times \$5 = \10 we have given the factors 2 and \$5, from which to find the product \$10.

In division we have given the product \$10 and the factor 2, to find the other factor \$5; or the factor \$5, to find the other factor 2.

Upon the question of abstract and concrete numbers the teacher should consult § 19.

29. Division. If the product and one of two factors are given, the process of finding the other factor is called *division*.

The symbol for division is $\div$. It is read *divided by*.

For example, $15 \div 5 = 3$. This is also indicated by another symbol, thus: $\frac{15}{5} = 3$. That is, as operations to be performed, $15 \div 5$ and $\frac{15}{5}$ have the same meaning.

30. Dividend. The given product, that is, the number to be divided, is called the *dividend*.

31. Divisor. The given factor, that is, the number by which we divide, is called the *divisor*.

32. Quotient. The required factor, that is, the number found by division, is called the *quotient*.

33. Remainder. Often a divisor does not exactly divide the dividend. Then that part of the dividend which is left is called the *remainder*.

Thus $14 \div 3 = 4$ with remainder 2. The quotient is usually written $4\frac{2}{3}$, meaning 4 with 2 still to be divided into 3 equal parts.

Such terms have long been familiar to you, and they are given at this time only for the purpose of review.

34. Two Kinds of Division. Since $2 \times \$4 = \8 , we see that $\$8 \div \$4 = 2$ and $\$8 \div 2 = \4 . That is,

If the dividend and the divisor are concrete, they must be like numbers, and the quotient is abstract.

If the dividend is concrete and the divisor abstract, the quotient is concrete and like the dividend.

Concerning this distinction, however, the teacher should consult § 19.

35. Check. To check the work in division, we multiply the quotient by the divisor; this product added to the remainder, if any, should equal the dividend.

For example, $45 \div 5 = 9$; then $5 \times 9 = 45$. Again, $48 \div 5 = 9$ with remainder 3; then $5 \times 9 = 45$, and $45 + 3 = 48$.

36. Short Division. When the divisor is so small that the work may be done mentally, the process is called *short division*.

1. Required to divide 342 by 3.

If 342 is separated as shown, we can divide each part by 3. This is what we really do mentally. We write the numbers as shown; then 3 is contained in 3 once (that is, 100 times in 300); 3 is contained in 4 once with remainder 1 (that is, 10 times in 40 with remainder 10); this 1 is 10 units, and with the 2 it makes 12. Then 3 is contained in 12 four times. Therefore the quotient is 114. We think, "3 in 3, 1; in 4, 1; in 12, 4."

$$\begin{array}{r} 3 \overline{) 300 + 30 + 12} \\ 100 + 10 + 4 = 114 \end{array}$$

2. Required to divide 42,364 by 7.

Here 7 is not contained in 4, but 7 is contained in 42 six times. Then 7 is not contained in 3, so we write a 0 below the 3. Then 7 is contained in 36 five times with remainder 1, and in 14 twice. Therefore the quotient is 6052.

$$\begin{array}{r} 3 \overline{) 342} \\ 114 \end{array}$$

$$\begin{array}{r} 7 \overline{) 42364} \\ 6052 \end{array}$$

3. Required to divide 35,282 by 9.

Here 9 is contained in 35 three times with remainder 8; in 82 nine times with remainder 1; in 18 twice; in 2 no times with remainder 2. Therefore the quotient is 3920 with remainder 2, and this is written $3920\frac{2}{9}$.

$$\begin{array}{r} 9 \overline{)35282} \\ \underline{3920} \\ 3920\frac{2}{9} \end{array}$$

4. Required to divide \$27.53 by 4.

Dividing as before, we have $\$27 \div 4 = \6 with \$3 remaining; $\$3.50 \div 4 = \0.8 with \$0.30 remaining; $\$0.33 \div 4 = \0.08 with \$0.01 remaining. Therefore the quotient is $\$6.88\frac{1}{4}$.

$$\begin{array}{r} 4 \overline{)\$27.53} \\ \underline{\$6.88} \phantom{\frac{1}{4}} \\ \$6.88\frac{1}{4} \end{array}$$

ORAL EXERCISE

Divide the following :

- | | | | |
|------------------|-------------------|-------------------|-------------------|
| 1. $48 \div 2$. | 6. $84 \div 7$. | 11. $52 \div 4$. | 16. $33 \div 2$. |
| 2. $39 \div 3$. | 7. $96 \div 8$. | 12. $75 \div 5$. | 17. $41 \div 3$. |
| 3. $84 \div 4$. | 8. $54 \div 9$. | 13. $78 \div 6$. | 18. $53 \div 4$. |
| 4. $65 \div 5$. | 9. $38 \div 2$. | 14. $91 \div 7$. | 19. $63 \div 5$. |
| 5. $72 \div 6$. | 10. $54 \div 3$. | 15. $96 \div 6$. | 20. $75 \div 6$. |

WRITTEN EXERCISE

Divide the following :

- | | | |
|---------------------|------------------------|------------------------|
| 1. $8242 \div 2$. | 11. $\$38.72 \div 4$. | 21. $42,345 \div 8$. |
| 2. $4122 \div 3$. | 12. $\$92.75 \div 5$. | 22. $76,209 \div 9$. |
| 3. $7004 \div 4$. | 13. $\$41.30 \div 6$. | 23. $\$35.21 \div 2$. |
| 4. $4015 \div 5$. | 14. $\$99.47 \div 7$. | 24. $\$72.43 \div 3$. |
| 5. $4032 \div 6$. | 15. $\$65.68 \div 8$. | 25. $\$61.23 \div 4$. |
| 6. $3073 \div 7$. | 16. $\$83.61 \div 9$. | 26. $\$72.36 \div 5$. |
| 7. $8368 \div 8$. | 17. $81,004 \div 4$. | 27. $\$82.02 \div 6$. |
| 8. $4019 \div 9$. | 18. $39,008 \div 5$. | 28. $\$42.07 \div 7$. |
| 9. $7555 \div 2$. | 19. $41,269 \div 6$. | 29. $\$75.20 \div 8$. |
| 10. $9015 \div 3$. | 20. $80,047 \div 7$. | 30. $\$73.18 \div 9$. |

37. Dividing by 10 and its Powers. To divide by 10 is merely to find how many tens there are in the number.

Thus in 1230 there are 123 tens and no units. In 2373 there are 237 tens and 3 units remainder; that is, $2373 \div 10 = 237 \text{ r } 3$.

To divide an integer by 10, cut off the right-hand figure of the dividend. The result is the quotient, and the right-hand figure expresses the remainder.

To divide an integer by any power of 10, cut off as many figures from the right of the dividend as there are zeros at the right of the divisor. The result is the quotient, and the right-hand figures express the remainder.

Thus $12,400 \div 100 = 124$; $736,117 \div 1000 = 736 \text{ r } 117$.

38. Dividing by Multiples of Powers of 10. In this case we divide by the power of 10, as in § 37, and then by the number indicating the multiple.

For example, to divide 42,800 by 200, we cut off two zeros from the divisor and the dividend and then divide by 2.

So, to divide 27,407 by 300, we cut off two zeros from the divisor and two figures from the right of the dividend. We then divide by 3, and the quotient is 91 with remainder 1 in the hundreds' place, and to this we join the 07 cut off, obtaining the remainder 107.

$$200 \overline{) 42800}$$

$$214$$

$$300 \overline{) 27407}$$

$$91 \frac{107}{300}$$

WRITTEN EXERCISE

Find the quotient, and the remainder if any:

1. $860 \div 10$.

5. $950 \div 50$.

9. $450 \div 90$.

2. $940 \div 20$.

6. $720 \div 60$.

10. $810 \div 90$.

3. $630 \div 30$.

7. $840 \div 70$.

11. $375 \div 10$.

4. $840 \div 40$.

8. $960 \div 80$.

12. $625 \div 20$.

Find the quotient, and the remainder if any:

- | | | |
|---------------------|-------------------------|---------------------------|
| 13. $625 \div 30$. | 21. $41,300 \div 100$. | 29. $378,367 \div 900$. |
| 14. $475 \div 40$. | 22. $37,800 \div 200$. | 30. $438,000 \div 3000$. |
| 15. $336 \div 50$. | 23. $54,300 \div 300$. | 31. $228,000 \div 4000$. |
| 16. $527 \div 70$. | 24. $92,400 \div 400$. | 32. $745,000 \div 5000$. |
| 17. $642 \div 70$. | 25. $72,501 \div 500$. | 33. $663,600 \div 6000$. |
| 18. $495 \div 80$. | 26. $67,803 \div 600$. | 34. $569,800 \div 7000$. |
| 19. $873 \div 90$. | 27. $49,521 \div 700$. | 35. $848,623 \div 8000$. |
| 20. $888 \div 90$. | 28. $32,449 \div 800$. | 36. $729,542 \div 9000$. |

37. There being 60 minutes in an hour, how many hours are there in 1080 minutes?

38. There being 60 seconds in a minute, how many minutes are there in 1680 seconds?

39. At \$40 each, how many harnesses must a manufacturer sell to receive \$1240?

40. A farmer pays \$6750 for some land at \$75 an acre. How many acres does he buy?

41. A dealer sells some city lots for \$44,100 at \$700 a lot. How many lots does he sell?

42. An investor has \$62,750 with which to buy some bonds at \$500 each. What is the greatest number he can buy, and how much money will he have left?

43. An automobile dealer has \$133,000 to invest in automobiles at \$1200 each. What is the greatest number he can buy for this sum, and how much money will he have left?

44. A ton is 2000 pounds. If a coal pocket contains 597,350 pounds of coal, how many tons does it contain, and how many pounds are left over?

39. Long Division. When the divisor has two or more figures, except in such cases as are given in §§ 37 and 38, it is customary to use a process known as *long division*.

We shall first consider the simplest case, that in which the units' figure is 1.

Required to divide 7182 by 21.

If all the work is written, it would appear as in the first of these forms, but for practical purposes the second is used.

(1)	(2)
$\begin{array}{r} 342 \\ 21 \overline{)7182} \\ \underline{6300} = 300 \text{ times the divisor} \\ 882 \\ \underline{840} = 40 \text{ times the divisor} \\ 42 \\ \underline{42} = 2 \text{ times the divisor} \end{array}$	$\begin{array}{r} 342 \\ 21 \overline{)7182} \\ \underline{63} \\ 88 \\ \underline{84} \\ 42 \\ \underline{42} \end{array}$

First we will consider (1), the more complete form. We see that 21 is evidently not contained in 7, so there are no thousands in the quotient. The divisor is contained three times in 71, and since the 71 is hundreds, the quotient figure 3 is written over the hundreds' place. Then $300 \times 21 = 6300$, and there remains 882 still to be divided; 21 is contained in 88 four times, and since the 88 is tens, the 4 of the quotient is written over the tens' place. Then $40 \times 21 = 840$, and there remains 42 to be divided; 21 is contained in 42 twice, and 2 is written over the units' place. The quotient is therefore 342. It will be seen that 6300, 840, and 42 make up 342×21 , as should be the case.

Consider now the second form, which is always used in practice. Since at any time in this example we need only two figures of the dividend or two figures of such a partial dividend as 882, we may omit the zeros in 6300 and 840, and use only the 88 in the remainder 882, thus making the work shorter.

We may check our result by multiplying 342 by 21. Then $21 \times 342 = 7182$, the dividend.

Required to divide 8547 by 21.

Here we have $85 \div 21 = 4$, and since the 84 is hundreds, the 4 is written over the hundreds' place. Then $4 \times 21 = 84$, and when 84 is subtracted, there is a remainder 1. Bringing down one figure as before, we have 14 tens to be divided by 21. Since 21 is not contained in 14, we write 0 in the tens' place to show that there are no tens, and bring down the next figure, 7. Then $147 \div 21 = 7$, and $7 \times 21 = 147$, exactly, so there is no remainder.

$$\begin{array}{r} 407 \\ 21 \overline{)8547} \\ \underline{84} \\ 147 \\ \underline{147} \\ 0 \end{array}$$

WRITTEN EXERCISE

Divide the following :

- | | | |
|---------------------|------------------------|------------------------|
| 1. $462 \div 11$. | 19. $1098 \div 61$. | 37. $41,004 \div 51$. |
| 2. $561 \div 11$. | 20. $1353 \div 41$. | 38. $49,410 \div 61$. |
| 3. $693 \div 11$. | 21. $1633 \div 71$. | 39. $59,696 \div 41$. |
| 4. $979 \div 11$. | 22. $2673 \div 81$. | 40. $52,582 \div 61$. |
| 5. $869 \div 11$. | 23. $3367 \div 91$. | 41. $62,267 \div 71$. |
| 6. $968 \div 11$. | 24. $2331 \div 21$. | 42. $63,048 \div 71$. |
| 7. $748 \div 11$. | 25. $2583 \div 21$. | 43. $71,280 \div 81$. |
| 8. $924 \div 11$. | 26. $3472 \div 31$. | 44. $90,090 \div 91$. |
| 9. $273 \div 21$. | 27. $2121 \div 21$. | 45. $90,909 \div 91$. |
| 10. $294 \div 21$. | 28. $6789 \div 31$. | 46. $50,337 \div 51$. |
| 11. $462 \div 21$. | 29. $8547 \div 21$. | 47. $50,898 \div 51$. |
| 12. $693 \div 21$. | 30. $3317 \div 31$. | 48. $62,937 \div 81$. |
| 13. $798 \div 21$. | 31. $10,731 \div 21$. | 49. $71,928 \div 81$. |
| 14. $819 \div 21$. | 32. $14,322 \div 31$. | 50. $60,939 \div 61$. |
| 15. $987 \div 21$. | 33. $16,461 \div 31$. | 51. $34,587 \div 61$. |
| 16. $899 \div 31$. | 34. $25,297 \div 41$. | 52. $73,629 \div 81$. |
| 17. $779 \div 41$. | 35. $18,879 \div 31$. | 53. $65,448 \div 81$. |
| 18. $816 \div 51$. | 36. $28,741 \div 41$. | 54. $73,619 \div 91$. |

40. Long Division Continued. We shall now consider the more general case of a two-figure divisor.

Required to divide 19,224 by 54.

The divisor is not contained in 1 or in 19, so there are no ten-thousands and no thousands in the quotient. But since $3 \times 54 = 162$, the divisor is contained three times in 192 with remainder 30. Since this is hundreds, the quotient figure 3 is written over the hundreds' place. Bringing down the next figure of the dividend, 2, we have 302 tens to be divided. Since $5 \times 54 = 270$, the divisor is contained five times in 302 with remainder 32. Since this is tens, the quotient figure 5 is written over the tens' place. Bringing down the next figure of the dividend, 4, we have 324 still to be divided. Since 324 is exactly 6×54 , the units' figure is 6 and there is no remainder. Hence the quotient is 356.

$$\begin{array}{r}
 356 \\
 54 \overline{)19224} \\
 \underline{162} \\
 302 \\
 \underline{270} \\
 324 \\
 \underline{324} \\
 0
 \end{array}$$

41. Product Tables. It is sometimes helpful for beginners to prepare a table of products of the divisor, as follows:

$$\begin{array}{llll}
 2 \times 54 = 108 & 4 \times 54 = 216 & 6 \times 54 = 324 & 8 \times 54 = 432 \\
 3 \times 54 = 162 & 5 \times 54 = 270 & 7 \times 54 = 378 & 9 \times 54 = 486
 \end{array}$$

With such a table made out in advance, we can tell at once, without making several trials, what the next figure of the quotient will be.

WRITTEN EXERCISE

Divide the following:

- | | | |
|---------------------|------------------------|-------------------------|
| 1. $3848 \div 52$. | 7. $2793 \div 57$. | 13. $63,204 \div 69$. |
| 2. $8556 \div 92$. | 8. $6006 \div 77$. | 14. $74,907 \div 87$. |
| 3. $6572 \div 62$. | 9. $2470 \div 19$. | 15. $89,001 \div 99$. |
| 4. $4599 \div 73$. | 10. $8580 \div 66$. | 16. $41,890 \div 59$. |
| 5. $8815 \div 43$. | 11. $22,311 \div 67$. | 17. $301,184 \div 64$. |
| 6. $2275 \div 35$. | 12. $31,950 \div 75$. | 18. $891,891 \div 99$. |

WRITTEN EXERCISE

1. If \$1548 is divided equally among a dozen persons, how much is the share of each?
2. If \$2193.75 is divided equally among 15 persons, how much is the share of each?
3. If a piece of land containing 289 acres is cut into 17 equal lots, what is the area of each?
4. If the dividend is 420, the quotient 32, and the remainder 4, what is the divisor?
5. If the quotient is 24, the divisor 12, and the remainder 4, what is the dividend?
6. At 260 mi. a day, how long would it take a steamer to go from San Francisco to Guam, 5200 mi.?
7. At a certain temperature sound travels 1132 ft. a second. How long will it take it to travel 23,772 ft.?
8. The product of two numbers is 98,441, and one of them is 49. What is the other number?
9. A man paid \$20,280 for a farm, the price being \$65 an acre. How many acres did he buy?
10. A canning factory ships 30,000 cans of fruit in boxes containing 4 dozen cans each. How many boxes does it require?
11. There being 27 cu. ft. in a cubic yard, how many loads (cubic yards) of earth must be removed to excavate 108,675 cu. ft.?
12. One of the routes around the world is 18,424 mi. long. If a man makes the trip in 56 days, what is the average distance traveled per day?
13. If the circumference of a wagon wheel is 15 ft., and there are 5280 ft. in a mile, how many times will the wheel turn in going 9 mi.? how many times in going 25 mi.?

42. Long Division Continued. We may now consider a case like that of 341,597 divided by 248.

The divisor is not contained in 3 or in 34, so there are no hundred-thousands or ten-thousands in the quotient. But it is contained in 341 once, with remainder 93. Since this is thousands, the quotient figure, 1, is written over the thousands' place. Bringing down the next figure of the dividend, 5, we have 935 hundreds to be divided. It might seem as if the next figure of the quotient would be 4, and here a product table (§ 41) would be helpful; but $4 \times 248 = 992$, and this is larger than 935. Trying 3 for the next figure, we have $3 \times 248 = 744$ with 191 remaining. Since we have divided hundreds by 248, the 3 is hundreds and is written over the hundreds' place. Bringing down the next figure, we find that $1919 \div 248 = 7$ with 183 remaining. Since the 7 is tens, we write it over the tens' place. Then, bringing down the last figure, we have $1837 \div 248 = 7$ with 101 remaining. Therefore the quotient is $1377\frac{1}{2}\frac{1}{8}$.

$$\begin{array}{r}
 1377 \\
 248 \overline{)341597} \\
 \underline{248} \\
 935 \\
 \underline{744} \\
 1919 \\
 \underline{1736} \\
 1837 \\
 \underline{1736} \\
 101
 \end{array}$$

We may check the work by adding 101 to the product of 248×1377 , the result being 341,597, the dividend.

43. General Directions. Therefore, in long division

Write the divisor to the left of the dividend.

Take for the first partial dividend the least number of left-hand figures that will contain the divisor, and write the quotient over the right-hand figure of this partial dividend.

Multiply the divisor by this quotient, and place the product under the partial dividend used.

Subtract this product, and to the remainder annex the next figure of the dividend for a new partial dividend.

Divide as before, and continue this process until all the figures of the dividend have been used.

WRITTEN EXERCISE

Divide the following :

- | | |
|-------------------------|----------------------------|
| 1. $25,585 \div 301.$ | 26. $7753 \div 69.$ |
| 2. $52,373 \div 631.$ | 27. $8598 \div 76.$ |
| 3. $47,971 \div 623.$ | 28. $8585 \div 23.$ |
| 4. $153,397 \div 211.$ | 29. $76,452 \div 56.$ |
| 5. $976,950 \div 501.$ | 30. $18,753 \div 64.$ |
| 6. $972,471 \div 111.$ | 31. $71,797 \div 57.$ |
| 7. $161,784 \div 321.$ | 32. $19,140 \div 46.$ |
| 8. $324,360 \div 306.$ | 33. $26,523 \div 47.$ |
| 9. $140,850 \div 225.$ | 34. $27,559 \div 59.$ |
| 10. $183,750 \div 175.$ | 35. $47,078 \div 69.$ |
| 11. $148,555 \div 407.$ | 36. $50,602 \div 82.$ |
| 12. $234,901 \div 503.$ | 37. $101,160 \div 232.$ |
| 13. $208,575 \div 309.$ | 38. $110,370 \div 124.$ |
| 14. $870,672 \div 204.$ | 39. $252,150 \div 412.$ |
| 15. $367,848 \div 786.$ | 40. $120,830 \div 653.$ |
| 16. $102,465 \div 495.$ | 41. $260,380 \div 422.$ |
| 17. $472,932 \div 604.$ | 42. $192,892 \div 232.$ |
| 18. $349,206 \div 407.$ | 43. $211,428 \div 5873.$ |
| 19. $117,440 \div 320.$ | 44. $287,712 \div 5328.$ |
| 20. $419,640 \div 538.$ | 45. $313,205 \div 8465.$ |
| 21. $287,712 \div 216.$ | 46. $411,075 \div 5481.$ |
| 22. $615,000 \div 328.$ | 47. $616,693 \div 8009.$ |
| 23. $685,542 \div 286.$ | 48. $965,496 \div 9852.$ |
| 24. $997,108 \div 478.$ | 49. $1,599,560 \div 2836.$ |
| 25. $965,496 \div 852.$ | 50. $2,402,728 \div 6257.$ |

WRITTEN EXERCISE

1. If 11 acres of land are worth \$1485, what is the average price per acre? How much are 4 acres worth?
2. If a dealer pays \$336 a dozen for suits of ready-made clothes, how much does he pay per suit? for 7 suits?
3. A man pays \$1276 for the rent of a house in the city for 11 months. At this rate, how much does he pay for 5 months?
4. A wagon dealer pays \$1215 for 9 wagons. At this rate, how much do 4 wagons cost? How much do 7 wagons cost?
5. At the rate of \$784 for 8 acres of land, how much will a man have to pay for 3 acres? for 5 acres? for 100 acres?
6. If a stock dealer buys some cattle for \$72,012, paying \$51 a head, how many head of cattle does he buy?
7. If a jobber sells during the year \$82,411 worth of shoes at \$61 a case, how many cases does he sell?
8. If a dealer sells \$164,598 worth of linseed oil during a year at \$42 a barrel, how many barrels does he sell?
9. If a factory sells 3684 ready-made suits of a certain grade for \$92,100, what is the price per suit?
10. If a state pays \$2,325,798 for constructing 471 miles of model road, what is the average cost per mile?
11. If it costs \$5,236,602 to construct 287 miles of railroad, what is the average cost per mile?
12. If a company employs 8009 men on the same schedule of wages, and the pay roll for these men is \$6,247,020 a year, what is the annual wages of each?
13. If a company employs 250 men on the same schedule of wages, and the pay roll for these men is \$270,000 a year, what is the monthly wages of each?

44. Long Division of United States Money. Division of United States money does not involve any new principle. We divide as with other numbers, placing the decimal point in the quotient above that in the dividend.

Required to divide \$21,375.12 by 372.

Since \$2, \$21, and \$213 cannot be divided by 372 with an integer for a quotient, we divide \$2137 by 372, and the quotient is \$5 with remainder \$277. Since the \$2137 is in the tens' place of dollars, we write the \$5 over the tens' place of the dividend.

The rest of the division is as in cases already studied. The decimal point in the quotient is placed over that in the dividend, and the quotient is \$57.46.

$$\begin{array}{r}
 \$57.46 \\
 372 \overline{) \$21375.12} \\
 \underline{1860} \\
 2775 \\
 \underline{2604} \\
 1711 \\
 \underline{1488} \\
 2232 \\
 \underline{2232} \\
 0000
 \end{array}$$

WRITTEN EXERCISE

Divide the following :

- | | |
|----------------------|--------------------------|
| 1. \$973.47 ÷ 111. | 10. \$39,913.56 ÷ 81. |
| 2. \$573.12 ÷ 199. | 11. \$27,891.25 ÷ 125. |
| 3. \$1967.50 ÷ 50. | 12. \$14,225.40 ÷ 420. |
| 4. \$6882.92 ÷ 11. | 13. \$734,986.12 ÷ 836. |
| 5. \$12,018.30 ÷ 21. | 14. \$501,738.80 ÷ 538. |
| 6. \$96,899.23 ÷ 23. | 15. \$102,465.00 ÷ 2070. |
| 7. \$36,147.06 ÷ 53. | 16. \$692,356.00 ÷ 8764. |
| 8. \$33,328.80 ÷ 54. | 17. \$616,693.00 ÷ 8009. |
| 9. \$33,747.20 ÷ 64. | 18. \$388,591.00 ÷ 9037. |

19. A shop has 664 men on its pay roll, each receiving the same wages. For 6 days this pay roll amounts to \$9960. How much does each man receive a day?

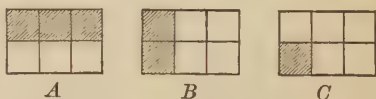
II. COMMON FRACTIONS

45. What you have Learned. You have already learned about such fractions as $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$, but you would have some trouble to find the cost of $2\frac{1}{2}$ lb. of tea at $37\frac{1}{2}\text{¢}$ a pound. Before you can do work like this you must study more about fractions.

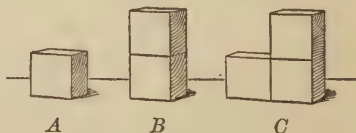
ORAL EXERCISE

1. When anything is divided into 3 equal parts, what is each part called? What is each part called when the object is divided into 4 equal parts? into 5 equal parts?

2. In these figures, what part of *A* is shaded? What part of *B* is shaded? What part of *C* is shaded? From *A* we see that $\frac{1}{2}$ equals how many sixths? From *B* we see that $\frac{1}{3}$ equals how many sixths?



3. In these figures of cubes, *A* is what part as large as *B*? *B* is what part as large as *C*? If *C* is called 1, what are *A*



and *B*? If *B* is called 1, what are *A* and *C*? What is $A + C$?

4. Each of these small rectangles, or oblongs, is what part of the whole rectangle? Each shaded triangle is what part of the whole rectangle? What does this tell about $\frac{1}{2}$ of $\frac{1}{9}$? What does it tell about $\frac{1}{9}$ of $\frac{1}{2}$?



The pupils should see that a fraction is an expression of a part of one thing (as $\frac{1}{3}$ of an orange) or of a group of things (as $\frac{1}{3}$ of 16 oranges). They should also see that it is an expression of division.

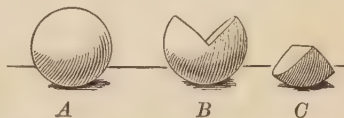
46. Unit. Any one thing is called a *unit*.

47. Fraction. One or more of the equal parts of a unit is called a *fraction*.

48. Denominator. The number which shows into how many equal parts a unit has been divided is called the *denominator*.

For example, a sphere has been divided into 4 equal parts, and B is $\frac{3}{4}$ of it. Then 4 is the denominator.

We read $\frac{1}{3}$ ft. "one third of a foot."



49. Numerator. The number which shows how many equal parts have been taken to make a fraction is called the *numerator*. In the fraction $\frac{3}{4}$, since 3 shows the number of fourths that have been taken, then 3 is the numerator.

50. Terms. The numerator and the denominator together are called the *terms* of the fraction.

51. Common Fraction. A fraction which has both terms expressed is called a *common fraction*.

52. Proper Fraction. A fraction whose numerator is less than the denominator is called a *proper fraction*.

53. Unit Fraction. If the numerator is 1, the fraction is called a *unit fraction*.

54. Improper Fraction. A fraction whose numerator equals or exceeds the denominator is called an *improper fraction*.

For example, A is $\frac{1}{4}$ of a sphere, and $A + C$ is $\frac{5}{4}$ of a sphere.

55. Mixed Number. The sum of a whole number (often called an *integer*) and a fraction is called a *mixed number*.

For example, if the above sphere A is 1, $A + B$ is $1\frac{3}{4}$ and $A + C$ is $1\frac{1}{4}$.

ORAL EXERCISE

Read aloud the following :

- | | | | | | |
|--------------------|-----------------|-----------------|-----------------|-----------------|------------------|
| 1. $\frac{3}{8}$ | $\frac{7}{8}$ | $\frac{1}{16}$ | $\frac{3}{5}$ | $\frac{7}{16}$ | $\frac{9}{16}$ |
| 2. $\frac{5}{8}$ | $\frac{2}{3}$ | $\frac{3}{16}$ | $\frac{15}{16}$ | $\frac{11}{16}$ | $\frac{7}{32}$ |
| 3. $2\frac{4}{5}$ | $3\frac{7}{8}$ | $9\frac{2}{3}$ | $8\frac{3}{16}$ | $2\frac{5}{32}$ | $25\frac{2}{3}$ |
| 4. $12\frac{1}{2}$ | $33\frac{1}{3}$ | $15\frac{4}{5}$ | $9\frac{3}{32}$ | $6\frac{7}{64}$ | $157\frac{7}{8}$ |
5. An inch is $\frac{1}{12}$ ft., $\frac{1}{36}$ yd., or $\frac{1}{198}$ rd.

Fractions having denominators larger than 8 are rarely used in computation, but in measuring we use fractions with larger denominators, usually not exceeding 64.

WRITTEN EXERCISE

Write the following fractions :

- | | |
|-----------------------------|--------------------------|
| 1. Seven sixteenths. | 5. Seven thirty-seconds. |
| 2. Nine sixteenths. | 6. Seventeen eightieths. |
| 3. Nineteen thirty-seconds. | 7. Nine sixty-fourths. |
| 4. Eight ninths. | 8. Five thirty-seconds. |

Write in words the following fractions or mixed numbers :

- | | | | | | | | |
|-------------------|---------------|----------------|----------------|-----------------|-----------------|-----------------|-----------------|
| 9. $\frac{1}{2}$ | $\frac{1}{8}$ | $\frac{5}{8}$ | $\frac{7}{8}$ | $\frac{5}{16}$ | $\frac{5}{24}$ | $\frac{15}{16}$ | $\frac{17}{32}$ |
| 10. $\frac{1}{6}$ | $\frac{1}{4}$ | $\frac{3}{4}$ | $\frac{3}{8}$ | $\frac{7}{16}$ | $\frac{23}{24}$ | $\frac{19}{32}$ | $\frac{21}{64}$ |
| 11. $\frac{1}{3}$ | $\frac{2}{3}$ | $1\frac{2}{3}$ | $5\frac{4}{5}$ | $62\frac{7}{8}$ | $27\frac{1}{2}$ | $6\frac{2}{3}$ | $15\frac{5}{8}$ |

In practical business $\frac{1}{2}\phi$ and over, in the final result, is called 1ϕ ; less than $\frac{1}{2}\phi$ is neglected in many cases, although in making out bills any fraction of a cent usually counts as a cent.

Write the following in words, and also write the number of dollars and cents that would be taken in making out bills :

- | | | | |
|----------------------------|-----------------------|------------------------|------------------------|
| 12. \$125.12 $\frac{1}{2}$ | \$6.66 $\frac{2}{3}$ | \$333.33 $\frac{1}{3}$ | \$275.16 $\frac{2}{3}$ |
| 13. \$135.87 $\frac{1}{2}$ | \$14.37 $\frac{1}{2}$ | \$112.16 $\frac{7}{8}$ | \$235.45 $\frac{1}{3}$ |

ORAL EXERCISE

1. Look at figure *A* and state how many sixths you see in one half.

2. Look at figure *B* and state how many eighths you see in one half; in three quarters.

*A**B**C*

3. Look at figure *C* and state how many tenths you see in one half; in one fifth; in four fifths.

4. How many fifths do you see in two tenths? in four tenths? in six tenths? in eight tenths?

56. Reducing Fractions. We see that $\frac{2}{4}$ may be obtained from $\frac{1}{2}$ by multiplying both terms of the fraction by 2, and that $\frac{1}{2}$ may be obtained from $\frac{5}{10}$ by dividing both terms by 5. That is,

Both terms of a fraction may be multiplied by the same number without changing the value of the fraction.

Both terms of a fraction may be divided by the same number without changing the value of the fraction.

When we change the terms without changing the value of a fraction we *reduce* the fraction.

When both terms cannot be divided by the same number, the fraction is said to be in *lowest terms*.

To reduce a fraction to lowest terms, divide by the largest number that will divide both terms without a remainder.

$$\frac{8}{12} = \frac{8 \div 4}{12 \div 4} = \frac{2}{3}, \text{ lowest terms}$$

In this example the common factor 4 is said to be *canceled* from both terms when the work is written as here shown.

$\frac{2}{\cancel{8}} = \frac{2}{3}$

ORAL EXERCISE

1. Calling the large rectangle *one*, how many halves do you see in 1?



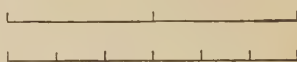
2. How many fourths do you see in $\frac{1}{2}$? in 1?



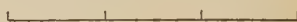
3. How many eighths do you see in $\frac{1}{4}$? in $\frac{1}{2}$? in 1?



4. Calling the line *one*, how many sixths do you see in $\frac{1}{2}$? in 1?



5. $1\frac{1}{2}$ = how many sixths?



6. How many sixths do you see in $\frac{1}{3}$? in $\frac{2}{3}$? in $\frac{3}{3}$? in 1?

WRITTEN EXERCISE

1. Reduce to halves:

$$\frac{2}{4} \quad \frac{4}{8} \quad \frac{8}{8} \quad \frac{12}{8} \quad \frac{16}{8} \quad \frac{24}{8} \quad \frac{32}{8} \quad \frac{40}{8}$$

2. Reduce to fourths:

$$\frac{1}{2} \quad \frac{2}{8} \quad \frac{4}{8} \quad \frac{16}{8} \quad \frac{10}{8} \quad \frac{20}{8} \quad \frac{18}{8} \quad \frac{22}{8}$$

3. Reduce to eighths:

$$\frac{1}{2} \quad \frac{1}{4} \quad \frac{3}{4} \quad \frac{4}{4} \quad \frac{6}{4} \quad \frac{7}{2} \quad \frac{2}{16} \quad \frac{4}{16}$$

4. Reduce to twelfths:

$$\frac{1}{2} \quad \frac{1}{4} \quad \frac{3}{4} \quad \frac{1}{3} \quad \frac{2}{3} \quad \frac{1}{6} \quad \frac{5}{6} \quad \frac{2}{24}$$

5. Reduce to lowest terms:

$$\frac{2}{4} \quad \frac{4}{8} \quad \frac{10}{8} \quad \frac{8}{10} \quad \frac{3}{6} \quad \frac{4}{6} \quad \frac{6}{8} \quad \frac{5}{10}$$

6. Express $\frac{1}{3}$ in. and $\frac{1}{2}$ in. as sixths of an inch, and tell which is the greater.

7. Express $\frac{1}{3}$ in. and $\frac{1}{4}$ in. as twelfths of an inch, and tell which is the greater.

ORAL EXERCISE

1. How many halves do you see in 1? in $1\frac{1}{2}$? in 2? Express $\frac{3}{2}$ as a mixed number. Express $\frac{4}{2}$ as a whole number.



2. How many thirds do you see in 1? in $1\frac{1}{3}$? in $1\frac{2}{3}$? in 2? Express $\frac{4}{3}$ and $\frac{5}{3}$ as mixed numbers; $\frac{6}{3}$ as a whole number.



WRITTEN EXERCISE

1. How many fourths are there in 1? in $1\frac{1}{4}$? in $1\frac{3}{4}$? in 2? Write the results in this way: $1 = \frac{4}{4}$, $1\frac{1}{4} = \frac{5}{4}$, and so on.

2. How many fifths are there in 1? in $1\frac{1}{5}$? in $1\frac{2}{5}$? in $1\frac{3}{5}$? in $1\frac{4}{5}$? in $1\frac{5}{5}$, or 2?

3. How many sixths are there in 1? in $1\frac{1}{6}$? in $1\frac{5}{6}$? in 2? in $2\frac{1}{6}$? in $2\frac{5}{6}$? in $2\frac{6}{6}$, or 3?

4. How many sevenths are there in 1? in $1\frac{1}{7}$? in $1\frac{2}{7}$? in $1\frac{3}{7}$? in $1\frac{4}{7}$? in $1\frac{5}{7}$? in $1\frac{6}{7}$? in 2?

5. How many eighths are there in 1? in 2? in $2\frac{1}{8}$? in $2\frac{3}{8}$? in $2\frac{5}{8}$? in $2\frac{7}{8}$? in 3?

6. How many ninths are there in 1? in 2? in 3? in 4? in $1\frac{1}{9}$? in $2\frac{2}{9}$? in $2\frac{8}{9}$?

7. How many tenths are there in 1? in 2? in 3? in $3\frac{1}{10}$? in $3\frac{3}{10}$? in $3\frac{7}{10}$? in $3\frac{9}{10}$?

8. How many twelfths are there in 1? in 2? in 3? in 4? in $2\frac{1}{12}$? in $3\frac{5}{12}$? in $4\frac{7}{12}$? in $4\frac{11}{12}$?

ORAL EXERCISE

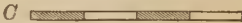
1. If A is 1, B shows that there are how many halves in 1?

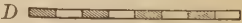
2. C shows that there are how many fourths in 1? in $\frac{1}{2}$?

3. D shows that $\frac{1}{4}$ equals how many eighths? $\frac{1}{2} = \frac{?}{8}$.

4. E shows that $\frac{1}{3} = \frac{?}{6}$, $\frac{2}{3} = \frac{?}{6}$, $\frac{1}{3} = \frac{?}{9}$, A 
 $\frac{2}{3} = \frac{?}{9}$, and $\frac{3}{6} = \frac{?}{2}$. B 

5. Point to the twelfths in E . Tell how many twelfths equal a certain number of ninths, sixths, thirds, and halves.

C 

D 

6. Looking at F , tell as you did in E the relations between the fifths and the tenths; the fifths and the fifteenths.

E 

F 

57. Multiplying or Dividing Terms. We see (pages 39-42) that

Multiplying or dividing both terms of a fraction by the same number does not change the value of the fraction.

For example, $\frac{2}{3} = \frac{2 \times 2}{3 \times 2} = \frac{4}{6}$, and $\frac{8}{10} = \frac{8 \div 2}{10 \div 2} = \frac{4}{5}$.

58. Canceling. Dividing both terms of a fraction by the same factor is called *canceling* the factor.

59. Reduction. Changing the form of a fraction without changing the value is called *reducing* the fraction.

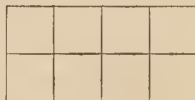
60. Reduction to Lower Terms. A fraction is said to be *reduced to lower terms* when the terms are made smaller without changing the value of the fraction.

61. Reduction to Higher Terms. A fraction is said to be *reduced to higher terms* when the terms are made larger without changing the value of the fraction.

ORAL EXERCISE

1. From the figure, $\frac{1}{2}$ equals how many eighths? how many fourths?

2. From the figure, $\frac{6}{8}$ equals how many fourths?



3. In what cases, in Exs. 1 and 2, did we reduce to lower terms, and in what cases to higher terms?

Reduce as indicated:

- | | | |
|--------------------------------|--------------------------|---------------------------|
| 4. $\frac{1}{2}$ to sixteenths | $\frac{1}{2}$ to sixths | $\frac{4}{5}$ to tenths |
| 5. $\frac{1}{4}$ to twelfths | $\frac{1}{4}$ to eighths | $\frac{3}{4}$ to twelfths |
| 6. $\frac{3}{8}$ to sixteenths | $\frac{2}{3}$ to fourths | $\frac{6}{16}$ to eighths |

WRITTEN EXERCISE

1. Reduce to any lower terms:

$$\frac{4}{8} \quad \frac{6}{16} \quad \frac{12}{16} \quad \frac{8}{10} \quad \frac{8}{12} \quad \frac{14}{32} \quad \frac{16}{32} \quad \frac{24}{32}$$

2. Reduce to sixteenths:

$$\frac{3}{8} \quad \frac{3}{4} \quad \frac{7}{8} \quad \frac{24}{32} \quad \frac{30}{32} \quad \frac{16}{64} \quad \frac{20}{64} \quad \frac{12}{64}$$

3. Reduce to thirty-seconds:

$$\frac{1}{4} \quad \frac{3}{4} \quad \frac{5}{8} \quad \frac{7}{8} \quad \frac{15}{16} \quad \frac{18}{64} \quad \frac{32}{64} \quad \frac{24}{64}$$

4. Reduce to sixty-fourths:

$$\frac{3}{8} \quad \frac{5}{8} \quad \frac{7}{16} \quad \frac{9}{16} \quad \frac{9}{32} \quad \frac{15}{32} \quad \frac{16}{32} \quad \frac{128}{128}$$

5. Reduce to twelfths:

$$\frac{1}{2} \quad \frac{1}{3} \quad \frac{1}{4} \quad \frac{2}{3} \quad \frac{3}{4} \quad \frac{1}{6} \quad \frac{12}{24} \quad \frac{10}{24}$$

6. Reduce to twenty-fourths:

$$\frac{3}{8} \quad \frac{1}{2} \quad \frac{7}{12} \quad \frac{1}{3} \quad \frac{2}{3} \quad \frac{6}{8} \quad \frac{5}{12} \quad \frac{11}{12}$$

ORAL EXERCISE

1. How many half spheres in 1 sphere? in $1\frac{1}{2}$ spheres?
2. How many halves in 2? in $2\frac{1}{2}$? in 3? in 5? in $5\frac{1}{2}$?
3. How many fourths in 1? in $1\frac{1}{4}$? in $1\frac{1}{2}$? in $1\frac{3}{4}$? in $5\frac{3}{4}$?

Express as thirds:

4. 2 4 5 8 $1\frac{1}{3}$ $2\frac{1}{3}$ $3\frac{2}{3}$ $10\frac{1}{3}$

Express as fourths:

5. 6 8 $2\frac{1}{4}$ 5 $3\frac{1}{4}$ $3\frac{3}{4}$ $8\frac{1}{4}$ 12
6. 7 9 $1\frac{3}{4}$ $2\frac{3}{4}$ $4\frac{1}{4}$ $4\frac{3}{4}$ $5\frac{1}{4}$ $5\frac{3}{4}$

Express as eighths:

7. 10 7 $6\frac{1}{8}$ $6\frac{3}{8}$ $7\frac{1}{8}$ $7\frac{5}{8}$ $8\frac{1}{8}$ $8\frac{7}{8}$

Reduce $14\frac{5}{12}$ to twelfths.

Since $1 = \frac{12}{12}$, we see that $14 = \frac{14 \times 12}{12} = \frac{168}{12}$.

Adding, $14\frac{5}{12} = \frac{168}{12} + \frac{5}{12} = \frac{173}{12}$.

WRITTEN EXERCISE

Reduce as indicated:

- | | |
|-------------------------------|---------------------------------|
| 1. 69 to eighths. | 8. $96\frac{2}{9}$ to ninths. |
| 2. 91 to thirds. | 9. 341 to tenths. |
| 3. $98\frac{1}{2}$ to halves. | 10. $73\frac{2}{5}$ to fifths. |
| 4. $17\frac{4}{5}$ to fifths. | 11. $48\frac{2}{3}$ to thirds. |
| 5. $83\frac{2}{3}$ to thirds. | 12. $39\frac{3}{4}$ to fourths. |
| 6. $66\frac{5}{6}$ to sixths. | 13. $27\frac{1}{8}$ to eighths. |
| 7. $77\frac{1}{9}$ to ninths. | 14. $33\frac{3}{8}$ to eighths. |

62. Reduction to an Improper Fraction. Reduce $5\frac{7}{8}$ to an improper fraction.

Since $1 = \frac{8}{8}$, we see that $5 = \frac{5 \times 8}{8} = \frac{40}{8}$.

Then $5\frac{7}{8} = \frac{40}{8} + \frac{7}{8} = \frac{47}{8}$.

ORAL EXERCISE

1. How many half hours in an hour? in $1\frac{1}{2}$ hours?
2. How many quarter hours in an hour? in $1\frac{1}{4}$ hours?
3. How many halves in 1? in 3? in $7\frac{1}{2}$? in $9\frac{1}{2}$? in 12?

Express as fourths:

4. 1 $1\frac{1}{2}$ $2\frac{1}{4}$ 3 $5\frac{3}{4}$ 7 $8\frac{1}{4}$ 10 50

Express as halves:

5. 1 $2\frac{1}{2}$ $3\frac{1}{2}$ 4 $4\frac{1}{2}$ 6 15 $37\frac{1}{2}$ $62\frac{1}{2}$

WRITTEN EXERCISE

Reduce as indicated:

- | | | |
|-------------------|------------------|---------------|
| 1. 7 to sixths | 10 to sevenths | 14 to tenths |
| 2. 8 to elevenths | 11 to sixths | 13 to fourths |
| 3. 9 to halves | 10 to twentieths | 15 to thirds |

Reduce to improper fractions:

- | | | | | |
|---------------------|------------------|------------------|------------------|------------------|
| 4. $5\frac{2}{3}$ | $6\frac{7}{8}$ | $7\frac{2}{3}$ | $8\frac{9}{10}$ | $6\frac{2}{3}$ |
| 5. $3\frac{7}{8}$ | $4\frac{5}{8}$ | $6\frac{4}{5}$ | $9\frac{3}{16}$ | $7\frac{3}{8}$ |
| 6. $8\frac{5}{8}$ | $9\frac{7}{8}$ | $8\frac{4}{5}$ | $7\frac{5}{16}$ | $8\frac{3}{8}$ |
| 7. $16\frac{2}{3}$ | $12\frac{1}{2}$ | $33\frac{1}{3}$ | $66\frac{2}{3}$ | $21\frac{3}{16}$ |
| 8. $333\frac{1}{3}$ | $166\frac{2}{3}$ | $666\frac{2}{3}$ | $125\frac{1}{2}$ | $43\frac{7}{16}$ |

63. Reduction to an Integer or a Mixed Number. Reduce $1\frac{124}{5}$ to an integer or a mixed number.

Since $\frac{1}{5}$ means that 1 has been divided into 5 equal parts, $1\frac{124}{5}$ means that 124×1 has been divided into 5 equal parts.

Therefore $1\frac{124}{5} = 124 \div 5 = 24\frac{4}{5}$.

A fraction is an indicated division.

To reduce an improper fraction to an integer or a mixed number, divide the numerator by the denominator.

ORAL EXERCISE

1. Express $2\frac{1}{2}$ as halves; $\frac{5}{2}$ as a mixed number.

2. In \$5 there are how many half dollars? Express 5 as halves. Express $\frac{1}{2}$ as an integer. Express $\frac{15}{5}$, $\frac{20}{4}$, $\frac{30}{3}$, and $\frac{40}{10}$ as integers.

Express as integers:

3. $\frac{12}{6}$ $\frac{15}{3}$ $\frac{16}{2}$ $\frac{81}{9}$ $\frac{25}{5}$ $\frac{125}{25}$ $\frac{64}{4}$ $\frac{48}{12}$

Express as mixed numbers:

4. $\frac{15}{7}$ $\frac{17}{8}$ $\frac{20}{3}$ $\frac{32}{10}$ $\frac{37}{7}$ $\frac{78}{12}$ $\frac{41}{5}$ $\frac{63}{8}$

WRITTEN EXERCISE

Reduce to integers or mixed numbers:

1. $\frac{33}{3}$	$\frac{125}{24}$	$\frac{632}{32}$	$\frac{127}{4}$	$\frac{81}{8}$	$\frac{107}{8}$
2. $\frac{47}{6}$	$\frac{298}{16}$	$\frac{190}{12}$	$\frac{87}{12}$	$\frac{92}{12}$	$\frac{116}{12}$
3. $\frac{42}{6}$	$\frac{236}{32}$	$\frac{425}{82}$	$\frac{93}{32}$	$\frac{76}{32}$	$\frac{160}{32}$
4. $\frac{78}{5}$	$\frac{349}{32}$	$\frac{276}{16}$	$\frac{198}{16}$	$\frac{725}{32}$	$\frac{525}{32}$
5. $\frac{65}{8}$	$\frac{427}{12}$	$\frac{629}{24}$	$\frac{332}{16}$	$\frac{342}{12}$	$\frac{353}{12}$
6. $\frac{69}{8}$	$\frac{625}{12}$	$\frac{221}{12}$	$\frac{625}{24}$	$\frac{896}{16}$	$\frac{257}{16}$
7. $\frac{29}{3}$	$\frac{175}{32}$	$\frac{113}{8}$	$\frac{125}{8}$	$\frac{441}{8}$	$\frac{549}{8}$

64. Lowest Terms. A fraction is reduced to *lowest terms* when both terms cannot be divided by the same number (§ 56).

For example, $\frac{4}{8} = \frac{2}{4} = \frac{1}{2}$, and $\frac{4}{8}$ is reduced to lowest terms when reduced to $\frac{1}{2}$.

In this case the terms are sometimes said to be *prime to each other*, but this expression is going out of use.

65. Reduction to Lowest Terms. *To reduce a fraction to lowest terms, cancel all common factors from both terms.*

Common factors are the factors found in both numbers. In all practical cases we can tell what factors to try, for they are usually 2, 3, 5, or 7. Cancel as large factors as possible.

Teachers should recognize the fact that, practically, we do not meet with fractions whose terms are large numbers, like $\frac{3}{59}\frac{5}{64}$, decimal fractions having taken their place.

The fractions most commonly used in commercial matters have as denominators 2, 3, 4, 8, 12, and 16; those most commonly used in connection with the machine and other industries have, in addition to these, the denominators 24, 48, 32, 64, and 96.

WRITTEN EXERCISE

Reduce to lowest terms:

- | | | | | | | |
|------------------|----------------|----------------|----------------|----------------|-----------------|-----------------|
| 1. $\frac{4}{8}$ | $\frac{4}{6}$ | $\frac{3}{9}$ | $\frac{3}{6}$ | $\frac{4}{12}$ | $\frac{6}{12}$ | $\frac{9}{12}$ |
| 2. $\frac{2}{6}$ | $\frac{2}{4}$ | $\frac{2}{12}$ | $\frac{3}{12}$ | $\frac{3}{15}$ | $\frac{6}{15}$ | $\frac{9}{15}$ |
| 3. $\frac{2}{8}$ | $\frac{4}{10}$ | $\frac{5}{10}$ | $\frac{6}{10}$ | $\frac{8}{10}$ | $\frac{10}{12}$ | $\frac{12}{12}$ |

Simplify the following fractions met in quotients:

- | | | | | | |
|---------------------|------------------|------------------|-------------------|-------------------|-------------------|
| 4. $\frac{72}{104}$ | $\frac{60}{105}$ | $\frac{72}{144}$ | $\frac{111}{123}$ | $\frac{65}{70}$ | $\frac{385}{700}$ |
| 5. $\frac{12}{144}$ | $\frac{75}{125}$ | $\frac{35}{30}$ | $\frac{130}{195}$ | $\frac{147}{350}$ | $\frac{231}{308}$ |

It must be understood that such fractions as those given in Exs. 4 and 5 are not used in business. We meet them in division, however, and for this reason it is necessary to know how to reduce them to lowest terms, although the decimal fraction is now making even this kind of reduction less important.

66. Similar Fractions. If several fractions have the same denominator, they are said to be *similar fractions* and to have a *common denominator*.

67. Least Common Denominator. If the common denominator of several fractions is the smallest possible, the fractions are said to have the *least common denominator* (L.C.D.).

For example, $\frac{4}{8}$ and $\frac{2}{3}$ have a common denominator, but they may be reduced to $\frac{2}{4}$ and $\frac{1}{4}$, 4 being the least common denominator.

68. Reduction to L.C.D. To reduce $\frac{1}{4}$, $\frac{3}{8}$, and $\frac{5}{12}$ to fractions having the least common denominator.

Since the required denominator must contain 4, 8, and 12, it must contain all their factors.

We see that 4, 8, and 12 all contain 4, that 8 also contains 2, and that 12 contains 3. Therefore the least common denominator is $4 \times 2 \times 3$, or 24.

Since $24 \div 4 = 6$, the terms of $\frac{1}{4}$ must be multiplied by 6. Similarly the other multipliers are found.

$\frac{6 \times 1}{6 \times 4}$	$= \frac{6}{24}$
$\frac{3 \times 3}{3 \times 8}$	$= \frac{9}{24}$
$\frac{2 \times 5}{2 \times 12}$	$= \frac{10}{24}$

ORAL EXERCISE

- Express $\frac{3}{4}$ as eighths. Add $\frac{3}{4}$ and $\frac{1}{8}$. Add $\frac{3}{4}$ and $\frac{7}{8}$.
- In adding $\frac{3}{4}$ in. and $\frac{1}{8}$ in., we first reduce $\frac{3}{4}$ in. to a fraction with what denominator?
- In subtracting $\frac{3}{8}$ in. from $\frac{1}{2}$ in., we first make what reduction?
- Before we add or subtract fractions, what must be the nature of the denominators?

Express as fractions with the same denominator:

5. $\frac{1}{2}, \frac{1}{4}$.

6. $\frac{1}{4}, \frac{1}{8}$.

7. $\frac{1}{2}, \frac{1}{8}$.

8. $\frac{1}{2}, \frac{5}{8}$.

ORAL EXERCISE

Reduce the following to similar fractions :

- | | | | |
|----------------------------------|-----------------------------------|-------------------------------------|-----------------------------------|
| 1. $\frac{2}{3}, \frac{1}{2}$. | 4. $\frac{2}{5}, \frac{7}{10}$. | 7. $\frac{3}{5}, \frac{4}{10}$. | 10. $\frac{6}{8}, \frac{9}{10}$. |
| 2. $\frac{3}{5}, \frac{2}{3}$. | 5. $\frac{5}{15}, \frac{2}{30}$. | 8. $\frac{3}{8}, \frac{4}{4}$. | 11. $\frac{2}{3}, \frac{3}{30}$. |
| 3. $\frac{4}{16}, \frac{2}{8}$. | 6. $\frac{1}{7}, \frac{2}{14}$. | 9. $\frac{10}{20}, \frac{15}{30}$. | 12. $\frac{5}{7}, \frac{6}{14}$. |

Reduce to fractions having the least common denominator :

- | | | | |
|-----------------------------------|------------------------------------|---|--|
| 13. $\frac{1}{2}, \frac{2}{3}$. | 16. $\frac{3}{7}, \frac{2}{3}$. | 19. $\frac{7}{12}, \frac{1}{4}$. | 22. $\frac{2}{9}, \frac{2}{3}, \frac{5}{6}$. |
| 14. $\frac{3}{5}, \frac{4}{7}$. | 17. $\frac{7}{8}, \frac{1}{16}$. | 20. $\frac{2}{7}, \frac{4}{5}$. | 23. $\frac{3}{8}, \frac{3}{5}, \frac{3}{10}$. |
| 15. $\frac{8}{10}, \frac{3}{5}$. | 18. $\frac{1}{36}, \frac{5}{32}$. | 21. $\frac{2}{3}, \frac{3}{4}, \frac{1}{6}$. | 24. $\frac{7}{8}, \frac{1}{5}, \frac{1}{40}$. |

WRITTEN EXERCISE

Reduce to fractions having the least common denominator :

- | | | | |
|----------------------------------|--------------------------------------|--------------------------------------|--|
| 1. $\frac{1}{2}, \frac{5}{6}$. | 11. $\frac{5}{12}, \frac{7}{24}$. | 21. $\frac{5}{8}, \frac{7}{24}$. | 31. $\frac{7}{8}, \frac{9}{64}$. |
| 2. $\frac{1}{4}, \frac{5}{6}$. | 12. $\frac{3}{16}, \frac{5}{24}$. | 22. $\frac{7}{8}, \frac{21}{24}$. | 32. $\frac{5}{16}, \frac{11}{64}$. |
| 3. $\frac{3}{4}, \frac{1}{6}$. | 13. $\frac{7}{8}, \frac{5}{12}$. | 23. $\frac{5}{12}, \frac{3}{32}$. | 33. $\frac{11}{32}, \frac{15}{64}$. |
| 4. $\frac{1}{8}, \frac{5}{6}$. | 14. $\frac{5}{8}, \frac{11}{24}$. | 24. $\frac{7}{12}, \frac{5}{32}$. | 34. $\frac{1}{12}, \frac{3}{64}$. |
| 5. $\frac{5}{8}, \frac{3}{4}$. | 15. $\frac{7}{8}, \frac{13}{24}$. | 25. $\frac{11}{12}, \frac{15}{32}$. | 35. $\frac{5}{12}, \frac{7}{64}$. |
| 6. $\frac{7}{8}, \frac{1}{3}$. | 16. $\frac{7}{8}, \frac{5}{32}$. | 26. $\frac{1}{24}, \frac{1}{32}$. | 36. $\frac{1}{24}, \frac{1}{64}$. |
| 7. $\frac{5}{6}, \frac{2}{3}$. | 17. $\frac{5}{8}, \frac{11}{32}$. | 27. $\frac{5}{24}, \frac{3}{32}$. | 37. $\frac{7}{24}, \frac{25}{64}$. |
| 8. $\frac{7}{8}, \frac{2}{3}$. | 18. $\frac{7}{16}, \frac{5}{32}$. | 28. $\frac{7}{24}, \frac{9}{32}$. | 38. $\frac{3}{8}, \frac{5}{16}, \frac{7}{24}$. |
| 9. $\frac{3}{5}, \frac{2}{3}$. | 19. $\frac{1}{16}, \frac{3}{32}$. | 29. $\frac{7}{24}, \frac{4}{48}$. | 39. $\frac{7}{8}, \frac{1}{12}, \frac{1}{24}$. |
| 10. $\frac{4}{5}, \frac{3}{8}$. | 20. $\frac{13}{16}, \frac{29}{32}$. | 30. $\frac{11}{24}, \frac{13}{48}$. | 40. $\frac{5}{8}, \frac{7}{12}, \frac{15}{32}$. |

Reduce the fractional parts of the following mixed numbers to fractions having the least common denominator :

- | | | |
|--------------------------------------|--------------------------------------|--|
| 41. $4\frac{7}{12}, 6\frac{9}{16}$. | 42. $5\frac{3}{16}, 7\frac{5}{24}$. | 43. $9\frac{11}{24}, 7\frac{15}{32}$. |
|--------------------------------------|--------------------------------------|--|

69. Adding Numbers having the Same Name. If we ask for the sum of 3 boys and 2 girls, the answer cannot be boys alone or girls alone. But we can say,

$$\begin{array}{r} 3 \text{ boys} = 3 \text{ children} \\ 2 \text{ girls} = 2 \text{ children} \\ \hline 5 \text{ children, all together} \end{array}$$

In adding, we think of things as having the same name.

In the same way, if we wish to find the sum of $\frac{3}{4}$ and $\frac{1}{2}$, we must think of these as $\frac{3}{4}$ and $\frac{2}{4}$, the answer being $\frac{5}{4}$, or $1\frac{1}{4}$. *In adding fractions we think of them as having the same name, that is, as similar fractions.*

Numerous illustrations of fractions have been given on pages 36-43. The same idea should be carried throughout, but it acquires more force if, from now on, the objective work is done by the pupil. Addition and subtraction of fractions should be taught exactly as addition and subtraction of integers were taught in the second grade — objectively, as suggested in the oral work on this page. Paper cutting, and diagrams of circles, rectangles, and lines should precede the use of figures. This work is best done by the teacher and pupils working together, the textbook furnishing the examples for practice.

ORAL EXERCISE

7

1. Express $\frac{1}{4}$ as eighths. To the result add $\frac{5}{8}$.
2. If you fasten a strip of molding $\frac{1}{4}$ in. thick to a strip of wood $\frac{5}{8}$ in. thick, how thick are the two together?
3. Express $\frac{4}{5}$ as tenths. To the result add $\frac{3}{10}$.
4. If you lay an arithmetic that is $\frac{4}{5}$ in. thick on a notebook that is $\frac{3}{10}$ in. thick, how thick are the two together?
5. Express $\frac{2}{3}$ as sixths. To the result add $\frac{1}{6}$.
6. If you sew a piece of cloth $\frac{2}{3}$ yd. wide to a strip $\frac{1}{6}$ yd. wide, how wide will the new piece be?

70. Addition of Fractions. As in § 69, we see that if we add 3 marbles and 5 pencils, we have as a sum neither marbles alone nor pencils alone, but just 8 things, of two different kinds. So if we add $\frac{3}{4}$ and $\frac{5}{6}$ we shall have as a sum neither fourths nor sixths, but just 8 parts of a unit, of two different sizes.

Therefore, to add fractions that are not similar and obtain a definite sum, we must first reduce them to similar fractions.

Suppose that we wish to find the sum of $\frac{3}{5}$, $\frac{9}{10}$, and $\frac{7}{12}$.

The L.C.D. is evidently $5 \times 2 \times 2 \times 3 = 60$. Dividing 60 by the three denominators, we have

$$60 \div 5 = 12, \text{ and } \frac{12 \times 3}{12 \times 5} = \frac{36}{60};$$

$$60 \div 10 = 6, \text{ and } \frac{6 \times 9}{6 \times 10} = \frac{54}{60};$$

$$\text{and } 60 \div 12 = 5, \text{ and } \frac{5 \times 7}{5 \times 12} = \frac{35}{60}.$$

$$\text{Furthermore, } \frac{36}{60} + \frac{54}{60} + \frac{35}{60} = \frac{125}{60} = 2\frac{5}{6} = 2\frac{1}{12}.$$

Therefore the sum of $\frac{3}{5}$, $\frac{9}{10}$, and $\frac{7}{12}$ is $2\frac{1}{12}$.

To add fractions, reduce them to fractions having the least common denominator, and add the numerators for the numerator of the sum, writing this numerator over the least common denominator. Then reduce to lowest terms.

ORAL EXERCISE

1. How many twelfths are $\frac{7}{12} + \frac{1}{12}$? How many thirds?

State the sum of each of the following pairs of fractions:

- | | | | | |
|--------------------------------|-----------------------------|-----------------------------|-----------------------------|--------------------------------|
| 2. $\frac{1}{2} + \frac{1}{4}$ | $\frac{1}{2} + \frac{3}{4}$ | $\frac{1}{2} + \frac{1}{8}$ | $\frac{1}{2} + \frac{7}{8}$ | $\frac{1}{12} + \frac{5}{12}$ |
| 3. $\frac{1}{4} + \frac{3}{4}$ | $\frac{3}{8} + \frac{5}{8}$ | $\frac{2}{3} + \frac{5}{6}$ | $\frac{1}{8} + \frac{3}{8}$ | $\frac{1}{12} + \frac{11}{12}$ |
| 4. $\frac{1}{8} + \frac{5}{8}$ | $\frac{1}{5} + \frac{4}{5}$ | $\frac{1}{5} + \frac{2}{5}$ | $\frac{1}{4} + \frac{1}{8}$ | $\frac{5}{12} + \frac{7}{12}$ |

WRITTEN EXERCISE

Add the following fractions :

1. $\frac{1}{2} + \frac{1}{3}$.

6. $\frac{1}{2} + \frac{4}{5}$.

11. $\frac{3}{4} + \frac{7}{8}$.

16. $\frac{1}{2} + \frac{5}{6}$.

2. $\frac{1}{2} + \frac{2}{3}$.

7. $\frac{1}{2} + \frac{3}{4}$.

12. $\frac{4}{5} + \frac{1}{8}$.

17. $\frac{3}{4} + \frac{5}{6}$.

3. $\frac{1}{2} + \frac{1}{5}$.

8. $\frac{1}{2} + \frac{3}{8}$.

13. $\frac{3}{5} + \frac{3}{8}$.

18. $\frac{1}{6} + \frac{1}{8}$.

4. $\frac{1}{2} + \frac{2}{5}$.

9. $\frac{1}{2} + \frac{7}{8}$.

14. $\frac{2}{5} + \frac{7}{8}$.

19. $\frac{1}{6} + \frac{3}{8}$.

5. $\frac{1}{2} + \frac{3}{5}$.

10. $\frac{3}{4} + \frac{5}{8}$.

15. $\frac{4}{5} + \frac{7}{8}$.

20. $\frac{5}{6} + \frac{7}{8}$.

21. A piece of cardboard $\frac{1}{4}$ in. thick is laid on a book $\frac{5}{8}$ in. thick. How thick are the two together ?

Add the following fractions :

22. $\frac{1}{2} + \frac{1}{4} + \frac{1}{8}$.

25. $\frac{1}{3} + \frac{1}{4} + \frac{1}{12}$.

28. $\frac{1}{3} + \frac{1}{8} + \frac{1}{12}$.

23. $\frac{1}{2} + \frac{3}{4} + \frac{1}{8}$.

26. $\frac{1}{3} + \frac{3}{4} + \frac{1}{12}$.

29. $\frac{2}{3} + \frac{1}{8} + \frac{1}{12}$.

24. $\frac{1}{2} + \frac{3}{4} + \frac{5}{8}$.

27. $\frac{2}{3} + \frac{3}{4} + \frac{7}{12}$.

30. $\frac{2}{3} + \frac{5}{8} + \frac{7}{12}$.

31. Add $\frac{1}{2}$ in., $\frac{1}{3}$ in., $\frac{1}{4}$ in., $\frac{1}{6}$ in., and $\frac{1}{12}$ in.

32. If you lay a notebook $\frac{1}{2}$ in. thick on a book $\frac{7}{8}$ in. thick, how thick are the two together ?

33. Some plaster $\frac{3}{8}$ in. thick is coated with a finer plaster $\frac{3}{16}$ in. thick. How thick is the plaster then ?

34. A plate of brass $\frac{1}{32}$ in. thick is laid on a plate of iron $\frac{1}{8}$ in. thick. What is the total thickness ?

35. An iron rod $\frac{7}{8}$ in. in diameter is covered with a brass plating $\frac{1}{32}$ in. thick. What is the diameter of the rod then ?

Add the following fractions :

36. $\frac{1}{4}, \frac{5}{24}$.

40. $\frac{3}{16}, \frac{1}{24}$.

44. $\frac{1}{2}, \frac{1}{48}$.

48. $\frac{5}{8}, \frac{1}{48}$.

52. $\frac{1}{8}, \frac{5}{64}$.

37. $\frac{3}{4}, \frac{11}{24}$.

41. $\frac{5}{16}, \frac{5}{24}$.

45. $\frac{3}{4}, \frac{1}{48}$.

49. $\frac{7}{8}, \frac{23}{48}$.

53. $\frac{5}{12}, \frac{1}{64}$.

38. $\frac{3}{8}, \frac{7}{24}$.

42. $\frac{7}{16}, \frac{1}{24}$.

46. $\frac{3}{4}, \frac{15}{28}$.

50. $\frac{1}{16}, \frac{1}{48}$.

54. $\frac{7}{12}, \frac{9}{64}$.

39. $\frac{5}{8}, \frac{11}{24}$.

43. $\frac{9}{16}, \frac{13}{24}$.

47. $\frac{3}{8}, \frac{13}{48}$.

51. $\frac{7}{16}, \frac{31}{48}$.

55. $\frac{15}{16}, \frac{63}{64}$.

71. Addition of Mixed Numbers. To add $4\frac{2}{3}$ and $2\frac{2}{3}$ we first write the numbers as here shown.

We then add the thirds, thus:

$$\frac{2}{3} + \frac{2}{3} = \frac{4}{3} = 1\frac{1}{3}.$$

We then write $\frac{1}{3}$ below the fractions and add the 1 to the next column.

We then have $1 + 2 + 4 = 7$, and we write the 7 below the line, beneath the 2 and 4. The sum is $7\frac{1}{3}$.

$4\frac{2}{3}$
$2\frac{2}{3}$
$7\frac{1}{3}$

WRITTEN EXERCISE

Add the following:

- | | | | | |
|---|--|---|--|---|
| 1.
$7\frac{2}{3}$
$\frac{1}{3}$
<hr/> $7\frac{3}{3} = 8$ | 2.
$7\frac{2}{3}$
$2\frac{1}{3}$
<hr/> $9\frac{3}{3} = 10$ | 3.
$7\frac{2}{3}$
$2\frac{2}{3}$
<hr/> $9\frac{4}{3} = 10\frac{1}{3}$ | 4.
$9\frac{1}{2}$
$3\frac{1}{2}$
<hr/> $12\frac{2}{2} = 13$ | 5.
$7\frac{1}{2}$
$4\frac{1}{2}$
<hr/> $11\frac{2}{2} = 12$ |
| 6.
$12\frac{1}{2}$
$2\frac{1}{2}$
<hr/> $14\frac{2}{2} = 15$ | 7.
$14\frac{1}{3}$
$2\frac{2}{3}$
<hr/> $16\frac{3}{3} = 17$ | 8.
$22\frac{1}{3}$
$12\frac{1}{3}$
<hr/> $34\frac{2}{3}$ | 9.
$15\frac{2}{3}$
$4\frac{1}{3}$
<hr/> $20\frac{3}{3} = 21$ | 10.
$17\frac{2}{3}$
$4\frac{2}{3}$
<hr/> $22\frac{4}{3} = 23\frac{1}{3}$ |
| 11.
$12\frac{1}{2}$
$14\frac{1}{2}$
$16\frac{1}{2}$
$22\frac{1}{2}$
<hr/> $64\frac{4}{2} = 66$ | 12.
$24\frac{1}{3}$
$16\frac{1}{3}$
$17\frac{2}{3}$
$5\frac{2}{3}$
<hr/> $62\frac{6}{3} = 66$ | 13.
$12\frac{2}{3}$
$3\frac{2}{3}$
$4\frac{2}{3}$
$3\frac{1}{3}$
<hr/> $23\frac{7}{3} = 24\frac{1}{3}$ | 14.
$16\frac{2}{3}$
$14\frac{2}{3}$
$15\frac{2}{3}$
56
<hr/> $51\frac{6}{3} = 57$ | 15.
$28\frac{1}{3}$
$14\frac{2}{3}$
$15\frac{2}{3}$
$32\frac{1}{3}$
<hr/> $69\frac{6}{3} = 73$ |
| 16.
$41\frac{1}{2}$
27
$33\frac{1}{2}$
42
<hr/> $143\frac{2}{2} = 144$ | 17.
$62\frac{1}{2}$
$17\frac{1}{2}$
12
$15\frac{1}{2}$
<hr/> $106\frac{2}{2} = 107$ | 18.
$39\frac{1}{2}$
$12\frac{1}{2}$
$33\frac{1}{2}$
$40\frac{1}{2}$
<hr/> $124\frac{2}{2} = 125$ | 19.
$37\frac{1}{3}$
$29\frac{1}{3}$
16
$4\frac{1}{3}$
<hr/> $86\frac{2}{3}$ | 20.
$57\frac{1}{3}$
$22\frac{2}{3}$
$33\frac{2}{3}$
$4\frac{1}{3}$
<hr/> $97\frac{6}{3} = 102$ |

72. Addition of Mixed Numbers Continued. In adding mixed numbers we may proceed as in the following example:

Required the sum of $3\frac{4}{5}$, $5\frac{7}{8}$, and $16\frac{3}{4}$.

The L.C.D. is $2 \times 2 \times 2 \times 5$, or 40. Reducing the fractions to similar fractions with denominator 40,

we have

$$3\frac{4}{5} = 3\frac{32}{40}$$

$$5\frac{7}{8} = 5\frac{35}{40}$$

and

$$16\frac{3}{4} = 16\frac{30}{40}$$

Therefore

$$\text{the sum} = 24\frac{97}{40} = 24 + 2\frac{17}{40} = 26\frac{17}{40}$$

Therefore, *to add mixed numbers, add separately the integers and the fractions and find the sum of the results.*

WRITTEN EXERCISE

Add the following:

1. $1\frac{1}{2} + \frac{1}{4} + \frac{1}{8}$.

7. $3\frac{1}{2} + 2\frac{3}{4} + 1\frac{5}{8}$.

13. $3\frac{1}{4} + 4\frac{1}{8} + 3\frac{1}{16}$.

2. $2\frac{1}{3} + \frac{1}{6} + \frac{5}{9}$.

8. $1\frac{2}{3} + 2\frac{5}{6} + \frac{8}{9}$.

14. $1\frac{1}{5} + 2\frac{1}{3} + \frac{1}{15}$.

3. $2\frac{3}{4} + 1\frac{7}{8} + 4\frac{1}{16}$.

9. $4\frac{2}{3} + 3\frac{3}{4} + \frac{5}{12}$.

15. $5\frac{7}{8} + 8\frac{2}{5} + 1\frac{7}{10}$.

4. $2\frac{1}{2} + 1\frac{3}{4} + \frac{1}{8}$.

10. $2\frac{1}{2} + 4\frac{3}{4} + 5\frac{7}{8}$.

16. $1\frac{3}{4} + 3\frac{5}{8} + 4\frac{5}{16}$.

5. $3\frac{2}{3} + \frac{5}{6} + \frac{7}{9}$.

11. $3\frac{2}{3} + 4\frac{5}{6} + 1\frac{2}{9}$.

17. $1\frac{2}{5} + 3\frac{2}{3} + \frac{4}{15}$.

6. $3\frac{3}{4} + 2\frac{5}{8} + 3\frac{3}{16}$.

12. $5\frac{2}{5} + 3\frac{1}{4} + \frac{3}{10}$.

18. $2\frac{3}{5} + 3\frac{1}{6} + \frac{7}{15}$.

19. A teacher's desk is $4\frac{1}{3}$ ft. long and $3\frac{1}{12}$ ft. wide. What is the perimeter (distance around)?

20. Some boys built a hut $7\frac{3}{4}$ ft. long and $5\frac{3}{8}$ ft. wide. What was the perimeter?

21. The three sides of a triangle are $3\frac{2}{3}$ in., $4\frac{1}{2}$ in., and $4\frac{5}{16}$ in. What is the perimeter?

22. If a boy weighs $56\frac{7}{8}$ lb., his books $1\frac{3}{4}$ lb., and his dog $20\frac{1}{2}$ lb., how much do all together weigh?

73. Subtracting Fractions. From our work in adding fractions we learn how to subtract.

To subtract fractions, reduce them to fractions having the least common denominator, and subtract the numerators for the numerator of the difference. Then reduce to lowest terms.

For example, we subtract $\frac{3}{10}$ from $\frac{1}{2}$ as follows:

$$\begin{array}{r} \frac{1}{2} = \frac{5}{10} \\ \frac{3}{10} = \frac{3}{10} \\ \hline \frac{2}{10} = \frac{1}{5} \end{array}$$

Subtracting,

In case the pupil has any difficulty, the teacher should resort to objective illustration, using paper folding or some other simple device.

ORAL EXERCISE

Subtract the following :

- | | | | |
|----------------------------------|----------------------------------|-----------------------------------|------------------------------------|
| 1. $\frac{1}{2} - \frac{1}{4}$. | 5. $\frac{5}{6} - \frac{1}{3}$. | 9. $\frac{2}{5} - \frac{1}{10}$. | 13. $\frac{1}{6} - \frac{1}{12}$. |
| 2. $\frac{3}{4} - \frac{1}{2}$. | 6. $\frac{5}{6} - \frac{1}{2}$. | 10. $\frac{1}{3} - \frac{1}{6}$. | 14. $\frac{5}{6} - \frac{1}{12}$. |
| 3. $\frac{3}{4} - \frac{1}{8}$. | 7. $\frac{4}{9} - \frac{2}{9}$. | 11. $\frac{1}{4} - \frac{1}{8}$. | 15. $\frac{5}{6} - \frac{5}{12}$. |
| 4. $\frac{7}{8} - \frac{1}{8}$. | 8. $\frac{7}{9} - \frac{4}{9}$. | 12. $\frac{8}{9} - \frac{2}{9}$. | 16. $\frac{5}{6} - \frac{2}{3}$. |

WRITTEN EXERCISE

Subtract the following :

- | | | | |
|-----------------------------------|------------------------------------|------------------------------------|-------------------------------------|
| 1. $\frac{1}{2} - \frac{5}{12}$. | 7. $\frac{7}{8} - \frac{5}{24}$. | 13. $\frac{6}{7} - \frac{2}{3}$. | 19. $\frac{8}{21} - \frac{3}{14}$. |
| 2. $\frac{7}{8} - \frac{1}{16}$. | 8. $\frac{3}{4} - \frac{7}{12}$. | 14. $\frac{8}{9} - \frac{1}{6}$. | 20. $\frac{7}{9} - \frac{2}{3}$. |
| 3. $\frac{3}{8} - \frac{1}{4}$. | 9. $\frac{5}{12} - \frac{1}{4}$. | 15. $\frac{7}{11} - \frac{1}{2}$. | 21. $\frac{2}{3} - \frac{1}{2}$. |
| 4. $\frac{2}{3} - \frac{5}{12}$. | 10. $\frac{1}{4} - \frac{1}{12}$. | 16. $\frac{5}{12} - \frac{3}{8}$. | 22. $\frac{6}{7} - \frac{1}{2}$. |
| 5. $\frac{3}{4} - \frac{3}{16}$. | 11. $\frac{7}{16} - \frac{1}{8}$. | 17. $\frac{9}{10} - \frac{3}{5}$. | 23. $\frac{5}{8} - \frac{3}{32}$. |
| 6. $\frac{5}{8} - \frac{1}{2}$. | 12. $\frac{8}{9} - \frac{2}{3}$. | 18. $\frac{5}{9} - \frac{1}{12}$. | 24. $\frac{7}{11} - \frac{1}{8}$. |

25. If you live $\frac{7}{8}$ mi. from school, how far have you still to go when you have gone $\frac{3}{4}$ mi.?

74. Subtracting a Mixed Number. To subtract $3\frac{2}{3}$ from 12 we first write the numbers as here shown.

We see that we cannot take $\frac{2}{3}$ from nothing; that is, there is no number which, added to $\frac{2}{3}$, makes 0.

We therefore think of 12 as $11\frac{3}{3}$, which we can do because $\frac{3}{3} = 1$. We then subtract in the usual way, $\frac{3}{3} - \frac{2}{3} = \frac{1}{3}$, and $11 - 3 = 8$.

Therefore the difference is $8\frac{1}{3}$.

We check the work by seeing that $8\frac{1}{3} + 3\frac{2}{3} = 12$.

12
$3\frac{2}{3}$
$8\frac{1}{3}$

WRITTEN EXERCISE

Subtract the following:

1.

$$\begin{array}{r} 434\frac{1}{2} \\ 203 \\ \hline \end{array}$$

2.

$$\begin{array}{r} 573\frac{1}{2} \\ 392\frac{1}{2} \\ \hline \end{array}$$

3.

$$\begin{array}{r} 603\frac{1}{2} \\ 296 \\ \hline \end{array}$$

4.

$$\begin{array}{r} 729\frac{1}{2} \\ 346\frac{1}{2} \\ \hline \end{array}$$

5.

$$\begin{array}{r} 653\frac{2}{3} \\ 245\frac{1}{3} \\ \hline \end{array}$$

6.

$$\begin{array}{r} 478\frac{1}{3} \\ 199 \\ \hline \end{array}$$

7.

$$\begin{array}{r} 282\frac{2}{3} \\ 165 \\ \hline \end{array}$$

8.

$$\begin{array}{r} 302\frac{2}{3} \\ 169\frac{1}{3} \\ \hline \end{array}$$

9.

$$\begin{array}{r} 308 \\ 184\frac{1}{2} \\ \hline \end{array}$$

10.

$$\begin{array}{r} 482 \\ 362\frac{1}{2} \\ \hline \end{array}$$

11.

$$\begin{array}{r} 445 \\ 381\frac{1}{2} \\ \hline \end{array}$$

12.

$$\begin{array}{r} 998 \\ 285\frac{1}{2} \\ \hline \end{array}$$

13.

$$\begin{array}{r} 389 \\ 227\frac{1}{3} \\ \hline \end{array}$$

14.

$$\begin{array}{r} 495 \\ 345\frac{1}{3} \\ \hline \end{array}$$

15.

$$\begin{array}{r} 975 \\ 226\frac{1}{3} \\ \hline \end{array}$$

16.

$$\begin{array}{r} 487 \\ 364\frac{1}{3} \\ \hline \end{array}$$

17.

$$\begin{array}{r} 668\frac{2}{3} \\ 549\frac{1}{3} \\ \hline \end{array}$$

18.

$$\begin{array}{r} 826\frac{1}{3} \\ 398 \\ \hline \end{array}$$

19.

$$\begin{array}{r} 472\frac{1}{3} \\ 298\frac{1}{3} \\ \hline \end{array}$$

20.

$$\begin{array}{r} 668\frac{1}{2} \\ 345\frac{1}{2} \\ \hline \end{array}$$

75. Subtracting from a Mixed Number. From a mixed number we may have to subtract a whole number, a fraction, or a mixed number.

1. From $17\frac{1}{3}$ to subtract 8.

We have $\frac{1}{3} - 0 = \frac{1}{3}$, and $17 - 8 = 9$. Therefore the result is $9\frac{1}{3}$.

$$\begin{array}{r} 17\frac{1}{3} \\ 8 \\ \hline 9\frac{1}{3} \end{array}$$

2. From $17\frac{1}{3}$ to subtract $\frac{5}{6}$.

We may think of $17\frac{2}{6}$ as $16\frac{8}{6}$, and $16\frac{8}{6} - \frac{5}{6} = 16\frac{3}{6} = 16\frac{1}{2}$.

$$\begin{array}{r} 17\frac{1}{3} = 17\frac{2}{6} \\ \phantom{17\frac{2}{6}} \frac{5}{6} \\ \hline 16\frac{1}{2} \end{array}$$

3. From $17\frac{1}{3}$ to subtract $8\frac{5}{6}$.

$$17\frac{1}{3} - 8\frac{5}{6} = 17\frac{2}{6} - 8\frac{5}{6}.$$

We may now think of $17\frac{2}{6}$ as $16\frac{8}{6}$, and $16\frac{8}{6} - 8\frac{5}{6} = 8\frac{3}{6} = 8\frac{1}{2}$.

WRITTEN EXERCISE

Subtract the following :

1. $91\frac{7}{8} - 19.$

6. $8\frac{1}{4} - 5\frac{7}{8}.$

11. $8\frac{5}{12} - \frac{7}{8}.$

2. $3\frac{1}{8} - \frac{3}{4}.$

7. $120\frac{3}{4} - 69.$

12. $16\frac{1}{3} - 7\frac{3}{4}.$

3. $9\frac{1}{2} - 6\frac{2}{3}.$

8. $7\frac{1}{12} - \frac{3}{8}.$

13. $200\frac{7}{12} - 96.$

4. $83\frac{5}{8} - 27.$

9. $15\frac{2}{5} - 9\frac{3}{8}.$

14. $6\frac{2}{3} - \frac{4}{5}.$

5. $5\frac{3}{8} - \frac{1}{2}.$

10. $103\frac{5}{6} - 78.$

15. $25\frac{1}{5} - 16\frac{7}{8}.$

16. If one book is $7\frac{1}{4}$ in. long and another is $6\frac{1}{16}$ in. long, what is the difference in their lengths?

17. If we have a piece of board $17\frac{1}{8}$ in. long and we need a piece only $15\frac{3}{4}$ in. long, how much shall we cut off?

18. If a lady has $14\frac{1}{4}$ yd. of cloth and uses $8\frac{3}{4}$ yd., how much cloth has she left?

19. How much must you add to $26\frac{7}{8}$ in. to make a yard?

20. How much must you take from 1 yd. to leave $16\frac{3}{8}$ in.?

WRITTEN EXERCISE

1. Frank is $4\frac{1}{8}$ ft. tall, and Rob lacks $\frac{1}{4}$ in. of being as tall as Frank. How tall is Rob?

2. John is $11\frac{2}{3}$ yr. old, Mary is $\frac{1}{2}$ yr. older, and Tom is $1\frac{1}{4}$ yr. younger than Mary. How old is Tom?

3. If my finger nail is $\frac{7}{16}$ in. long, and the rest of my finger is $3\frac{3}{8}$ in. long, what is the total length of my finger?

4. If you weigh $56\frac{3}{4}$ lb., and your dog weighs $18\frac{2}{3}$ lb., and your books weigh $3\frac{1}{2}$ lb., what is the total weight?

5. Harry had a kite string $198\frac{2}{3}$ yd. long. His kite was up in a shower and the string shrank $1\frac{7}{8}$ yd. How long was it then?

6. A plate of glass $23\frac{1}{4}$ in. by $37\frac{1}{8}$ in. was set in a picture frame that covered it $\frac{3}{8}$ in. from each edge. What are the dimensions of the glass not covered by the frame?

7. In making a dress ruffle $3\frac{7}{8}$ in. wide, enough cloth must be allowed to turn in $\frac{7}{8}$ in. on one side and $1\frac{1}{4}$ in. on the other. What is the total width of cloth needed?

8. In running a gas pipe into a schoolroom the gas fitter found he needed three pieces of pipe of these lengths: 6 ft. $2\frac{1}{2}$ in., 5 ft. $3\frac{7}{8}$ in., and 10 ft. 2 in. He also needed a fourth piece half as long as the third one. What was the total length required?

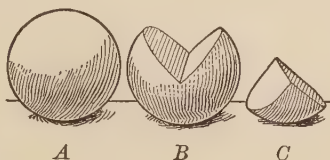
9. A tenpenny nail is $2\frac{3}{4}$ in. long, and a fourpenny nail is $1\frac{3}{8}$ in. shorter; how long is a fourpenny nail? A sixpenny nail is $\frac{5}{8}$ in. longer than a fourpenny nail; how long is a sixpenny nail?

10. Five cans of sirup were measured very carefully and the first was found to contain $2\frac{1}{4}$ qt., the second $2\frac{3}{8}$ qt., the third $2\frac{7}{16}$ qt., the fourth $2\frac{5}{32}$ qt., and the fifth $2\frac{2}{3}$ qt. How much sirup was there in all?

From time to time teachers should read the note on page 22 and remind their pupils of its importance.

ORAL EXERCISE

- How much is $\frac{1}{2}$ of \$4? $\frac{1}{2}$ of 4 fifths? $\frac{1}{2}$ of $\frac{4}{7}$?
- How much is $\frac{1}{3}$ of 6 bu.? $\frac{1}{3}$ of 6 sevenths? $\frac{1}{3}$ of $\frac{6}{11}$?
- How much is $\frac{1}{4}$ of $\frac{16}{7}$? $\frac{1}{4}$ of $\frac{8}{15}$? $\frac{1}{4}$ of $\frac{24}{5}$? $\frac{1}{3}$ of $\frac{6}{5}$?
- In each of these cases in which you found a fraction of a fraction, how did you proceed?



5. In the above figure, *A* represents a whole sphere. What represents $\frac{3}{4}$ of a sphere? What represents $\frac{1}{3}$ of $\frac{3}{4}$ of a sphere?

State the values of:

- | | | | |
|-------------------------------------|----------------------------------|----------------------------------|----------------------------------|
| 6. $\frac{1}{2}$ of $\frac{2}{3}$ | $\frac{1}{5}$ of $\frac{15}{16}$ | $\frac{1}{6}$ of $\frac{12}{13}$ | $\frac{1}{2}$ of $\frac{8}{9}$ |
| 7. $\frac{1}{8}$ of $\frac{16}{21}$ | $\frac{1}{9}$ of $\frac{18}{5}$ | $\frac{1}{10}$ of $\frac{20}{1}$ | $\frac{1}{12}$ of $\frac{24}{5}$ |

8. Draw on the blackboard a rectangle $2\frac{1}{2}$ ft. long and 1 ft. high; mark it off into fifths; mark off each fifth into halves. Then how much is $\frac{1}{2}$ of $\frac{1}{5}$? How much is $\frac{1}{5}$ of $\frac{1}{2}$?

9. Draw another rectangle as in Ex. 8, marking off each fifth into thirds. How much is $\frac{1}{3}$ of $\frac{1}{5}$?

State the values of:

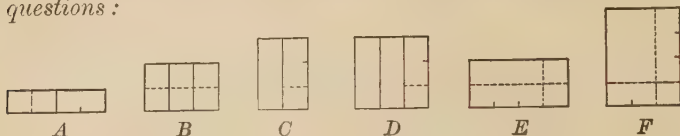
- | | | | |
|------------------------------------|---------------------------------|---------------------------------|--------------------------------|
| 10. $\frac{1}{3}$ of $\frac{1}{5}$ | $\frac{1}{5}$ of $\frac{1}{3}$ | $\frac{1}{5}$ of $\frac{1}{4}$ | $\frac{1}{4}$ of $\frac{1}{5}$ |
| 11. $\frac{1}{2}$ of $\frac{1}{5}$ | $\frac{1}{2}$ of $\frac{1}{10}$ | $\frac{1}{3}$ of $\frac{1}{10}$ | $\frac{1}{2}$ of $\frac{1}{3}$ |

12. In Exs. 10 and 11 what did you do to the two denominators?

Pages 59 and 60 contain development work for the multiplication of fractions begun on page 61.

ORAL EXERCISE

Draw these figures on the blackboard, and then answer the following questions :



1. What part of A is $\frac{1}{2}$ of $\frac{1}{2}$ of A ?
2. What part of B is $\frac{1}{2}$ of $\frac{1}{3}$ of B ?
3. Show that $\frac{1}{2}$ of $\frac{1}{3}$ of B is the same as $\frac{1}{3}$ of $\frac{1}{2}$ of B .
4. What part of C is $\frac{1}{3}$ of $\frac{1}{2}$ of C ?
5. What part of D is $\frac{1}{3}$ of $\frac{1}{3}$ of D ?
6. Point to $\frac{1}{2}$ of E ; to $\frac{1}{4}$ of $\frac{1}{2}$ of E . Also point to $\frac{1}{2}$ of $\frac{1}{4}$ of E . What part of E is $\frac{1}{2}$ of $\frac{1}{4}$ of E , or $\frac{1}{4}$ of $\frac{1}{2}$ of E ?
7. Point to $\frac{1}{2}$ of $\frac{3}{4}$ of E ; to $\frac{3}{4}$ of $\frac{1}{2}$ of E . What part of E is $\frac{3}{4}$ of $\frac{1}{2}$ of E ?
8. Point to $\frac{1}{3}$ of F ; to $\frac{1}{4}$ of $\frac{1}{3}$ of F . Also point to $\frac{1}{4}$ of F , and to $\frac{1}{3}$ of $\frac{1}{4}$ of F . Each result is what part of F ?

WRITTEN EXERCISE

Draw rectangles to show :

- | | |
|--|---|
| 1. $\frac{1}{5}$ of $\frac{1}{3}$, and $\frac{1}{3}$ of $\frac{1}{5}$. | 7. $\frac{1}{2}$ of $\frac{1}{6}$, and $\frac{1}{6}$ of $\frac{1}{2}$. |
| 2. $\frac{2}{3}$ of $\frac{1}{5}$, and $\frac{1}{5}$ of $\frac{2}{3}$. | 8. $\frac{1}{2}$ of $\frac{5}{6}$, and $\frac{5}{6}$ of $\frac{1}{2}$. |
| 3. $\frac{3}{5}$ of $\frac{2}{3}$, and $\frac{2}{3}$ of $\frac{3}{5}$. | 9. $\frac{4}{5}$ of $\frac{1}{6}$, and $\frac{1}{6}$ of $\frac{4}{5}$. |
| 4. $\frac{1}{8}$ of $\frac{1}{2}$, and $\frac{1}{2}$ of $\frac{1}{8}$. | 10. $\frac{3}{8}$ of $\frac{1}{2}$, and $\frac{1}{2}$ of $\frac{3}{8}$. |
| 5. $\frac{5}{8}$ of $\frac{1}{5}$, and $\frac{1}{5}$ of $\frac{5}{8}$. | 11. $\frac{2}{5}$ of $\frac{1}{2}$, and $\frac{1}{2}$ of $\frac{2}{5}$. |
| 6. $\frac{1}{10}$ of $\frac{1}{2}$, and $\frac{1}{2}$ of $\frac{1}{10}$. | 12. $\frac{3}{10}$ of $\frac{1}{2}$, and $\frac{1}{2}$ of $\frac{3}{10}$. |

ORAL EXERCISE

1. How much is 2 times 3 in. ? 2 times $\frac{3}{7}$? 5 times $\frac{2}{11}$? How did you multiply these fractions by an integer ?

2. How much is 4 times $\frac{1}{4}$? 4 times $\frac{3}{4}$? 2 times $\frac{3}{4}$? 5 times $\frac{7}{10}$? 7 times $\frac{4}{9}$? How did you multiply these fractions by an integer ?

76. Multiplication of a Fraction by a Whole Number. From the answers to the above questions we see that the following is true :

To multiply a fraction by a whole number, multiply the numerator by the multiplier.

Before actually multiplying, indicate the multiplication and cancel common factors if possible.

For example, to multiply $1\frac{1}{3}$ by 18. Since we have 11 sixty-thirds, if we have 18 times as much we shall have

$$\frac{18 \times 11}{\cancel{63}^2 \cancel{7}^1}, \text{ or } \frac{22}{7}, \text{ or } 3\frac{1}{7}.$$

Multiply the following, using cancellation whenever possible :

- | | | | |
|-------------------------|--------------------------|--------------------------|---------------------------|
| 3. $\frac{5}{9}$ by 9. | 7. $\frac{5}{8}$ by 80. | 11. $\frac{4}{5}$ by 5. | 15. $\frac{9}{10}$ by 5. |
| 4. $\frac{3}{4}$ by 2. | 8. $\frac{3}{8}$ by 8. | 12. $\frac{7}{8}$ by 4. | 16. $\frac{3}{10}$ by 5. |
| 5. $\frac{3}{8}$ by 16. | 9. $\frac{4}{5}$ by 10. | 13. $\frac{5}{8}$ by 24. | 17. $\frac{3}{16}$ by 48. |
| 6. $\frac{5}{8}$ by 40. | 10. $\frac{7}{8}$ by 24. | 14. $\frac{5}{8}$ by 48. | 18. $\frac{7}{12}$ by 24. |

WRITTEN EXERCISE

Multiply the following, using cancellation whenever possible :

- | | | | |
|--------------------------|---------------------------|---------------------------|-----------------------------|
| 1. $\frac{4}{25}$ by 35. | 4. $\frac{7}{16}$ by 24. | 7. $\frac{11}{64}$ by 36. | 10. $\frac{5}{48}$ by 64. |
| 2. $\frac{4}{8}$ by 84. | 5. $\frac{7}{32}$ by 28. | 8. $\frac{7}{64}$ by 96. | 11. $\frac{9}{32}$ by 48. |
| 3. $\frac{7}{72}$ by 16. | 6. $\frac{5}{12}$ by 132. | 9. $\frac{7}{36}$ by 27. | 12. $\frac{11}{64}$ by 192. |

ORAL EXERCISE

1. How much is $\frac{1}{2}$ of 6 ? $\frac{1}{2}$ of 60 ? $\frac{1}{2}$ of 600 ?
2. How much is $\frac{1}{3}$ of 6 ? $\frac{2}{3}$ of 6 ? $\frac{1}{3}$ of 60 ? $\frac{2}{3}$ of 60 ?
3. How much is $\frac{1}{4}$ of 8 ? $\frac{3}{4}$ of 8 ? $\frac{1}{4}$ of 80 ? $\frac{3}{4}$ of 80 ?
4. How much is $\frac{1}{5}$ of 25 ? $\frac{2}{5}$ of 25 ? $\frac{3}{5}$ of 25 ? $\frac{4}{5}$ of 25 ?

Find the following parts of numbers :

- | | | | |
|------------------------|---------------------|---------------------|---------------------|
| 5. $\frac{1}{8}$ of 24 | $\frac{3}{8}$ of 24 | $\frac{5}{8}$ of 24 | $\frac{7}{8}$ of 24 |
| 6. $\frac{1}{6}$ of 30 | $\frac{5}{6}$ of 30 | $\frac{1}{6}$ of 48 | $\frac{5}{6}$ of 48 |
| 7. $\frac{1}{9}$ of 72 | $\frac{2}{9}$ of 72 | $\frac{4}{9}$ of 72 | $\frac{5}{9}$ of 72 |
8. How do you find a fractional part of a number ?

WRITTEN EXERCISE

1. Find $\frac{1}{8}$ of 736. Then find $\frac{3}{8}$ of 736.
2. Find $\frac{1}{12}$ of 1728. Then find $\frac{5}{12}$ of 1728.

Find the following parts of numbers :

- | | | |
|--------------------------|-----------------------|------------------------|
| 3. $\frac{3}{8}$ of 1768 | $\frac{5}{8}$ of 1768 | $\frac{5}{8}$ of 2576 |
| 4. $\frac{7}{8}$ of 2576 | $\frac{1}{6}$ of 1740 | $\frac{5}{6}$ of 1752 |
| 5. $\frac{5}{6}$ of 1764 | $\frac{5}{6}$ of 1776 | $\frac{1}{16}$ of 3376 |
| 6. $\frac{3}{8}$ of 3376 | $\frac{5}{8}$ of 3392 | $\frac{7}{16}$ of 3408 |
| 7. $\frac{5}{8}$ of 6752 | $\frac{5}{8}$ of 6784 | $\frac{5}{32}$ of 6816 |

8. A man has to travel 117 mi. After traveling $\frac{5}{9}$ of the distance, how many miles has he still to travel ?

9. A man owns 176 head of cattle. If he sells $\frac{3}{8}$ of them, how many does he sell ? how many has he left ?

10. A man owns 336 acres of land. If he sells $\frac{3}{8}$ of it, how many acres does he sell ? how many acres has he left ?

77. Multiplication by a Fraction. To take a fractional part of a number is called *multiplying by a fraction*.

We write " $\frac{1}{2}$ of 4" or " $\frac{1}{2} \times 4$," the $\times$ being read "of."

78. Multiplication of a Fraction by a Fraction. Suppose that we have to multiply $\frac{4}{5}$ by $\frac{2}{3}$.



In this figure let the line AB be divided into five equal parts, and let each of the five parts be divided into three equal parts. Then each part evidently is $\frac{1}{15}$ of AB .

That is, $\frac{1}{3}$ of $\frac{1}{5} = \frac{1}{15}$ of the whole line.

Then $\frac{1}{3}$ of $\frac{4}{5}$ must be 4 times as much, or $\frac{4}{15}$,

and $\frac{2}{3}$ of $\frac{4}{5}$ must be twice $\frac{4}{15}$, or $\frac{8}{15}$.

Therefore, *to multiply a fraction by a fraction, find the product of the numerators for the required numerator and the product of the denominators for the required denominator.*

Always cancel if possible. For example,

$$\frac{11}{12} \text{ of } \frac{15}{22} = \frac{11 \times 15}{12 \times 22} = \frac{5}{8}.$$

$\begin{matrix} 5 \\ 4 \end{matrix}$
 $\begin{matrix} 2 \\ 2 \end{matrix}$

WRITTEN EXERCISE

Multiply the following:

- | | | | |
|---|---|---|---|
| 1. $\frac{3}{4} \times \frac{8}{9}$. | 6. $\frac{3}{4} \times \frac{5}{24}$. | 11. $\frac{3}{16} \times \frac{8}{9}$. | 16. $\frac{1}{2} \times \frac{4}{5}$. |
| 2. $\frac{2}{3} \times \frac{9}{10}$. | 7. $\frac{1}{8} \times \frac{4}{5}$. | 12. $\frac{7}{16} \times \frac{8}{21}$. | 17. $\frac{1}{5} \times \frac{5}{8}$. |
| 3. $\frac{2}{3} \times \frac{15}{16}$. | 8. $\frac{3}{8} \times \frac{7}{8}$. | 13. $\frac{1}{32} \times \frac{4}{5}$. | 18. $\frac{2}{5} \times \frac{5}{8}$. |
| 4. $\frac{2}{3} \times \frac{11}{24}$. | 9. $\frac{7}{8} \times \frac{4}{7}$. | 14. $\frac{1}{48} \times \frac{24}{25}$. | 19. $\frac{4}{5} \times \frac{5}{8}$. |
| 5. $\frac{2}{3} \times \frac{25}{32}$. | 10. $\frac{5}{8} \times \frac{3}{10}$. | 15. $\frac{7}{48} \times \frac{16}{21}$. | 20. $\frac{7}{32} \times \frac{16}{21}$. |

Find the products of:

$$\checkmark 21. \frac{3}{5} \times \frac{5}{48}.$$

$$\checkmark 22. \frac{4}{5} \times \frac{25}{48}.$$

$$23. \frac{5}{9} \times \frac{81}{100}.$$

$$24. \frac{5}{8} \times \frac{24}{25}.$$

$$\checkmark 25. \frac{3}{8} \times \frac{16}{21}.$$

$$26. \frac{7}{8} \times \frac{32}{5}.$$

$$27. \frac{9}{16} \times \frac{4}{9}.$$

$$28. \frac{7}{16} \times \frac{5}{7}.$$

$$29. \frac{5}{16} \times \frac{345}{45}.$$

$$30. \frac{13}{16} \times \frac{2}{3}.$$

$$31. \frac{15}{16} \times \frac{2}{5}.$$

$$32. \frac{7}{12} \times \frac{4}{9}.$$

$$33. \frac{5}{12} \times \frac{3}{5}.$$

$$34. \frac{11}{12} \times \frac{4}{5}.$$

$$35. \frac{11}{16} \times \frac{4}{11}.$$

$$36. \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5}.$$

$$37. \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4}.$$

$$38. \frac{2}{3} \times \frac{2}{3} \times \frac{3}{4}.$$

$$39. \frac{3}{5} \times \frac{3}{4} \times \frac{1}{8}.$$

$$40. \frac{3}{4} \times \frac{3}{8} \times \frac{4}{5}.$$

$$41. \frac{5}{8} \times \frac{7}{8} \times \frac{3}{8}.$$

A few examples with denominators larger than 10 are given only for drill.

42. If an express train runs 1 mi. in $\frac{3}{4}$ min., how long will it take the train to run $\frac{7}{8}$ mi. at the same rate?

43. If a train runs $\frac{1}{3}$ of a certain distance in 1 hr., what part of the distance can it run in $\frac{3}{4}$ hr.?

44. If the distance to a certain house is $\frac{3}{8}$ mi., what fraction of a mile has a boy gone when he has covered $\frac{5}{16}$ of the distance?

45. If it takes a train $\frac{3}{4}$ hr. to reach a certain station, what fraction of an hour will it take to cover $\frac{3}{5}$ of the distance?

46. If you live $\frac{4}{5}$ mi. from school, and Jack lives $\frac{1}{8}$ as far, how far does he live from school?

47. If $\frac{3}{4}$ yd. of lace is needed to trim two neckbands, how much lace will be needed to trim one neckband?

48. If an automobile travels a mile in $\frac{59}{60}$ min. in a certain race, how long would it take to travel $\frac{3}{4}$ mi. at the same rate?

49. If Kate lives $\frac{7}{8}$ mi. from school, and Mary lives $\frac{1}{3}$ as far, how far from school does Mary live? If Sue lives $\frac{1}{2}$ as far as Mary, how far from school does Sue live?

50. A milliner used $\frac{7}{8}$ yd. of velvet for the upper brim and the crown of a hat, and $\frac{2}{5}$ as much for the lower brim. How much velvet did she use for the hat?

ORAL EXERCISE

1. How much is 4 times 5? 4 times $\frac{2}{9}$? 4 times $5\frac{2}{9}$?
2. How much is 2×8 ? $\frac{3}{4}$ of 8? $2\frac{3}{4}$ times 8?
3. How much is $\frac{1.0}{3}$ of $\frac{6}{5}$? Then how much is $3\frac{1}{3} \times 1\frac{1}{5}$?
4. How much is $\frac{1.5}{4}$ of $\frac{4}{3}$? Then how much is $3\frac{3}{4} \times 1\frac{1}{3}$?

79. Multiplication of Mixed Numbers. From these answers we see the truth of the following statements:

To multiply a mixed number by an integer, multiply the whole number and the fractional part separately and add the products.

For example, $6 \times 3\frac{2}{5} = 18\frac{12}{5} = 20\frac{2}{5}$.

To multiply an integer by a mixed number, multiply by the whole number and the fractional part separately and add the products.

For example, $3\frac{2}{5} \times 6 = 3 \times 6 + \frac{2}{5} \text{ of } 6 = 18\frac{12}{5} = 20\frac{2}{5}$.

To multiply a mixed number by a mixed number, reduce each to an improper fraction and multiply the results.

For example, $5\frac{2}{5} \times 3\frac{3}{4} = \frac{27}{5} \text{ of } \frac{15}{4} = \frac{81}{4} = 20\frac{1}{4}$.

WRITTEN EXERCISE

Multiply the following:

- | | | |
|---------------------------------|--|---|
| 1. $17 \times 21\frac{9}{4}$. | 8. $29\frac{4}{9} \times 36$. | 15. $9\frac{1}{3} \times 15\frac{7}{8}$. |
| 2. $27 \times 41\frac{5}{9}$. | 9. $49\frac{7}{8} \times 68$. | 16. $16\frac{3}{4} \times 14\frac{2}{3}$. |
| 3. $49 \times 81\frac{6}{7}$. | 10. $25\frac{2}{3} \times 18$. | 17. $15\frac{5}{9} \times 14\frac{3}{7}$. |
| 4. $81 \times 62\frac{5}{9}$. | 11. $47\frac{1}{10} \times 65$. | 18. $14\frac{3}{4} \times 16\frac{4}{5}$. |
| 5. $48 \times 37\frac{3}{10}$. | 12. $26\frac{1}{2} \times 63$. | 19. $18\frac{4}{7} \times 23\frac{7}{8}$. |
| 6. $26\frac{3}{4} \times 32$. | 13. $35\frac{7}{8} \times 36$. | 20. $18\frac{1}{3} \times 27\frac{3}{5}$. |
| 7. $37\frac{4}{5} \times 45$. | 14. $6\frac{7}{8} \times 7\frac{7}{8}$. | 21. $15\frac{3}{4} \times 28\frac{4}{9}$. |

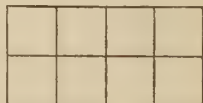
ORAL EXERCISE

State the answers to the following :

- | | | | |
|--------------------|------------------------|------------------------|------------------------|
| 1. 15 ft. $\div$ 5 | $\frac{15}{16} \div 5$ | $\frac{15}{82} \div 5$ | $\frac{15}{64} \div 5$ |
| 2. 25 pt. $\div$ 5 | $\frac{25}{4} \div 5$ | $\frac{25}{8} \div 5$ | $\frac{25}{2} \div 5$ |
| 3. 14 in. $\div$ 7 | $\frac{14}{5} \div 7$ | $\frac{14}{9} \div 7$ | $\frac{14}{3} \div 7$ |

4. Dividing the numerator of a fraction does what to the fraction itself? Give an example.

5. If you divide $\frac{1}{2}$ of this rectangle into two equal parts, each is what part of the rectangle? How much is $\frac{1}{2} \div 2$?



6. If you divide $\frac{1}{4}$ of the rectangle into two equal parts, each is what part of the rectangle? How much is $\frac{1}{4} \div 2$? What have you done to the denominator?

7. If you divide $\frac{3}{4}$ of the rectangle into two equal parts, how many eighths of the rectangle do you then have? How much is $\frac{3}{4} \div 2$? What have you done to the denominator?

8. Multiplying the denominator of a fraction does what to the fraction itself?

Divide the following fractions by dividing the numerators :

- | | | | | |
|--------------------------|------------------------|------------------------|------------------------|-------------------------|
| 9. $\frac{4}{5} \div 2$ | $\frac{15}{16} \div 3$ | $\frac{15}{16} \div 5$ | $\frac{25}{64} \div 5$ | $\frac{21}{32} \div 7$ |
| 10. $\frac{6}{7} \div 3$ | $\frac{12}{5} \div 6$ | $\frac{16}{5} \div 8$ | $\frac{27}{32} \div 9$ | $\frac{33}{64} \div 11$ |

Divide the following fractions by multiplying the denominators :

- | | | | | |
|--------------------------|----------------------|----------------------|----------------------|----------------------|
| 11. $\frac{1}{3} \div 5$ | $\frac{2}{3} \div 7$ | $\frac{3}{6} \div 4$ | $\frac{3}{8} \div 7$ | $\frac{3}{4} \div 5$ |
| 12. $\frac{4}{5} \div 9$ | $\frac{7}{8} \div 3$ | $\frac{3}{8} \div 2$ | $\frac{5}{8} \div 9$ | $\frac{3}{6} \div 7$ |

Divide the following :

- | | | | | |
|-----------------------------|------------------------|-----------------------|-----------------------|----------------------|
| 13. $\frac{4}{16} \div 2$ | $\frac{3}{6} \div 2$ | $\frac{7}{8} \div 3$ | $\frac{5}{8} \div 5$ | $\frac{7}{8} \div 2$ |
| 14. $\frac{15}{16} \div 10$ | $\frac{25}{32} \div 5$ | $\frac{5}{12} \div 4$ | $\frac{15}{3} \div 5$ | $\frac{7}{3} \div 5$ |

80. How to Divide a Fraction by a Whole Number. From the answers to the questions on page 66 we have the following:

Dividing the numerator by a whole number divides the fraction by that number.

Multiplying the denominator by a whole number divides the fraction by that number.

If we have a mixed number to be divided, we may reduce the mixed number to an improper fraction.

Thus $2\frac{1}{3} \div 3 = \frac{7}{3} \div 3 = \frac{7}{9}.$

Sometimes, however, it is easier to divide the mixed numbers. For example, to divide a number like $244\frac{1}{2}$ by 3 it is easier not to reduce to an improper fraction.

Thus $244\frac{1}{2} \div 3 = 81 + \frac{1}{3} \text{ of } 1\frac{1}{2} = 81 + \frac{1}{2} = 81\frac{1}{2}.$

WRITTEN EXERCISE

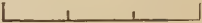
Divide the following :

- | | | | | |
|-------------------------|-----------------------|-----------------------|-----------------------|-----------------------|
| 1. $\frac{1}{2} \div 2$ | $\frac{1}{3} \div 2$ | $\frac{1}{4} \div 2$ | $\frac{3}{4} \div 2$ | $\frac{3}{8} \div 2$ |
| 2. $\frac{1}{2} \div 3$ | $\frac{2}{3} \div 3$ | $\frac{1}{4} \div 3$ | $\frac{3}{4} \div 3$ | $\frac{3}{8} \div 3$ |
| 3. $\frac{2}{5} \div 3$ | $\frac{4}{5} \div 3$ | $\frac{1}{8} \div 2$ | $\frac{6}{5} \div 3$ | $\frac{6}{5} \div 4$ |
| 4. $\frac{5}{8} \div 5$ | $\frac{5}{8} \div 2$ | $\frac{5}{8} \div 3$ | $\frac{5}{8} \div 4$ | $\frac{7}{8} \div 7$ |
| 5. $\frac{6}{7} \div 2$ | $\frac{6}{7} \div 3$ | $\frac{6}{7} \div 5$ | $\frac{6}{7} \div 6$ | $\frac{3}{8} \div 6$ |
| 6. $\frac{5}{8} \div 5$ | $3\frac{3}{4} \div 5$ | $7\frac{1}{2} \div 3$ | $4\frac{1}{5} \div 7$ | $4\frac{2}{3} \div 7$ |

Divide the whole part and the fractional part separately, and add the results :

- | | | | | |
|---------------------------|------------------------|------------------------|------------------------|-------------------------|
| 7. $12\frac{3}{4} \div 3$ | $21\frac{1}{2} \div 3$ | $36\frac{3}{5} \div 3$ | $24\frac{4}{5} \div 4$ | $35\frac{1}{16} \div 5$ |
| 8. $23\frac{1}{3} \div 7$ | $25\frac{1}{2} \div 8$ | $28\frac{1}{5} \div 9$ | $28\frac{3}{4} \div 5$ | $134\frac{1}{2} \div 5$ |

ORAL EXERCISE

1. How many quarters of a dollar in \$1?
2. How many thirds of an inch in 1 in.? Then $1 \div \frac{1}{3}$ are how many? What can you say about the values of $1 \div \frac{1}{3}$ and 3×1 ? 
3. How many thirds in 1? in 2? What can you say about the values of $2 \div \frac{1}{3}$ and 3×2 ?
4. How many eighths of an inch in 1 in.? What can you say about the values of $1 \div \frac{1}{8}$ and 8×1 ?
5. How many fifths of an inch in 1 in.? Then $1 \div \frac{1}{5}$ are how many? What can you say about the values of $1 \div \frac{1}{5}$ and 5×1 ? 
6. Tell the values of
 $1 \div \frac{1}{8}$ $1 \div \frac{1}{9}$ $1 \div \frac{1}{7}$ $1 \div \frac{1}{10}$ $1 \div \frac{1}{20}$
7. How do you divide by a unit fraction?
8. If a sheet of veneer is $\frac{1}{16}$ in. thick, how many sheets pressed together will have a thickness of $\frac{3}{16}$ in.?
9. If a sheet of felt is $\frac{1}{16}$ in. thick, how many sheets are there in a pile $1\frac{1}{16}$ in. thick?
10. If a bicycle is traveling at the rate of a mile in $\frac{1}{8}$ hr., how many miles will it travel in $3\frac{1}{8}$ hr.?
11. How many weights of $\frac{1}{16}$ lb. (or 1 oz.) each will it take to weigh $2\frac{1}{16}$ lb.?
12. How many pints ($\frac{1}{8}$ gal.) are there in 3 gal.? How many pints are there in $2\frac{1}{2}$ gal.?
13. How many inches ($\frac{1}{12}$ ft.) are there in 3 ft.? How many inches are there in 10 ft.?
14. If each book is $\frac{1}{2}$ in. thick, how many books are there in a pile that is $3\frac{1}{2}$ in. high?

81. Dividing by a Unit Fraction. From the work given on page 68 it is clear that division by a unit fraction is simple.

To divide a number by a unit fraction, multiply the number by the denominator of the fraction.

Thus	$1 \div \frac{1}{8} = 1 \times 8 = 8,$
	$3 \div \frac{1}{8} = 3 \times 8 = 24,$
and	$\frac{3}{4} \div \frac{1}{8} = \frac{3}{4} \times 8 = 6.$

WRITTEN EXERCISE

Divide the following :

- | | | |
|----------------------------|-----------------------------|--|
| 1. $25 \div \frac{1}{3}.$ | 11. $16 \div \frac{1}{9}.$ | 21. $39 \div \frac{1}{20}.$ |
| 2. $32 \div \frac{1}{4}.$ | 12. $23 \div \frac{1}{7}.$ | 22. $3\frac{1}{2} \div \frac{1}{16}.$ |
| 3. $37 \div \frac{1}{5}.$ | 13. $32 \div \frac{1}{6}.$ | 23. $7\frac{3}{8} \div \frac{1}{32}.$ |
| 4. $43 \div \frac{1}{6}.$ | 14. $54 \div \frac{1}{4}.$ | 24. $9\frac{7}{16} \div \frac{1}{64}.$ |
| 5. $51 \div \frac{1}{7}.$ | 15. $29 \div \frac{1}{8}.$ | 25. $\frac{2}{3} \div \frac{1}{2}.$ |
| 6. $63 \div \frac{1}{8}.$ | 16. $48 \div \frac{1}{5}.$ | 26. $\frac{4}{5} \div \frac{1}{10}.$ |
| 7. $27 \div \frac{1}{9}.$ | 17. $63 \div \frac{1}{7}.$ | 27. $\frac{3}{8} \div \frac{1}{32}.$ |
| 8. $36 \div \frac{1}{12}.$ | 18. $27 \div \frac{1}{6}.$ | 28. $\frac{3}{4} \div \frac{1}{8}.$ |
| 9. $51 \div \frac{1}{16}.$ | 19. $17 \div \frac{1}{12}.$ | 29. $\frac{5}{6} \div \frac{1}{12}.$ |
| 10. $81 \div \frac{1}{8}.$ | 20. $97 \div \frac{1}{12}.$ | 30. $2\frac{1}{2} \div \frac{1}{2}.$ |

31. If a sheet of veneer is $\frac{1}{16}$ in. thick, how many sheets pressed together will have a thickness of $1\frac{1}{8}$ in. ?

32. If a sheet of blotting paper is $\frac{1}{32}$ in. thick, how many sheets are there in a pile that is $15\frac{5}{8}$ in. thick ?

33. How many weights of $\frac{1}{16}$ lb. (or 1 oz.) each will it take to weigh $17\frac{3}{4}$ lb. ?

34. If a wagon wheel makes $\frac{1}{12}$ of a revolution in going 1 foot, how many feet will it go in making $17\frac{3}{4}$ revolutions ?

ORAL EXERCISE

1. Divide 6ϕ by 2ϕ ; 6 in. by 2 in.; $\frac{6}{3}$ by $\frac{2}{3}$.
2. Divide $\frac{8}{9}$ by $\frac{4}{9}$; $\frac{8}{7}$ by $\frac{4}{7}$; $\frac{8}{5}$ by $\frac{4}{5}$.
3. Divide $\frac{6}{5}$ by $\frac{2}{5}$. Multiply $\frac{6}{5}$ by $\frac{5}{2}$.
4. Show that $\frac{4}{5} \div \frac{2}{5}$ is the same as $\frac{5}{2} \times \frac{4}{5}$.

To divide a number by a common fraction, multiply the number by the fraction inverted.

5. Study the following:

$$15 \div \frac{3}{5} = \overset{5}{15} \times \frac{5}{3} = 25.$$

6. Study the following:

$$\frac{3}{8} \div \frac{5}{16} = \frac{3}{8} \times \overset{2}{16} \div \frac{5}{5} = 1\frac{1}{5}.$$

WRITTEN EXERCISE

Divide the following:

- | | | | | |
|------------------------------------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| 1. $12 \div \frac{2}{3}$ | $9 \div \frac{3}{4}$ | $4 \div \frac{4}{5}$ | $9 \div \frac{3}{5}$ | $8 \div \frac{4}{5}$ |
| 2. $15 \div \frac{2}{3}$ | $15 \div \frac{3}{4}$ | $16 \div \frac{4}{5}$ | $21 \div \frac{3}{5}$ | $16 \div \frac{2}{5}$ |
| 3. $1\frac{2}{3} \div \frac{3}{4}$ | $\frac{3}{4} \div \frac{2}{3}$ | $\frac{3}{8} \div \frac{5}{8}$ | $\frac{7}{8} \div \frac{5}{8}$ | $\frac{5}{8} \div \frac{7}{8}$ |
| 4. $2\frac{1}{3} \div \frac{3}{4}$ | $3\frac{2}{3} \div \frac{3}{4}$ | $5\frac{1}{8} \div \frac{1}{2}$ | $5\frac{3}{8} \div \frac{3}{4}$ | $2\frac{7}{8} \div \frac{3}{8}$ |

5. How many times is $\frac{7}{8}$ in. contained in 14 in.?
6. How many paper pads, each $\frac{5}{16}$ in. thick, will it take to make a package 5 in. thick?
7. How many notebooks, each $\frac{7}{16}$ in. thick, will it take to make a pile 14 in. high?
8. How many thicknesses of board, each $\frac{3}{4}$ in. thick, will it take to make a pile $9\frac{3}{4}$ in. high?

82. Dividing by a Fraction. From page 70 we have found how to divide by a fraction. We shall now look a little more carefully at the reason. We found on page 69 that when we have to divide by $\frac{1}{3}$ we may multiply by 3. We also know that when we multiply the divisor by 2 we divide the quotient by 2.

That is, $6 \div \frac{1}{3} = 3 \times 6,$
 and $6 \div \frac{2}{3} = \text{half as much} = \frac{3 \times 6}{2} = 9.$

That is, $6 \div \frac{2}{3} = \frac{3}{2} \text{ of } 6.$

Therefore, *to divide by a common fraction, multiply by that fraction inverted.*

That is, $15 \div \frac{3}{5} = 15 \times \frac{5}{3} = 25.$

Similarly, $\frac{3}{8} \div \frac{5}{6} = \frac{3}{8} \times \frac{6}{5} = \frac{9}{20},$

and $4\frac{2}{3} \div 2\frac{4}{5} = \frac{14}{3} \div \frac{14}{5} = \frac{14}{3} \times \frac{5}{14} = \frac{5}{3} = 1\frac{2}{3}.$

An inverted fraction is also called the *reciprocal* of the fraction. That is, the reciprocal of $\frac{5}{6}$ is $\frac{6}{5}.$

WRITTEN EXERCISE

Divide the following :

1. $\frac{7}{8} \div \frac{2}{3}.$

7. $\frac{5}{64} \div \frac{7}{32}.$

13. $9\frac{7}{12} \div 5\frac{3}{4}.$

2. $\frac{5}{6} \div \frac{3}{4}.$

8. $\frac{9}{24} \div \frac{5}{16}.$

14. $5\frac{1}{4} \div 18\frac{3}{8}.$

3. $\frac{9}{10} \div \frac{4}{5}.$

9. $\frac{3}{40} \div \frac{6}{25}.$

15. $3\frac{9}{20} \div 2\frac{3}{10}.$

4. $\frac{5}{12} \div \frac{7}{8}.$

10. $\frac{7}{48} \div \frac{7}{12}.$

16. $4\frac{5}{9} \div 13\frac{2}{3}.$

5. $\frac{3}{8} \div \frac{5}{12}.$

11. $\frac{3}{4} \div 1\frac{1}{16}.$

17. $6\frac{3}{10} \div 1\frac{4}{5}.$

6. $\frac{7}{16} \div \frac{3}{32}.$

12. $2\frac{7}{16} \div 3\frac{1}{8}.$

18. $4\frac{9}{10} \div 12\frac{5}{6}.$

ORAL EXERCISE

Divide the following :

1. $\frac{1}{2} \div 2$

$\frac{1}{2} \div \frac{1}{2}$

$\frac{1}{2} \div \frac{1}{4}$

$\frac{1}{2} \div \frac{1}{8}$

$\frac{1}{2} \div \frac{1}{3}$

2. $\frac{1}{2} \div \frac{3}{4}$

$\frac{1}{2} \div \frac{3}{8}$

$\frac{1}{2} \div \frac{5}{8}$

$\frac{1}{2} \div \frac{7}{8}$

$\frac{1}{2} \div \frac{1}{16}$

3. $\frac{1}{4} \div \frac{1}{8}$

$\frac{3}{4} \div \frac{1}{8}$

$\frac{3}{4} \div \frac{5}{8}$

$\frac{3}{4} \div \frac{7}{8}$

$\frac{3}{4} \div \frac{9}{8}$

4. Divide $\frac{1}{8}$ by 2; 2 by $\frac{1}{8}$; 2 by 3; 3 by 2.

5. Divide $\frac{3}{4}$ by 3; 3 by $\frac{3}{4}$; 3 by $\frac{3}{5}$; 3 by $\frac{3}{8}$.

6. Divide $\frac{4}{5}$ by 2; 2 by $\frac{4}{5}$; 2 by $\frac{2}{3}$; $\frac{2}{3}$ by 2.

7. Divide $\frac{3}{8}$ by $\frac{5}{8}$; $\frac{5}{8}$ by $\frac{3}{8}$; $\frac{3}{4}$ by $\frac{5}{4}$; $\frac{5}{4}$ by $\frac{3}{4}$.

8. How many half-yard pieces can be made by cutting 5 yd. of ribbon? by cutting 7 yd. of ribbon?

9. If your recipe for making cake calls for $\frac{1}{2}$ lb. of sugar, 6 lb. of sugar is enough for how many times as much cake?

WRITTEN EXERCISE

Divide the following :

1. $\frac{2}{3} \div \frac{3}{4}$.

8. $\frac{4}{5} \div \frac{5}{8}$.

15. $\frac{5}{9} \div \frac{4}{5}$.

22. $5\frac{1}{4} \div \frac{5}{8}$.

2. $\frac{3}{4} \div \frac{2}{3}$.

9. $\frac{7}{8} \div \frac{3}{4}$.

16. $\frac{4}{5} \div \frac{5}{8}$.

23. $3\frac{3}{4} \div \frac{7}{8}$.

3. $\frac{2}{3} \div \frac{4}{5}$.

10. $\frac{3}{4} \div \frac{7}{8}$.

17. $\frac{3}{5} \div \frac{2}{3}$.

24. $8\frac{1}{3} \div \frac{5}{6}$.

4. $\frac{4}{5} \div \frac{2}{3}$.

11. $\frac{4}{5} \div \frac{7}{8}$.

18. $\frac{2}{3} \div \frac{3}{6}$.

25. $3\frac{5}{8} \div \frac{3}{4}$.

5. $\frac{3}{8} \div \frac{3}{4}$.

12. $\frac{7}{8} \div \frac{4}{5}$.

19. $\frac{5}{12} \div \frac{2}{3}$.

26. $3\frac{5}{8} \div \frac{3}{5}$.

6. $\frac{3}{4} \div \frac{3}{8}$.

13. $\frac{5}{9} \div \frac{2}{3}$.

20. $\frac{2}{3} \div \frac{5}{12}$.

27. $4\frac{7}{8} \div \frac{3}{8}$.

7. $\frac{5}{8} \div \frac{4}{5}$.

14. $\frac{2}{3} \div \frac{5}{9}$.

21. $5 \div \frac{5}{8}$.

28. $5\frac{4}{9} \div 2\frac{1}{3}$.

29. How many mats, each $\frac{3}{4}$ yd. long, can be made from a strip of carpet 9 yd. long, making no allowance for turning in or hemming?

30. How many towels, each $\frac{5}{8}$ yd. long, can be made from a strip of toweling $11\frac{1}{4}$ yd. long, making no allowance for hemming?

WRITTEN EXERCISE

Divide the following :

1. $21 \div 5\frac{1}{4}$.

12. $17\frac{1}{8} \div 2\frac{1}{2}$.

23. $27\frac{1}{2} \div 33\frac{1}{3}$.

2. $77 \div 9\frac{1}{3}$.

13. $12\frac{1}{2} \div 3\frac{1}{2}$.

24. $40\frac{3}{4} \div 6\frac{1}{3}$.

3. $69 \div 5\frac{3}{4}$.

14. $15\frac{1}{2} \div 3\frac{1}{3}$.

25. $28\frac{1}{6} \div 2\frac{1}{8}$.

4. $35 \div 5\frac{5}{8}$.

15. $78 \div 8\frac{1}{2}$.

26. $29\frac{1}{5} \div 58\frac{2}{5}$.

5. $21 \div 4\frac{2}{3}$.

16. $16\frac{4}{5} \div 4\frac{1}{5}$.

27. $20\frac{1}{8} \div 15\frac{1}{3}$.

6. $14 \div 2\frac{5}{8}$.

17. $79\frac{4}{5} \div 8\frac{1}{7}$.

28. $31\frac{5}{8} \div 15\frac{1}{16}$.

7. $61\frac{1}{4} \div 8\frac{3}{4}$.

18. $324 \div 7\frac{1}{6}$.

29. $13\frac{9}{10} \div 4\frac{2}{5}$.

8. $71 \div 8\frac{7}{8}$.

19. $25\frac{3}{5} \div 5\frac{1}{3}$.

30. $151\frac{7}{8} \div 15\frac{3}{16}$.

9. $17\frac{7}{8} \div 4\frac{7}{8}$.

20. $52\frac{2}{5} \div 8\frac{3}{16}$.

31. $876\frac{1}{2} \div 129\frac{5}{12}$.

10. $33 \div 7\frac{1}{5}$.

21. $61\frac{7}{8} \div 4\frac{1}{16}$.

32. $202\frac{1}{8} \div 14\frac{7}{16}$.

11. $15\frac{2}{3} \div 6\frac{1}{3}$.

22. $77 \div 6\frac{5}{12}$.

33. $171\frac{7}{8} \div 17\frac{3}{16}$.

34. How many strips of cloth, each $5\frac{3}{8}$ yd. long, can we get from a strip $37\frac{5}{8}$ yd. long?

35. How many city lots, each $31\frac{3}{4}$ ft. front, can a man get from a piece of land having $222\frac{1}{4}$ ft. front?

36. How many yards of cloth at $33\frac{1}{3}\phi$ a yard can be bought for \$7? for \$19? for \$27?

37. How many books, each $\frac{7}{8}$ in. thick, will it take to make a pile $21\frac{7}{8}$ in. high?

38. How many strips of wood, each $\frac{9}{16}$ in. thick, will it take to make a pile $19\frac{1}{8}$ in. high?

39. How many sheets of bookbinding board, each $\frac{3}{32}$ in. thick, will it take to make a pile $9\frac{3}{16}$ in. thick?

40. If the product of two numbers is 46, and one of the numbers is $2\frac{7}{8}$, what is the other?

ORAL EXERCISE

1. In line D , point to $\frac{3}{16}$ of the line; also to $\frac{6}{16}$. We see that $\frac{6}{16}$ is how many times $\frac{3}{16}$? How does multiplying the numerator by 2 change the value of the fraction? How does dividing the numerator by 2 change the value of the fraction?



A



B



D

These figures can be used more easily if drawn on the board.

2. Point to $\frac{1}{2}$ of square A ; also to $\frac{1}{4}$. We see that $\frac{1}{2}$ is how many times $\frac{1}{4}$? How does multiplying the denominator by 2 change the value of the fraction? How does dividing the denominator by 2 change the value of the fraction?

3. In rectangle B , show that $\frac{2}{3} = \frac{4}{6}$. Then does it change the value of a fraction to multiply or divide both terms by the same number? Show this with the circle C .

83. Review of Important Principles. From the above exercise we again see the truth of these important principles:

Multiplying the numerator of a fraction by any number multiplies the fraction by that number.

Dividing the numerator of a fraction by any number divides the fraction by that number.

Multiplying the denominator of a fraction by any number divides the fraction by that number.

Dividing the denominator of a fraction by any number multiplies the fraction by that number.

Dividing both terms of a fraction by the same number does not change the value of the fraction.

ORAL EXERCISE

1. How many cents in $\$ \frac{1}{8}$? Then 16 times $\$ \frac{1}{8}$ is the same as 16 times how many cents?

2. How many cents in $\$ \frac{1}{3}$? Then 9 times $\$ \frac{1}{3}$ is the same as 9 times how many cents?

3. How many cents in $\$ \frac{1}{6}$? Then 24 times $\$ \frac{1}{6}$ is the same as 24 times how many cents?

84. Use of Aliquot Parts in Multiplication. It is easier to multiply $\$ \frac{1}{8}$, $\$ \frac{1}{6}$, and $\$ \frac{1}{3}$ than it is to multiply $12\frac{1}{2}\phi$, $16\frac{2}{3}\phi$, and $33\frac{1}{3}\phi$. Such parts of a dollar or of any other unit are often called, as we found in § 26, aliquot parts.

At $33\frac{1}{3}\phi$ each, 9 neckties cost 9 times $\$ \frac{1}{3}$, or $\$3$.

At $16\frac{2}{3}\phi$ each, 12 bottles of ink cost 12 times $\$ \frac{1}{6}$, or $\$2$.

At $12\frac{1}{2}\phi$ each, 9 balls cost 9 times $\$ \frac{1}{8}$, or $\$1\frac{1}{8}$, or $\$1.12\frac{1}{2}$.

While goods are not marked $16\frac{2}{3}\phi$ or $\$ \frac{1}{6}$, they are often marked 6 for $\$1$, which is the same thing.

4. At $12\frac{1}{2}\phi$ a yard, what will 16 yd. of cloth cost?

5. At $16\frac{2}{3}\phi$ a yard, what will 18 yd. of cloth cost?

6. At $33\frac{1}{3}\phi$ a yard, what will 21 yd. of cloth cost?

7. At 8 doz. marbles for $\$1$, what will 6 doz. cost?

8. At $16\frac{2}{3}\phi$ each, find the cost of 9 water pitchers.

9. At the rate of 3 for $\$1$, what must the school pay for 15 arithmetics?

10. At $12\frac{1}{2}\phi$ each, find the cost of 10 glasses; of 4 lamp chimneys; of 20 cans of soup; of 32 yd. of calico.

11. How many jackets can be cut from $17\frac{1}{2}$ yd. of cloth, allowing $2\frac{1}{2}$ yd. to a jacket?

ORAL EXERCISE

1. How much is 8 times $\$ \frac{1}{8}$? $\$ 1 \frac{1}{8}$? $\$ 1.12 \frac{1}{2}$?
2. How much is 6 times $\$ \frac{1}{6}$? $\$ 1 \frac{1}{6}$? $\$ 1.16 \frac{2}{3}$?
3. How much is 3 times $\$ \frac{1}{3}$? $\$ 1 \frac{1}{3}$? $\$ 1.33 \frac{1}{3}$?
4. At $\$ 1.33 \frac{1}{3}$ a yard, how much will 15 yd. of lace cost?
5. At $\$ 1.16 \frac{2}{3}$ a yard, how much will 9 yd. of silk cost?
6. At $\$ 1.12 \frac{1}{2}$ a yard, how much will 6 yd. of velvet cost?
7. Find the cost of 12 books at $\$ 1.12 \frac{1}{2}$ each; at $\$ 1.16 \frac{2}{3}$ each.
8. At $\$ 1.33 \frac{1}{3}$ each, find the cost of 3 chairs; of 18 chairs.
9. At $\$ 1.16 \frac{2}{3}$ each, find the cost of 6 maps; of 12 maps; of 30 maps; of 54 maps.
10. At $\$ 1.12 \frac{1}{2}$ each, find the cost of 8 books; of 40 books; of 32 books; of 20 books.

At $\$ 1.12 \frac{1}{2}$ each, 7 books cost $\$ 7 + \$ \frac{7}{8}$, or $\$ 7.87 \frac{1}{2}$ ($\$ 7.88$).

At $\$ 1.16 \frac{2}{3}$ each, 5 books cost $\$ 5 + \$ \frac{5}{6}$, or $\$ 5.83 \frac{1}{3}$ ($\$ 5.83$).

At $\$ 1.33 \frac{1}{3}$ each, 8 lamps cost $\$ 8 + \$ \frac{8}{3}$, or $\$ 10 \frac{2}{3}$ ($\$ 10.67$).

WRITTEN EXERCISE

1. At $12 \frac{1}{2} \text{¢}$ each, find the cost of 147 balls.
2. At $16 \frac{2}{3} \text{¢}$ each, find the cost of 113 notebooks.
3. At $33 \frac{1}{3} \text{¢}$ each, find the cost of 101 handkerchiefs.
4. At $\$ 1.33 \frac{1}{3}$ a volume, find the cost of 165 books.
5. At $\$ 1.16 \frac{2}{3}$ a yard, find the cost of 142 yd. of lace.
6. At $\$ 1.12 \frac{1}{2}$ each, find the cost of 112 yd. of velvet.
7. At $\$ 2.16 \frac{2}{3}$ each, find the cost of 72 umbrellas.
8. What is the difference in cost of 144 books at $\$ 1.16 \frac{2}{3}$ a volume, and the same number at $\$ 1.12 \frac{1}{2}$ a volume?

III. DENOMINATE NUMBERS

ORAL EXERCISE

1. How many ounces in a pound? An ounce is what part of a pound?

2. State the number of pints in a quart and the number of quarts in a gallon. A quart is what part of a gallon?

3. State the number of inches in a foot and the number of feet in a yard. An inch is what part of a foot?

4. State the number of quarts in a peck and the number of pecks in a bushel. A peck is what part of a bushel? A quart is what part of a peck? A quart is what part of a bushel?

5. How many seconds in a minute? minutes in an hour? hours in a day? days in a week? days in a common year? days in a leap year? What months have 30 days?

Express the following as indicated:

6. 6 pt. as quarts. 10. 8 qt. as gallons. 14. 3 yd. as feet.

7. 14 da. as weeks. 11. 8 pk. as bushels. 15. 12 ft. as yards.

8. $1\frac{1}{2}$ lb. as ounces. 12. 15 pt. as quarts. 16. 4 wk. as days.

9. 48 hr. as days. 13. 32 oz. as pounds. 17. $2\frac{1}{2}$ qt. as pints.

18. How many pints in a quart? in $1\frac{1}{2}$ qt.? in 2 qt.? in a gallon? in 2 gal.? in $2\frac{1}{2}$ gal.?

19. How many quarts in a peck? in 2 pk.? in a bushel? in 2 bu.? in $2\frac{1}{2}$ bu.? in 10 bu.? in 100 bu.?

20. If you buy $\frac{1}{2}$ yd. of cloth, how many inches do you buy?

21. If you buy 72 in. of cloth, how many yards do you buy?

22. How many minutes are there from a quarter before ten to half past ten?

85. Denominate Numbers. Concrete numbers that denote measure (including weight) are called *denominate numbers*.

For example, \$2, 3 ft., 7 lb. 4 oz. are denominate numbers, but not 5 chairs or 8 rivers.

We change denominate numbers as shown in these examples:

1. Change $7\frac{1}{2}$ ft. to inches.

Since $1 \text{ ft.} = 12 \text{ in.},$
therefore $7\frac{1}{2} \text{ ft.} = 7\frac{1}{2} \times 12 \text{ in.} = 90 \text{ in.}$

2. Change 90 in. to feet.

Since $1 \text{ in.} = \frac{1}{12} \text{ ft.},$
therefore $90 \text{ in.} = 90 \times \frac{1}{12} \text{ ft.} = 7\frac{1}{2} \text{ ft.}$

ORAL EXERCISE

Recite the following tables of measures:

- | | | |
|---|-----------------|--------------------|
| 1. Length. | 3. Time. | 5. Liquid measure. |
| 2. Weight. | 4. Dry measure. | 6. Square measure. |
| 7. Name something that is sold by the pound; by the ton. | | |
| 8. Name something that is sold by the quart; by the bushel. | | |

WRITTEN EXERCISE

Copy and complete:

- | | |
|----------------------------------|----------------------------------|
| 1. 3 ft. = (36) in. | 8. 15 pt. = () qt. |
| 2. $2\frac{1}{2}$ ft. = (36) in. | 9. 17 gal. = () qt. |
| 3. 36 in. = (3) ft. | 10. 17 qt. = () gal. |
| 4. 54 in. = () yd. | 11. 17 qt. = () pt. |
| 5. 39 yd. = () ft. | 12. 32 pk. = () bu. |
| 6. 39 ft. = () yd. | 13. 32 bu. = () pk. |
| 7. 15 qt. = () pt. | 14. $2\frac{1}{2}$ yd. = () ft. |

Copy and complete :

- | | |
|-------------------------------------|--------------------------------------|
| 15. $2\frac{1}{2}$ yd. = () in. | 27. 120 min. = () sec. |
| 16. $1\frac{1}{2}$ mi. = () ft. | 28. $2\frac{1}{2}$ gal. = () qt. |
| 17. 29 ft. = () in. | 29. $2\frac{1}{2}$ qt. = () pt. |
| 18. 29 ft. = () yd. | 30. 640 oz. = () lb. |
| 19. 36 pk. = () bu. | 31. 640 lb. = () oz. |
| 20. 36 bu. = () pk. | 32. $12\frac{1}{2}$ ft. = () in. |
| 21. 48 lb. = () oz. | 33. $12\frac{1}{2}$ yd. = () ft. |
| 22. 48 oz. = () lb. | 34. $12\frac{1}{2}$ yd. = () in. |
| 23. $2\frac{1}{2}$ mi. = () rd. | 35. $2\frac{1}{2}$ gal. = () pt. |
| 24. $2\frac{1}{2}$ rd. = () ft. | 36. 4000 lb. = () T. |
| 25. 120 min. = () hr. | 37. 4000 T. = () lb. |
| 26. 120 hr. = () min. | 38. $2\frac{1}{2}$ hr. = () min. |

39. If Mary buys $1\frac{1}{2}$ qt. of milk, how many pints of milk does she buy?

40. If John feeds his horse 4 qt. of oats three times a day, how many days will 6 bu. of oats last?

41. If you live $\frac{1}{2}$ mi. from school, how many feet must you walk in coming to school?

42. In Ex. 41, how many yards must you walk?

43. When apples are worth \$1.20 a bushel, how much are they worth a peck?

44. If you reach school at 10 min. before nine, and leave at 10 min. after twelve, how many minutes are you in school?

45. If you reach school at 1.15 P.M. and leave at 3.45 P.M., how many minutes are you in school?

In the case of 1.15 P.M. the period is not a decimal point. The expression means 15 min. past 1 P.M. It is often written 1:15 P.M., or 1 15 P.M.

86. Solving Problems. The solution of a problem consists of four parts:

1. Reading, analyzing, understanding the problem, finding what it *gives* and what it *requires*.

2. Planning how to solve it; determining the processes to be used and choosing the most economical way of proceeding.

3. Executing the plan; forecasting the probable result, using the shortest roads to the answer by taking advantage of the relations existing between the numbers.

For example, if 3 yd. of cloth cost 90¢, $4\frac{1}{2}$ yd. will cost half as much more, or 45¢ more, or \$1.35.

4. Testing the work, checking the calculation, comparing the result with the statement of the problem.

The teacher should solve, on the blackboard, Exs. 1 and 2 of the following exercise, applying the above four statements.

WRITTEN EXERCISE

1. If 4 bu. of wheat cost \$3.60, what will 6 bu. cost?
2. If 7 bu. of oats cost \$3.50, what will $3\frac{1}{2}$ bu. cost?
3. If 9 yd. of ribbon cost \$1.08, what will $4\frac{1}{2}$ yd. cost?
4. If 5 qt. of milk cost 45¢, what will 15 qt. cost?
5. If 8 lb. of butter cost \$3.20, what will 16 lb. cost?
6. If 35 arithmetics cost \$14, what will 7 arithmetics cost?
7. If a man earns \$2.80 in 7 hr., how much will he earn in 3 hr. at the same rate?
8. If $\frac{2}{3}$ of a farm is worth \$4800, how much is $\frac{1}{3}$ of the farm worth? How much is the whole farm worth?
9. If John pays $\frac{2}{5}$ towards the cost of a canoe, and pays \$6, how much does the canoe cost?

IV. MEASURING AND DRAWING TO SCALE

WRITTEN EXERCISE

1. A farm is plotted as here shown. The scale is 1 in. to 100 rd. There being 160 sq. rd. in an acre, find the number of acres in the farm.

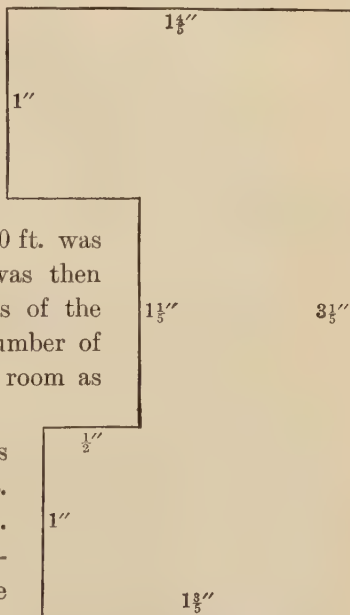
To draw to the scale 1 in. to 100 rd., we represent 100 rd. by 1 in. To draw to the scale $\frac{1}{12}$, we represent 12 in. by 1 in. We frequently write 2' 7" for 2 ft. 7 in.

2. A rectangular meadow is plotted to the scale of 1 in. to 50 rd. The plot is 2 in. by $3\frac{1}{5}$ in. Find the number of acres in the meadow.

3. A plan for a schoolroom drawn to the scale of 1 in. to 50 ft. was 1 in. long and $\frac{7}{10}$ in. wide. It was then decided to reduce the dimensions of the room one fifth. What was the number of square feet of floor space in the room as finally built?

4. A plan for a playhouse is drawn to the scale of 1 in. to $2\frac{1}{2}$ ft. The plan is $6\frac{2}{5}$ in. long and 4 in. wide, and the playhouse is rectangular. Compute the area of the playhouse.

5. The class has considered the question of a model kitchen in an apartment house. The pupils drew a plan to the scale $\frac{1}{60}$, representing the kitchen as 3 in. long and 2 in. wide. What was the floor space of the proposed kitchen?



WRITTEN EXERCISE

1. Measure this page of the arithmetic. Draw a plan of the page to the scale $\frac{1}{4}$.

2. Draw a plan of the same page to the scale $\frac{1}{2}$.

3. Measure the inside of the cover of this arithmetic. It is a little larger than the page. Draw a plan to the scale $\frac{1}{5}$.

4. Measure the top of your desk. Draw a plan to the scale of 1 in. to 1 ft.

5. Draw a plan of the top of your desk to the scale $\frac{1}{6}$.

6. Measure the length and width of the schoolroom. Draw a plan and write below it the scale that you have used.

7. Measure the size of the paper upon which you make your drawings. Draw a plan to the scale $\frac{1}{8}$.

8. Measure the size of any other book you are studying. Draw a plan of a page to the scale $\frac{1}{9}$.

9. A drawing has been made of a box cover. The cover is 14 in. long and the drawing is $3\frac{1}{2}$ in. long. What scale was used?

10. A drawing of a floor 24 ft. long and 20 ft. wide is 6 in. long and 5 in. wide. What scale was used?

11. A drawing is made of a rectangle. The drawing has an area of 6 sq. in. If a scale twice as large were used, what would be the area of the drawing? Draw a plan of the two rectangles, so as to show that the answer is correct.

12. A drawing is made of a lamp screen $18\frac{3}{4}$ in. high, and the drawing is $3\frac{1}{8}$ in. high. What scale was used?

Teachers should have measurements made of desks, doors, blackboards, and other objects in the schoolroom, and scale drawings made from these measurements. The pupils should gradually come to know what the working drawings of the carpenter, the builder, and the manufacturer mean.

REVIEW AND DRILL

87. Nature and Purpose of this Work. After the work of each half grade in this book there occurs a section like this one. The work is called Review and Drill, and it is optional with the teacher whether or not it shall be used and how much of it shall be taken. None of these sections is merely a review of the chapter immediately preceding it, for review work is already provided in the chapter itself; on the contrary, the review is general and covers all the important processes in arithmetic that have been studied. This renders possible a continual return to fundamental and first principles, a constant brushing up in the essentials of arithmetic. This constant brushing up, drilling, and reviewing of the essentials is necessary for the attainment of good results. It is easily made interesting, it contributes to efficiency in computation, and it will, if carried out in the right spirit, help to remove the charge that most pupils are not quick and accurate in their calculations.

When the length of the school year permits, the study of these Review and Drill sections is strongly recommended; but the work being supplementary and optional, it is possible, and doubtless will often be desirable, for each teacher to select from these exercises only what is specially adapted to his immediate needs, letting the rest go. A certain flexibility is thus offered that will make the textbook more adaptable to varying local requirements than has heretofore been the case.

Each section consists of three parts: first, a drill in operations, emphasizing the *mechanical* side of arithmetic; second, an exercise in problems without numbers, emphasizing the *analytical* side; and finally, live industrial problems that photograph real situations, emphasizing the *practical* side.

It will at once occur to every teacher that in these Review and Drill sections, with their threefold, inclusive emphasis, he has ready at hand a series of *efficiency tests* which will enable him at frequent intervals to check up the work of the school as well as the work of the individual pupils.

I. DRILL SECTION

ORAL EXERCISE

State the sum and also the difference of :

1. 98	73	66	80	88	$88\frac{1}{2}$	$44\frac{1}{3}$
<u>11</u>	<u>21</u>	<u>22</u>	<u>30</u>	<u>30</u>	<u>$31\frac{1}{2}$</u>	<u>$22\frac{3}{4}$</u>

2. 60	40	33	52	71	$83\frac{1}{8}$	$99\frac{1}{3}$
<u>12</u>	<u>15</u>	<u>14</u>	<u>13</u>	<u>60</u>	<u>$23\frac{1}{2}$</u>	<u>$11\frac{2}{3}$</u>

Multiply, in turn, by 2, 3, 4, and so on to 9 :

3. 20	30	40	70	60	$\frac{5}{8}$	$\frac{3}{4}$
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Divide, in turn, by 2, 3, 4, and so on to 9 :

4. 10	20	50	70	30	$\frac{2}{3}$	$\frac{7}{8}$
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WRITTEN EXERCISE

1. Prove that the following curious multiplications are correct:

12,345,679	1,122,334,455,667,789
<u>54</u>	<u>297</u>
666,666,666	333,333,333,333,333,333

2. Multiply 37 by 3, 6, 9, 12, 15, 18, 21, 24, and 27, and notice the curious results. Check each result by dividing the product by the multiplier.

3. Notice the peculiar results arising from multiplying 1221 by 91, and 900,991 by 123,321. Check each result by division.

4. Compare the products of 11×91 , 73×137 , and 11×9091 .

5. Notice the peculiar results arising from multiplying 12,345,679 by 3, 6, 15, and 66, in turn.

6. The product of $333 \times 111,222,333,444,555,666,777,889$ is peculiar. What is the product?

WRITTEN EXERCISE

1. Write the date of the present year, last year, and next year in Roman numerals.

2. Write in words the numbers 1,001,001 and 1,100,100.

	I	II	III	IV	V	VI	VII	VIII
<i>a</i>	11	$1\frac{1}{10}$	$\frac{1}{5}$	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{3}{8}$	$\frac{5}{6}$	$\frac{7}{8}$
<i>b</i>	31	$\frac{1}{5}$	$\frac{1}{8}$	$\frac{2}{3}$	$\frac{1}{5}$	$\frac{3}{4}$	$1\frac{1}{2}$	$1\frac{9}{16}$
<i>c</i>	17	$\frac{7}{8}$	$\frac{5}{8}$	$\frac{3}{8}$	$\frac{1}{8}$	$3\frac{3}{2}$	$1\frac{5}{2}$	$3\frac{7}{2}$
<i>d</i>	3317	$\frac{1}{6}$	$\frac{1}{4}$	$1\frac{1}{6}$	$\frac{7}{8}$	$\frac{1}{4}$	$\frac{2}{3}$	$\frac{1}{4}$
<i>e</i>	9130	$1\frac{7}{4}$	$\frac{1}{2}$	$3\frac{1}{2}$	$1\frac{5}{6}$	$\frac{7}{8}$	$1\frac{9}{10}$	$\frac{5}{6}$
<i>f</i>	7370	$1\frac{3}{8}$	$\frac{3}{8}$	$\frac{5}{8}$	$\frac{3}{4}$	$\frac{3}{4}$	$1\frac{1}{8}$	$\frac{1}{2}$
<i>g</i>	6789	$3\frac{3}{2}$	$1\frac{5}{6}$	$1\frac{3}{6}$	$\frac{1}{2}$	$1\frac{1}{6}$	$3\frac{1}{2}$	$\frac{1}{8}$

In each of the eight columns given above find the following results, thus making eight examples in each case :

3. $d + e$.	9. $a + g$.	15. $a \times e$.	21. $e \div a$.
4. $e - d$.	10. $b + f$.	16. $a \times f$.	22. $f \div a$.
5. $e + f$.	11. $c + e$.	17. $a \times g$.	23. $g \div b$.
6. $e - f$.	12. $a \times b$.	18. $d \times g$.	24. $d \div b$.
7. $f + g$.	13. $a \times c$.	19. $e \times f$.	25. $d \div c$.
8. $f - g$.	14. $a \times d$.	20. $e \times g$.	26. $e \div c$.

Add, and also subtract, thus making two examples in each of the following cases. Time yourself.

27.	28.	29.	30.	31.
$147\frac{2}{3}$	$147\frac{1}{3}$	$628\frac{2}{3}$	$408\frac{1}{3}$	$360\frac{1}{2}$
$29\frac{1}{3}$	$29\frac{2}{3}$	$342\frac{2}{3}$	$299\frac{1}{3}$	$196\frac{1}{2}$
32.	33.	34.	35.	36.
$420\frac{1}{2}$	342	$427\frac{1}{3}$	$326\frac{1}{2}$	$580\frac{1}{3}$
168	$199\frac{1}{2}$	$129\frac{2}{3}$	$129\frac{1}{2}$	$268\frac{2}{3}$

II. PROBLEMS WITHOUT NUMBERS

ORAL EXERCISE

1. How do you add a number of two figures to another number of two figures when the sum of the units is more than ten? Give your reasons.
2. How do you subtract a number of two figures from a greater number when the units' figure to be subtracted is greater in value than the units' figure of the minuend?
3. How do you multiply by a number of three figures, the middle figure being zero?
4. How do you check your work in division? How can you check your work in multiplication?
5. What do you mean by a power of a number? by the square of a number? by the cube of a number? Illustrate.
6. What is cancellation? What are its advantages? On what principle does cancellation depend?
7. If you know the cost of a certain number of acres of land, how do you find the average cost per acre?
8. If you know the weight of each member of your class, how do you find the average weight?
9. If you have the product of two numbers and one of the two numbers given, how do you find the other number?
10. If you have the dividend, remainder, and quotient given, how do you find the divisor?
11. If you have the multiplicand and product given, how do you find the multiplier?
12. If you know a workman's regular wages per hour and his wages per hour when he works overtime, how will you find the wages due him for a certain number of hours of regular work and a certain number of hours overtime?

13. What name do you give to numbers that are to be added?
What name do you give to the result?

14. What names do you give to numbers that are to be subtracted?
What name do you give to the result?

15. What names do you give to numbers that are to be multiplied?
What name do you give to the result?

16. What names do you give to numbers that are to be divided?
What name do you give to the result?

17. I have a fraction and I wish to reduce it to lowest terms.
How shall I proceed? Illustrate.

18. I have two fractions and I wish to multiply one by the
other. How shall I proceed? Illustrate.

19. I have two fractions and I wish to subtract one of them from
the other. How shall I proceed? Illustrate.

20. I have two fractions and I wish to divide one by the other.
How shall I proceed? Illustrate.

21. What characters do you use in writing Arabic numbers?
What characters do you use in writing Roman numbers? Write
the number one hundred twenty-five in each system.

22. A fraction is multiplied by a given number when you perform
what operation on the numerator? what operation on the
denominator? Illustrate both cases.

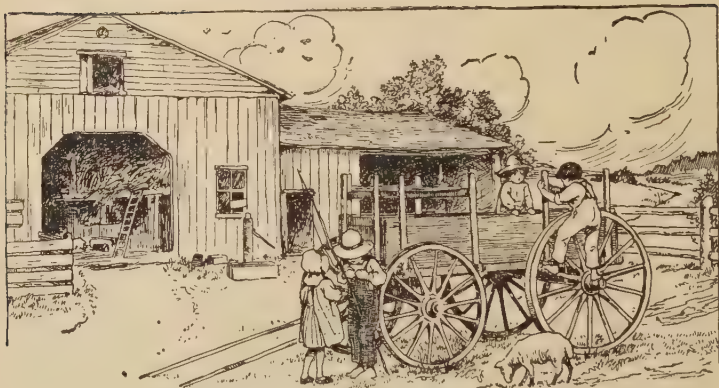
23. A fraction is divided by a given number when you perform
what operation on the numerator? what operation on the denomi-
nator? Illustrate both cases.

24. Is the value of a fraction changed by multiplying both terms
by the same number? by dividing both by the same number? by
adding the same number to each? Illustrate each answer.

25. Is the value of a fraction changed by subtracting the same
number from both terms? If so, is the value of the fraction in-
creased, or is it decreased? Illustrate by subtracting 1 from both
terms of the fractions $\frac{2}{3}$ and $\frac{3}{4}$.

III. INDUSTRIAL PROBLEMS

THE COST OF WASTEFULNESS



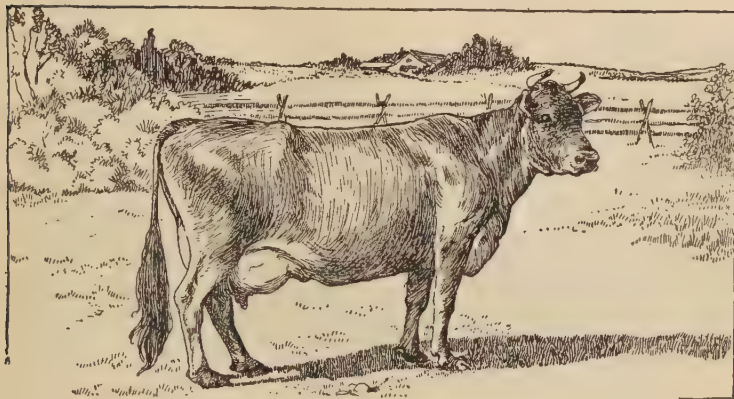
1. If a man who could earn \$1.50 a day is idle for 75 working days in a year, how much does he lose?

2. If a farm wagon that cost \$60 is left out in the yard instead of being kept in the shed, it will last about 6 yr., but if kept under cover, it will last about twice as long. How much does a farmer pay for his carelessness per year if he leaves the wagon out?

3. Each of two neighbors bought for his farm work a two-horse cultivator, and each paid \$25. One of them took good care of his cultivator, and at the end of 15 yr. it was still giving good service. The other left his out during all kinds of weather and had to buy a new one at the end of every 5 yr. How much did the careless one lose in 15 yr. by his neglect?

4. A farmer after threshing his wheat had 16 tons of straw left. This straw contains 12 lb. of nitrogen per ton, worth 20¢ a pound, and 10 lb. of potash and 2 lb. of phosphoric acid per ton, each worth 5¢ a pound. If the farmer burns the straw instead of using it on the soil as a fertilizer, how much money does he waste?

DOES BLOOD TELL IN THE DAIRY?



1. Ralph asked his father whether it really made any difference which cows they kept and which they sold. His father explained to him what was meant by grading the cows, keeping their records for milk, and disposing of those with poor records. He showed him the records of twelve farms in their state, as follows:

With Systematic Grading

Herd	Annual cost of food per cow	Annual profit per cow
<i>a</i>	\$34.28	\$34.02
<i>b</i>	33.37	36.19
<i>c</i>	47.11	25.83
<i>d</i>	36.72	38.27
<i>e</i>	31.19	48.86
<i>f</i>	31.57	42.81

Without Systematic Grading

Herd	Annual cost of food per cow	Annual profit per cow
<i>g</i>	\$31.65	\$13.34
<i>h</i>	40.58	8.13
<i>i</i>	38.30	22.22
<i>j</i>	37.40	22.33
<i>k</i>	33.78	38.93
<i>l</i>	32.88	14.91

Ralph and his father then figured out the average annual cost of food and profit per cow, in each class of farms. They did this by dividing each of the four totals by six. What were the results?

2. They then found the differences between the two cost averages and the two profit averages. What are these two differences?

BRAINS PAY



1. Joseph's father has told him he could grow some tomatoes on shares, the father furnishing the land and plants, and Joseph doing the work. Joseph was given the use of a rectangular plot of land 4 rd. by 5 rd. There being 160 sq. rd. to an acre, what part of an acre does Joseph have to cultivate?

2. Joseph reads in a farmers' bulletin that it is possible to average a crop worth \$51 an acre from plants started in a seed bed in the open ground, and as much as \$140 an acre for plants started in the most scientific way in a hotbed. How much money is there on Joseph's little plot of ground?

3. Joseph tried his best to reach the highest of these figures, but found that they were the result of more nearly perfect conditions than he met in his garden. He did, however, get $\frac{4}{5}$ of the highest figure. How much did that amount to on his plot of ground?

4. Joseph was given half of the money received. How much was his share worth? How much would it have been worth if he had raised tomatoes at the rate of 6 T. to the acre, selling the tomatoes at \$10 a ton? How much did he gain by using his brains?

RICHARD'S VEGETABLE GARDEN



1. Richard's father has rented him a vegetable garden. Richard is to work the garden and have the profits for a camping trip. On April 16 he plants some early potatoes. He counts on these potatoes being ready to dig in 79 da. When must he be home to begin to dig them?

2. If Richard allows 18 da. after the date found in Ex. 1 in which to let all the potatoes mature, and to sell them, what is the earliest day on which he can go camping?

3. On April 24 he sets out some early cauliflower, and he has to allow 72 da. for this to be ready to sell. When must he be on hand to sell the cauliflower?

4. Richard raised $4\frac{1}{2}$ bu. of early potatoes and sold them at \$1 a bushel. How much did he receive for the potatoes?

5. He sold 25 heads of early cauliflower at 10¢ a head. How much did he receive for the cauliflower?

6. Richard pays his father 75¢ rent for his garden and for his materials. Make out a statement showing Richard's income and expenses, and the amount that he has left for his camping trip.

WHAT DO PEACHES COST ?



1. Did you ever think of the labor that it takes to give you a peach to eat? Let us see some of the cost. In the first place, if a peach grower has 15 acres in his orchard, this land would rent for \$6 an acre each year. How much does this amount to for 15 acres?

2. He has to have the trees pruned. If he has 367 hr. of work done at 20¢ an hour, how much must he pay for pruning?

3. Then the peach grower must pay $1\frac{1}{4}$ ¢ a gallon for the spray and must use 100 gal. to an acre. How much does this cost him?

4. He must also have all the imperfect peaches picked off while they are young. Do you know why this "thinning," as it is called, is necessary? He pays for 113 hr. of work at 15¢ an hour. How much does this add to his expense account?

5. The peaches have to be picked and packed carefully. All this takes 964 hr. of work at 15¢ an hour. Find this part of the cost.

6. The other expenses amount to \$34.50. Add this to the grower's expense account. How much does this make?

7. The grower averages 142 bu. to the acre. Find how many bushels the grower gets, and how much they actually cost him.

GRADE V. SECOND HALF

I. REVIEW

WRITTEN EXERCISE

Add the following :

1. $273 + 421 + 68 + 9026 + 34,379 + 75 + 286.$
2. $3972 + 8298 + 4000 + 17,625 + 31 + 9 + 407.$
3. $4269 + 193,481 + 17,286 + 81,298 + 4876 + 923.$

Subtract the following :

- | | |
|-------------------------|-------------------------|
| 4. $876,481 - 298,327.$ | 7. $298,008 - 43,661.$ |
| 5. $173,209 - 106,492.$ | 8. $381,200 - 179,699.$ |
| 6. $400,100 - 199,298.$ | 9. $673,100 - 194,287.$ |

Multiply the following :

- | | | |
|-------------------------|-------------------------|-------------------------|
| 10. $628 \times 325.$ | 12. $872 \times 896.$ | 14. $469 \times 832.$ |
| 11. $1268 \times 3472.$ | 13. $2081 \times 6235.$ | 15. $4287 \times 3009.$ |

Divide the following :

- | | |
|---------------------------|---------------------------|
| 16. $65,664 \div 456.$ | 19. $77,112 \div 189.$ |
| 17. $221,336 \div 102.$ | 20. $328,320 \div 360.$ |
| 18. $7,605,920 \div 112.$ | 21. $7,913,576 \div 209.$ |

Besides the work given on pages 93-95, the teacher should introduce other review material from the preceding pages. There should be a careful and systematic review of addition, subtraction, multiplication, and division of common fractions, with much practice with mixed numbers. Abundant material for such work is given in the Review and Drill section on pages 83-92.

88. Making Change. If you owe 70¢ and give the merchant \$1, he says, "70 and 5 are 75, and 25 more makes a dollar," at the same time laying down 5¢ and 25¢. He thus sees that $\$1 - \$0.70 = \$0.30$.

Teachers may use real or toy money in teaching this subject, but pupils are now old enough to imagine the coins, as they will do in actual business methods.

ORAL EXERCISE

Make change for \$1, when you owe :

1. 25¢	50¢	96¢	85¢	63¢	39¢
2. 35¢	40¢	94¢	87¢	37¢	29¢

Make change for 50¢, when you owe :

3. 25¢	22¢	36¢	17¢	27¢	43¢
4. 35¢	32¢	38¢	16¢	19¢	24¢

Make change for 25¢, when you owe :

5. 10¢	8¢	7¢	6¢	21¢	13¢
6. 12¢	18¢	17¢	16¢	19¢	15¢

Make change for \$2, when you owe :

7. \$1	\$1.25	\$1.40	\$1.45	\$1.57
8. \$1.50	\$1.75	\$1.60	\$1.85	\$1.78

Imagine yourself buying goods at a store and handing the clerk the first amount given, the goods costing the second amount. State the amount of change due you.

9. \$1; 72¢.	13. \$4; \$3.41.	17. \$5; \$3.05.
10. \$2; \$1.37.	14. \$5; \$1.27.	18. \$5; \$1.70.
11. \$2; \$1.67.	15. \$5; \$2.40.	19. \$10; \$7.05.
12. \$3; \$2.17.	16. \$10; \$6.55.	20. \$10; \$6.05.

ORAL EXERCISE

1. At \$4 each, how much will 20 desks cost?
2. At \$9 each, how much will 30 tables cost?
3. At \$12 each, how much will 20 tables cost?

Multiply the following :

4. 60×8	70×9	40×11	50×7
5. 20×13	30×12	80×10	90×10

WRITTEN EXERCISE

Multiply, in turn, by 2, 20, and 200 :

1. 143	286	409	527	683
2. \$1.43	\$2.86	\$40.90	\$52.70	\$6.83

Multiply, in turn, by 3, 30, and 300 :

3. 482	525	633	840	900
4. \$4.82	\$52.50	\$63.30	\$8.40	\$9.00

Multiply, in turn, by 40, 400, 50, and 500 :

5. 736	821	963	742	893
6. \$5.35	\$8.93	\$7.34	\$6.81	\$9.75

7. Multiply \$234.65 by 9 ; by 90 ; by 900 ; by 982.

8. Multiply \$275.40 by 8 ; by 80 ; by 800 ; by 874.

9. Divide \$324.45 by 7 ; by 35 ; by $3\frac{1}{2}$.

10. Divide \$846.60 by 6 ; by 60 ; by 3 ; by $\frac{3}{4}$.

11. At \$85 each, what will 60 mules cost ?

12. At \$55 each, what will 70 cows cost ?

13. How many horses can be bought for \$12,285, if the average price of the horses is \$135 ?

WRITTEN EXERCISE

Add or subtract as indicated :

- | | |
|---|---|
| 1. $2\frac{1}{2} + 3\frac{1}{4} + 4\frac{3}{4}$. | 13. $7\frac{2}{3} + 8\frac{1}{2} + 12\frac{1}{4}$. |
| 2. $2\frac{1}{4} + 9\frac{3}{4} - \frac{1}{8}$. | 14. $15\frac{1}{2} - 2\frac{1}{4} - 1\frac{2}{3}$. |
| 3. $3\frac{1}{2} + 5\frac{1}{4} + 5\frac{3}{4}$. | 15. $5\frac{5}{6} + 12\frac{2}{3} + 21\frac{1}{4}$. |
| 4. $3\frac{1}{4} + 7\frac{1}{8} - 2\frac{1}{3}$. | 16. $22\frac{3}{4} - 3\frac{3}{8} - 2\frac{4}{5}$. |
| 5. $5\frac{1}{2} + 6\frac{3}{4} + 4\frac{3}{4}$. | 17. $7\frac{1}{6} + 18\frac{4}{5} + 23\frac{3}{8}$. |
| 6. $5\frac{1}{4} + 6\frac{1}{8} - 3\frac{3}{8}$. | 18. $17\frac{5}{6} - 2\frac{1}{12} - 3\frac{2}{3}$. |
| 7. $8\frac{1}{2} + 3\frac{3}{8} + 9\frac{3}{4}$. | 19. $8\frac{5}{6} + 21\frac{1}{12} + 20\frac{3}{4}$. |
| 8. $8\frac{7}{8} + 5\frac{3}{8} - 2\frac{5}{8}$. | 20. $23\frac{5}{6} - 3\frac{5}{12} - 2\frac{2}{3}$. |
| 9. $4\frac{1}{2} + 4\frac{7}{8} + 8\frac{3}{5}$. | 21. $7\frac{1}{6} + 18\frac{1}{12} + 22\frac{3}{4}$. |
| 10. $7\frac{7}{8} + 3\frac{5}{8} - 4\frac{1}{5}$. | 22. $33\frac{1}{6} - 2\frac{7}{12} - 3\frac{3}{4}$. |
| 11. $5\frac{1}{2} + 7\frac{5}{8} + 11\frac{1}{3}$. | 23. $9\frac{5}{6} + 15\frac{1}{12} + 13\frac{1}{3}$. |
| 12. $8\frac{7}{8} + 5\frac{2}{5} - 7\frac{1}{5}$. | 24. $41\frac{1}{6} - 3\frac{1}{12} - 2\frac{1}{3}$. |

25. How much greater is $57\frac{1}{4}$ mi. than $29\frac{7}{8}$ mi. ?

26. How much less is $36\frac{7}{8}$ mi. than $90\frac{1}{4}$ mi. ?

27. If a room is $16\frac{1}{8}$ ft. long and $14\frac{1}{2}$ ft. wide, what is the perimeter of the room ?

28. From a board 16 ft. long a man saws off a piece $2\frac{1}{4}$ ft. long and another piece $3\frac{1}{2}$ ft. long. How long is the remaining part ?

29. A boy puts in his basket $3\frac{1}{2}$ lb. of meat, 2 lb. of sugar, and $1\frac{1}{4}$ lb. of figs. The basket weighs $2\frac{1}{8}$ lb. What is the weight of the basket and contents ?

30. If a piece of ribbon $3\frac{3}{4}$ yd. long has another piece sewed to it so as to increase the total length by $3\frac{1}{2}$ yd., and then $5\frac{1}{4}$ yd. is cut off and used for trimming a hat, how many yards of ribbon are left ?

WRITTEN REVIEW DRILL

Add the following :

- | | | | |
|--------------------------------|---------------------------------|----------------------------------|----------------------------------|
| 1. $\frac{1}{2} + \frac{1}{3}$ | $47\frac{1}{2} + 93\frac{1}{3}$ | $72\frac{1}{4} + 83\frac{1}{6}$ | $29\frac{3}{4} + 47\frac{1}{5}$ |
| 2. $\frac{1}{5} + \frac{1}{3}$ | $62\frac{1}{5} + 35\frac{1}{3}$ | $85\frac{4}{5} + 16\frac{2}{3}$ | $87\frac{5}{6} + 19\frac{2}{3}$ |
| 3. $\frac{1}{2} + \frac{1}{6}$ | $67\frac{1}{2} + 29\frac{1}{6}$ | $89\frac{3}{8} + 23\frac{1}{2}$ | $67\frac{4}{5} + 49\frac{1}{2}$ |
| 4. $\frac{4}{5} + \frac{3}{4}$ | $39\frac{4}{5} + 28\frac{3}{4}$ | $87\frac{3}{5} + 62\frac{7}{10}$ | $37\frac{4}{5} + 18\frac{1}{10}$ |

Subtract the following :

- | | | | |
|--------------------------------|---------------------------------|---------------------------------|---------------------------------|
| 5. $\frac{4}{5} - \frac{1}{2}$ | $79\frac{4}{5} - 22\frac{1}{2}$ | $86\frac{2}{3} - 37\frac{1}{2}$ | $48\frac{3}{4} - 23\frac{7}{8}$ |
| 6. $\frac{3}{4} - \frac{2}{5}$ | $92\frac{3}{4} - 37\frac{2}{5}$ | $30\frac{2}{3} - 19\frac{7}{8}$ | $82\frac{1}{2} - 68\frac{7}{8}$ |
| 7. $\frac{2}{5} - \frac{1}{8}$ | $72\frac{2}{5} - 56\frac{1}{8}$ | $41\frac{3}{8} - 16\frac{5}{6}$ | $272 - 46\frac{2}{3}$ |

Multiply the following :

- | | | | |
|-------------------------------|-------------------------------------|------------------------------------|---------------------------------------|
| 8. $27 \times 14\frac{2}{3}$ | $2\frac{1}{2} \times 92\frac{1}{2}$ | $2\frac{1}{2} \times 3\frac{1}{4}$ | $74\frac{2}{5} \times 65$ |
| 9. $18 \times 96\frac{3}{4}$ | $5\frac{1}{2} \times 68$ | $4\frac{7}{8} \times 2\frac{1}{2}$ | $40\frac{1}{3} \times 96$ |
| 10. $85 \times 32\frac{2}{5}$ | $1\frac{3}{4} \times 96$ | $6\frac{3}{8} \times 4\frac{1}{2}$ | $21\frac{7}{8} \times 35\frac{1}{2}$ |
| 11. $10 \times 12\frac{1}{2}$ | $6\frac{1}{3} \times 7\frac{1}{8}$ | $5\frac{1}{3} \times 3\frac{1}{2}$ | $27\frac{1}{5} \times 4\frac{1}{4}$ |
| 12. $65 \times 94\frac{1}{5}$ | $29 \times 35\frac{1}{4}$ | $5\frac{1}{4} \times 6\frac{2}{3}$ | $42\frac{1}{2} \times 408\frac{2}{3}$ |

Divide the following :

- | | | | |
|-------------------------------------|---------------------------------|----------------------------------|----------------------------------|
| 13. $\frac{2}{3} \div \frac{3}{8}$ | $\frac{3}{8} \div \frac{2}{3}$ | $\frac{7}{8} \div \frac{4}{5}$ | $2\frac{4}{5} \div \frac{7}{8}$ |
| 14. $\frac{5}{8} \div \frac{3}{4}$ | $\frac{3}{4} \div \frac{5}{8}$ | $\frac{3}{4} \div \frac{7}{8}$ | $3\frac{3}{4} \div 5\frac{1}{2}$ |
| 15. $\frac{5}{8} \div 2\frac{1}{5}$ | $\frac{5}{8} \div \frac{2}{3}$ | $\frac{2}{3} \div 4\frac{2}{3}$ | $7\frac{3}{8} \div 1\frac{3}{4}$ |
| 16. $\frac{2}{3} \div 3\frac{1}{3}$ | $\frac{1}{3} \div \frac{2}{3}$ | $\frac{4}{5} \div 3\frac{1}{2}$ | $5\frac{5}{8} \div 1\frac{1}{8}$ |
| 17. $\frac{1}{5} \div \frac{1}{3}$ | $\frac{1}{3} \div 1\frac{1}{5}$ | $\frac{9}{10} \div 2\frac{1}{3}$ | $2\frac{4}{5} \div 3\frac{1}{2}$ |

Find the value of :

- | | |
|--|--|
| 18. $\frac{3}{5}$ of $\frac{4}{5}$ of $\frac{1}{2}$ of 1 cu. ft. | $\frac{5}{8}$ of $\frac{4}{5}$ of $\frac{2}{3}$ of 1 cu. in. |
| 19. $\frac{1}{2}$ of 5×8 sq. ft. | $\frac{3}{5}$ of 15×48 sq. ft. |
| 20. $\frac{7}{8}$ of 4×16 sq. in. | $\frac{3}{7}$ of 13×14 sq. in. |

ORAL EXERCISE

1. What part of 4 is 2? What part of 10 is 8? How do you find what part one number is of another?

2. What part of 7 is 5? of 9 is 4? of 7 is 6? of 16 is 12? of 25 is $12\frac{1}{2}$? of 100 is $33\frac{1}{3}$? of 100 is $16\frac{2}{3}$?

89. Fractional Parts. From these answers we have:

To find what part the second of two numbers is of the first, divide the second by the first.

For example, what part of 51 is 17? 17 is $\frac{1}{3}\frac{1}{7}$, or $\frac{1}{3}$, of 51.

WRITTEN EXERCISE

The second of the following numbers is what part of the first?

- | | | |
|---------------------------------------|---|---|
| 1. 14, 8. | 9. $66\frac{2}{3}$, $16\frac{2}{3}$. | 17. 225, $37\frac{1}{2}$. |
| 2. 18, 14. | 10. 243, 27. | 18. $22\frac{1}{2}$, $16\frac{2}{3}$. |
| 3. $\frac{7}{8}$, $\frac{2}{3}$. | 11. 100, $37\frac{1}{2}$. | 19. $40\frac{1}{8}$, $21\frac{2}{5}$. |
| 4. $\frac{2^2}{7}$, $\frac{6}{11}$. | 12. $45\frac{3}{5}$, $7\frac{3}{5}$. | 20. $37\frac{1}{2}$, $33\frac{1}{3}$. |
| 5. $3\frac{1}{3}$, $1\frac{2}{3}$. | 13. $16\frac{5}{8}$, $2\frac{3}{8}$. | 21. 421, $60\frac{1}{7}$. |
| 6. 17, $8\frac{1}{2}$. | 14. $42\frac{1}{2}$, $14\frac{1}{6}$. | 22. $33\frac{1}{3}$, $16\frac{2}{3}$. |
| 7. 82, $9\frac{1}{9}$. | 15. $62\frac{1}{8}$, $7\frac{1}{8}$. | 23. 427, $47\frac{4}{9}$. |
| 8. $41\frac{3}{5}$, $5\frac{1}{5}$. | 16. $66\frac{2}{3}$, $33\frac{1}{3}$. | 24. 325, $29\frac{6}{11}$. |

25. Frank is 12 yr. old and his brother is 18 yr. old. Frank's age is what part of his brother's?

26. The age of Frank's brother, who is 18 yr. old, is what part of his uncle's, who is 54 yr. old?

27. If Frank is 12 yr. old and his brother is 18 yr. old, Frank's age was what part of his brother's 6 yr. ago? It will be what part of his brother's age 6 yr. from now?

ORAL EXERCISE

1. 10 is $\frac{1}{2}$ of what number? $\frac{1}{3}$ of what number?
2. 12 is $\frac{1}{3}$ of what number? If 12 is $\frac{2}{3}$ of some number, what is $\frac{1}{3}$ of the number? What is $\frac{2}{3}$ of the number?
3. 15 is $\frac{3}{7}$ of some number. What is $\frac{1}{7}$ of the number? What is the number?

90. Given a Part of a Number to find the Number. If $33\frac{1}{3}$ is $\frac{2}{3}$ of some number, what is the number?

Because $\frac{2}{3}$ of some number $= 33\frac{1}{3} = \frac{100}{3}$,
 therefore $\frac{1}{3}$ of the number $= \frac{1}{2}$ of $\frac{100}{3} = \frac{50}{3}$,
 and $\frac{3}{3}$ of the number $= 3$ times $\frac{50}{3} = 50$.

Therefore $33\frac{1}{3}$ is $\frac{2}{3}$ of 50.

WRITTEN EXERCISE

Find the number of which:

- | | | |
|--------------------------|--------------------------------------|--------------------------------|
| 1. 23 is $\frac{1}{3}$. | 5. 45 is $\frac{9}{10}$. | 9. 144 is $\frac{9}{16}$. |
| 2. 38 is $\frac{2}{3}$. | 6. 150 is $\frac{3}{5}$. | 10. \$150 is $\frac{3}{5}$. |
| 3. 63 is $\frac{7}{8}$. | 7. 222 is $\frac{6}{7}$. | 11. \$2.25 is $\frac{5}{8}$. |
| 4. 75 is $\frac{3}{8}$. | 8. 125 is $\frac{2}{3}\frac{5}{2}$. | 12. \$10.25 is $\frac{5}{9}$. |
13. If there are 85 girls in a school, and they are $\frac{5}{9}$ of the total number of pupils, how many pupils are there?
 14. In my library there are 164 books on history, and these are $\frac{2}{17}$ of all my books. How many books have I?
 15. If a man spends \$1687.50 this year, and this is $\frac{3}{4}$ of his income, how much is his income?
 16. A man left to his son $\frac{3}{5}$ of his estate, this share amounting to \$3210. How much was the estate worth?

ORAL EXERCISE

1. What will $2\frac{1}{2}$ lb. of coffee cost at 24¢ a pound?
2. What will $\frac{3}{4}$ lb. of pepper cost at 24¢ a pound?
3. What will $2\frac{1}{2}$ lb. of butter cost at 30¢ a pound?
4. What will 5 oz. of cinnamon cost at 32¢ a pound?
5. What will 20 lb. of loaf sugar cost at $6\frac{1}{4}\text{¢}$ a pound?
6. A grocer buys 240 cans of corn and sells half of them. How many cans of corn does he sell?
7. A merchant buys 200 yd. of cloth and sells a quarter of it. How many yards of cloth does he sell?
8. What will $1\frac{1}{2}$ lb. of cocoa cost at 22¢ per half-pound can?

WRITTEN EXERCISE

1. At \$1.38 a yard, what will 37 yd. of carpet cost?
2. What is the cost of $6\frac{1}{4}$ lb. of mackerel @ 16¢ , $3\frac{1}{2}$ lb. of codfish @ 12¢ , and 4 cans of salmon at the rate of 2 for a quarter?
3. I bought $\frac{1}{2}$ gal. of olive oil at \$3.30 a gallon, but $\frac{1}{4}$ of what I bought leaked out. What rate per gallon do I pay for what is left?
4. In buying 48 cans of tomatoes, how much is gained by buying two cases of 2 doz. cans each, at \$2.89 a case, over buying at the rate of 3 cans for 40¢ ?
5. I bought 3 gal. of best maple sirup @ $\$1.12\frac{1}{2}$ a gallon, $2\frac{1}{2}$ gal. New Orleans molasses @ 62¢ a gallon, and gave the grocer a \$5 bill. How much change did I get?

Teachers should send the pupils to local stores to ascertain prices, and these prices should be used in the above and similar problems.

II. CANCELLATION

91. Cancellation. We have seen, on pages 39 and 42, that we may divide both terms of a fraction by the same number without changing the value of the fraction; that is, we may *cancel* any divisor that is found in both terms.

Teachers should make it clear that this is a case of *division*; that we cannot *subtract* the same number from both terms of the fraction.

In order to divide both terms of the fraction by the same number we must know more about different divisors.

92. Common Divisor. A factor that is common to two or more numbers is called a *common divisor*.

For example, 5 is a common divisor of 15 and 25. It might be called a common factor, but common divisor is the usual name.

93. Greatest Common Divisor. The greatest factor common to two or more numbers is called their *greatest common divisor* (G.C.D.). For example, although 3 is a common divisor of 12, 18, and 48, the number 6 is the *greatest* common divisor.

94. Finding the G.C.D. The greatest common divisor of such numbers as we have met or shall meet in fractions is easily found. For example, consider 12 and 30. We see that

$$12 = 2 \times 3 \times 2, \qquad 30 = 2 \times 3 \times 5,$$

and that 2 and 3 are the only common factors. Therefore 2×3 , or 6, is the greatest common divisor.

95. Prime Number. A number that has no factors except itself and 1 is called a *prime number*. For example, 7, 11, and 17 are prime numbers.

96. Composite Number. A number not prime is called *composite*. For example, 25, 39, and 51 are composite numbers.

97. Even Number. A number that contains the factor 2 is called an *even number*. For example, 20, 38, and 1112 are even numbers.

98. Odd Number. A number that is not even is called an *odd number*. For example, 21, 39, and 127 are odd numbers.

99. Prime Factor. A factor that is a prime number is called a *prime factor*. For example, the prime factors of 68 are 2, 2, and 17, and the prime factors of 25 are 5 and 5.

100. Mutually Prime Numbers. Numbers that have no common factors are said to be *prime to each other*. For example, 12 and 35.

ORAL EXERCISE

1. What do you mean by the factors of a number?
2. Name two factors of 35; of 77; of 26; of 34.
3. Name three factors of 30; of 70; of 66; of 110.
4. Name a factor that is common to 35 and 45.
5. Name two factors common to 30 and 48.
6. Find a common divisor of 20 and 22; of 16 and 36.
7. Find the greatest common divisor of 75 and 100.
8. Which of the following numbers have no factors except 1 and themselves: 4, 5, 6, 7, 10, 11, 12, 13?
9. In reducing $\frac{1\frac{2}{16}}$ to lowest terms is it sufficient to cancel the common divisor 2? What kind of common divisor must be canceled in reducing a fraction to lowest terms?
10. Find the odd numbers and the prime numbers from 1 to 30.
11. Find the prime factors of 20; of 48; of 60; of 77; of 50.
12. Find the greatest common divisor of 8, 12, 20, and 24.
13. Find the greatest common divisor of 9, 15, 21, and 60.

101. Divisible Numbers. When we speak of one number being *divisible* by another we mean that the quotient is a whole number.

102. Divisibility by 2. *A number is divisible by 2 if the units are so divisible.* For example, 64, or $60 + 4$, must be divisible by 2 if 4, the number of units, is divisible by 2, because any multiple of 10 is divisible by 2.

103. Divisibility by 5. *A number is divisible by 5 if it ends in 0 or 5.* For example, 180 and 275 are divisible by 5.

104. Divisibility by 3. *A number is divisible by 3 if the sum of its digits is so divisible.* For example, the sum of the digits of 411 is 6, and 6 is divisible by 3. Hence 411 is also divisible by 3.

ORAL EXERCISE

1. Is 2 a factor of 14? of 27? of 50? of 45? of 600? How do you tell whether 2 is a factor of a number?

2. Is 2 a factor of 10? of any number of 10's? Then is it a factor of any number if it is a factor of the units?

3. Is 5 a factor of 10? of any number of 10's? Then is it a factor of any number if it is a factor of the units?

WRITTEN EXERCISE

1. From this list select the numbers divisible by 2:

660 4907 6255 3027 1356 37,268

2. From this list select the numbers divisible by 3:

4867 7075 9636 8124 3672 12,475

3. From this list select the numbers divisible by 5:

1235 5778 9100 8328 3445 31,035

105. Other Tests of Divisibility. There are various tests of divisibility besides those on page 103. They are easily illustrated or explained, and may be given or not as the teacher prefers. The more important are the following:

A number is divisible by 4 if the number represented by its two right-hand figures is so divisible.

A number is divisible by 6 if it is even and if the sum of its digits is divisible by 3.

A number is divisible by 8 if the number represented by its three right-hand figures is so divisible.

A number is divisible by 9 if the sum of its digits is so divisible.

A number is divisible by 11 if the difference between the sums of the digits in its even and odd places is so divisible.

For example, 430,507 is divisible by 11, for $7 + 5 + 3 = 15$, $0 + 0 + 4 = 4$, and $15 - 4 = 11$.

ORAL EXERCISE

1. State the prime factors of 15, 18, 27, 35, 42, and 75.
2. State one factor of 395, 123, 777, 692, 1275, and 1263.
3. Is 5 a factor of 20? of 78? of 45? of 72? of 800?
4. Write all the odd numbers and all the prime numbers from 1 to 100.
5. Select the numbers divisible by 2, 3, and 5 respectively:
4263 7065 2022 3708 5445 13,005
6. Select the numbers divisible by 4, 6, and 8 respectively:
3104 3204 3304 7275 3116 54,012
7. Select the numbers divisible by 9 and 11 respectively:
5004 1001 9009 6264 2244 12,342

106. Cancellation Extended. Since a fraction is an indicated division, *equal factors may be canceled from both dividend and divisor without changing the quotient.*

1. Divide $2 \times 7 \times 15$ by $21 \times 5 \times 6$.

Writing this as a fraction, 2 is canceled from 2 and 6; 7 is canceled from 7 and 21; 3 is canceled from 3 (left from the 21) and 15; 5 is canceled from 5 (left from the 15) and 5.

Therefore the quotient is $\frac{1}{3}$, for evidently 1 is a factor of the numerator.

$$\frac{\cancel{2} \times \cancel{7} \times \overset{\cancel{3}}{\cancel{15}}}{\underset{\cancel{3}}{\cancel{21}} \times \cancel{5} \times \underset{\cancel{3}}{\cancel{6}}} = \frac{1}{3}$$

2. What is the product of $\frac{1}{15}$, $45\frac{1}{2}$, $33\frac{1}{3}$, and $16\frac{7}{8}$?

Reducing to improper fractions, $45\frac{1}{2} = \frac{91}{2}$, $33\frac{1}{3} = \frac{100}{3}$, $16\frac{7}{8} = \frac{135}{8}$. Proceeding as before, the product is $1706\frac{1}{4}$.

$$\frac{1 \times \overset{25}{91} \times \overset{27}{100} \times \overset{3}{135}}{\underset{\cancel{3}}{\cancel{15}} \times 2 \times \underset{2}{\cancel{3}} \times \underset{\cancel{8}}{\cancel{8}}} = \frac{6825}{4} = 1706\frac{1}{4}$$

WRITTEN EXERCISE

Divide, using cancellation:

- $7 \times 21 \times 77 \times 6$ by $49 \times 33 \times 42$.
- $20 \times 25 \times 36 \times 48$ by $27 \times 40 \times 15 \times 12$.
- $32 \times 27 \times 44 \times 39$ by $55 \times 72 \times 26 \times 8 \times 3$.
- $125 \times 108 \times 51 \times 8$ by $40 \times 45 \times 85 \times 12 \times 3$.

Multiply, using cancellation:

- $13\frac{1}{3} \times 32\frac{1}{5} \times 16\frac{7}{8}$.
- $27\frac{1}{5} \times 24\frac{1}{2} \times 37\frac{1}{2}$.
- $37\frac{1}{2} \times 22\frac{1}{4} \times 12\frac{4}{5}$.
- $15\frac{2}{3} \times 16\frac{7}{8} \times 33\frac{1}{3}$.
- $42\frac{2}{3} \times 66\frac{2}{3} \times 74\frac{1}{4}$.
- $7\frac{3}{5} \times 2\frac{1}{7} \times 5\frac{1}{4} \times 21\frac{1}{3}$.
- $6\frac{3}{4} \times 7\frac{3}{4} \times 9\frac{1}{3} \times 12\frac{1}{3}$.
- $9\frac{3}{5} \times 6\frac{2}{3} \times 3\frac{1}{2} \times 15\frac{2}{3}$.

Multiply, using cancellation when possible :

13. $18 \times \frac{3}{4}$.

18. $\frac{3}{8}$ of 26.

23. $\frac{3}{8}$ of $\frac{4}{9}$.

14. $27 \times \frac{5}{12}$.

19. $\frac{7}{8}$ of 34.

24. $\frac{7}{8}$ of $\frac{3}{4}$.

15. $32 \times \frac{7}{8}$.

20. $\frac{3}{5}$ of 35.

25. $\frac{7}{8}$ of $2\frac{1}{4}$.

16. $48 \times \frac{3}{8}$.

21. $\frac{3}{8}$ of 52.

26. $\frac{5}{6}$ of $\frac{3}{10}$.

17. $56 \times \frac{7}{12}$.

22. $\frac{5}{6}$ of 33.

27. $\frac{6}{7}$ of $\frac{7}{12}$.

Using cancellation when possible, find :

28. $\frac{2}{3}$ of $\frac{3}{8}$.

33. $\frac{3}{7}$ of $1\frac{4}{9}$.

38. $\frac{1}{5}$ of $1\frac{2}{3}$.

29. $\frac{5}{8}$ of $\frac{4}{9}$.

34. $\frac{6}{7}$ of $1\frac{4}{9}$.

39. $\frac{3}{5}$ of $6\frac{2}{3}$.

30. $\frac{3}{5}$ of $1\frac{5}{16}$.

35. $\frac{5}{6}$ of $\frac{3}{16}$.

40. $\frac{7}{8}$ of $4\frac{4}{5}$.

31. $\frac{4}{5}$ of $1\frac{5}{16}$.

36. $\frac{5}{6}$ of $2\frac{9}{10}$.

41. $\frac{5}{6}$ of $4\frac{3}{8}$.

32. $\frac{5}{8}$ of $1\frac{4}{5}$.

37. $\frac{5}{8}$ of $2\frac{4}{11}$.

42. $\frac{5}{8}$ of $9\frac{3}{5}$.

Find the products of :

43. $\frac{2}{3}$ of $\frac{3}{8}$ of $\frac{4}{5}$.

48. $\frac{7}{8}$ of $1\frac{1}{7} \times 3\frac{2}{3} \times \frac{3}{22}$.

44. $\frac{7}{8}$ of $\frac{4}{5}$ of $1\frac{5}{14}$.

49. $1\frac{2}{3} \times 3\frac{1}{5} \times 2\frac{1}{2} \times 5\frac{1}{4}$.

45. $\frac{5}{8}$ of $\frac{2}{5}$ of $\frac{4}{9}$ of $\frac{3}{5}$.

50. $1\frac{7}{8} \times 3\frac{1}{2} \times 2\frac{3}{5} \times 3\frac{1}{4}$.

46. $\frac{3}{8}$ of $\frac{2}{3}$ of $\frac{3}{5}$ of $\frac{5}{6}$.

51. $1\frac{2}{3} \times 2\frac{2}{5} \times 3\frac{1}{2} \times 4\frac{3}{5}$.

47. $\frac{5}{8}$ of $1\frac{3}{5} \times 2\frac{1}{2} \times \frac{3}{5}$.

52. $2\frac{2}{5} \times 5\frac{1}{2} \times 10 \times 3\frac{3}{8}$.

Using cancellation when possible, find :

53. $14 \div \frac{7}{8}$.

60. $\frac{4}{5} \div \frac{2}{3}$.

67. $3\frac{3}{5} \div 9$.

54. $25 \div 1\frac{5}{16}$.

61. $\frac{3}{5} \div 1\frac{9}{10}$.

68. $9 \div 7\frac{1}{5}$.

55. $\frac{3}{8} \div \frac{4}{5}$.

62. $\frac{7}{8} \div 1\frac{5}{16}$.

69. $18 \div 4\frac{1}{2}$.

56. $\frac{5}{9} \div 1\frac{5}{16}$.

63. $1\frac{5}{16} \div \frac{7}{8}$.

70. $12\frac{1}{2} \div 3\frac{1}{8}$.

57. $\frac{3}{4} \div 1\frac{9}{10}$.

64. $\frac{3}{8} \div 1\frac{9}{10}$.

71. $16\frac{2}{3} \div 4\frac{1}{6}$.

58. $70\frac{2}{3} \div 8$.

65. $\frac{5}{8} \div 25$.

72. $37\frac{1}{2} \div 12\frac{1}{2}$.

59. $\frac{5}{8} \div 1\frac{3}{16}$.

66. $25 \div \frac{5}{8}$.

73. $87\frac{1}{2} \div 37\frac{1}{2}$.

III. DECIMAL FRACTIONS

107. The Meaning of Decimals. You have learned that \$3.25 means 3 dollars + 25 cents, or $\$3 + \$\frac{25}{100}$.

Just as	\$3.25	means	$\$3 + \$\frac{25}{100}$,
so	3.25 ft.	means	$3 \text{ ft.} + \frac{25}{100} \text{ ft., or } 3\frac{1}{4} \text{ ft.};$
	4.75 in.	means	$4 \text{ in.} + \frac{75}{100} \text{ in., or } 4\frac{3}{4} \text{ in.};$
	2.5	means	$2 + \frac{5}{10}, \text{ or } 2\frac{1}{2};$
	.5, or 0.5,	means	$\frac{5}{10}, \text{ or } \frac{50}{100}, \text{ or } 0.50, \text{ or } \frac{1}{2};$
	.06, or 0.06,	means	$\frac{6}{100}, \text{ or } \frac{3}{50};$
and	.125, or 0.125,	means	$\frac{125}{1000}, \text{ or } \frac{1}{8}.$

108. Decimal Fractions. A fraction whose denominator is not written but is some power of 10 is called a *decimal fraction*.

Thus 0.5 (5 tenths) is a decimal fraction. If written in the form $\frac{5}{10}$ it becomes a common fraction.

The expression 0.5 means the same as .5. The 0 may be written or not, as we please. Those who have much computing to do usually find it better to write the 0 before .5, except where the numbers are written in columns, so there shall be no misunderstanding. It is perfectly proper to omit it, however, as is always done when we have columns that are ruled for dollars and cents, and as is often the case in printed numbers. You should accustom yourself to both forms, for you will see both in newspapers and books.

Decimal fractions are usually called *decimals*. The period placed at the left of tenths is called the *decimal point*.

We have seen that the decimal point is written between dollars and dimes, as in the case of \$2.75. Indeed, all the work with decimal fractions is just the same as that with which we are familiar in dealing with numbers representing money.

109. Reading Decimals. The names of the decimal places most frequently used are given in the following scheme :

Thousands	Hundreds	Tens	Units	(Decimal point)	Tenths	Hundredths	Thousandths	Ten-thousandths
8	3	9	5	.	2	7	6	5

This number is read "eight thousand, three hundred ninety-five and two thousand seven hundred sixty-five ten-thousandths." The orders beyond ten-thousandths are hundred-thousandths, millionths, ten-millionths, and so on. Practically, we do not have much need for decimals beyond thousandths.

Pupils should be given much exercise in reading such numbers from the blackboard, avoiding the use of "and" except between the units and tenths, and the vulgarism of "aught" for "naught."

110. Writing Decimals. Although we write a decimal fraction without its denominator, the denominator is shown by the decimal point and is indicated in reading.

Thus 0.9 has no printed denominator, but when we say "nine tenths" we indicate the fact that the denominator is ten, as well as state that the numerator is nine.

Therefore, *write only the numerator of the fraction, inserting such zeros as are necessary to indicate the denominator, and place the decimal point before the tenths.*

Thus, to write in figures two hundred one hundred-thousandths, we write 201 and prefix two zeros, preceded by a decimal point, so that the last figure is in hundred-thousandths' place. We then have 0.00201.

ORAL EXERCISE

Read the following numbers :

- | | | | |
|----------|------------|------------|--------------|
| 1. 1.25. | 4. 6.33. | 7. 0.375. | 10. 100.025. |
| 2. 3.78. | 5. 0.125. | 8. 67.005. | 11. 91.075. |
| 3. 2.95. | 6. 48.675. | 9. 32.375. | 12. 300.075. |

13. What is the name of the place to the right of tenths ? to the right of that ? What is the name of the fourth decimal place ? the fifth ? the sixth ?

Since we rarely read the names of numbers beyond three decimal places, even when we write them, the chief drill should be on reading decimal fractions like those in Exs. 1-12.

Read the following numbers :

- | | | | |
|-------------|--------------|---------------|----------------|
| 14. 16.004. | 16. 82.3745. | 18. 91.82734. | 20. 68.998232. |
| 15. 75.137. | 17. 12.3147. | 19. 16.42398. | 21. 14.763458. |

WRITTEN EXERCISE

Write the following numbers in figures :

1. Three tenths ; five tenths ; nine tenths.
2. Four and four tenths ; seven and eight tenths.
3. Seventy-five hundredths ; thirty-three hundredths.
4. One hundred forty-four thousandths ; six hundred seventy-five thousandths ; four ten-thousandths.
5. Five and seven thousandths ; eight and forty-two thousandths ; twenty-five thousandths ; one millionth.
6. Twelve hundredths ; fifty-five hundredths ; nine hundredths ; one hundredth ; twenty hundredths.
7. One hundred twenty-five thousandths ; forty-seven thousandths ; ten thousandths ; five thousandths.

111. Annexing Zeros to Decimals. Because we know that the fraction $\frac{5}{10} = \frac{50}{100}$, or $0.5 = 0.50$, we see that

Annexing a zero to a decimal fraction does not change its value.

Decimals that have the same number of decimal places are called *similar decimals*.

Therefore, *to change dissimilar decimals to similar decimals, give them the same number of decimal places by annexing or cutting off zeros.*

For example, change 0.25, 2.125, and 3.5 to similar decimals. We see that

$$0.25 = 0.250 \qquad 2.125 = 2.125, \qquad 3.5 = 3.500,$$

and 0.250, 2.125, and 3.500 are similar decimals.

WRITTEN EXERCISE

Change to similar decimals:

- | | | |
|-----------------|------------------|------------------|
| 1. 0.2, 0.17. | 8. 0.6, 0.123. | 15. 0.12, 0.103. |
| 2. 0.8, 0.08. | 9. 0.5, 0.700. | 16. 0.72, 0.008. |
| 3. 0.7, 0.002. | 10. 0.4, 0.050. | 17. 0.69, 0.070. |
| 4. 0.9, 0.130. | 11. 0.1, 0.357. | 18. 0.30, 0.700. |
| 5. 0.6, 0.600. | 12. 0.5, 0.50. | 19. 0.7, 0.70. |
| 6. 0.60, 0.600. | 13. 0.5, 0.500. | 20. 0.7, 0.700. |
| 7. 0.6, 0.60. | 14. 0.50, 0.500. | 21. 0.70, 0.700. |
22. Express as hundredths: 0.6; 0.7; 0.9; 0.750.
 23. Express as tenths: 0.70; 0.90; 0.500; 0.900; 0.800.
 24. Express as thousandths: 0.1; 0.3; 0.10; 0.30; 0.7; 0.70.
 25. Compare 0.2 and 0.20; 0.2 and 0.200; 0.20 and 0.200.
 26. Compare 2.5 and 2.50; $2\frac{1}{2}$ and 2.50; $2\frac{1}{2}$ and 2.500.

112. Changing Decimals to Common Fractions. Because the fraction $0.25 = \frac{25}{100}$, which equals $\frac{1}{4}$, therefore

To change a decimal to a common fraction, write the numerator and the denominator of the decimal fraction and then reduce the common fraction to lowest terms.

In a case like that of $0.33\frac{1}{3}$ we proceed in the same way, thus:

$$0.33\frac{1}{3} = \frac{33\frac{1}{3}}{100} = \frac{\frac{100}{3}}{100} = \frac{1}{3}.$$

ORAL EXERCISE

Read these decimals as common fractions in lowest terms:

- | | | | | |
|---------|----------|---------|-----------|-----------|
| 1. 0.1. | 4. 0.9. | 7. 0.2. | 10. 0.25. | 13. 0.40. |
| 2. 0.3. | 5. 0.11. | 8. 0.4. | 11. 0.5. | 14. 0.80. |
| 3. 0.7. | 6. 0.17. | 9. 0.6. | 12. 0.50. | 15. 0.75. |

WRITTEN EXERCISE

Change to common fractions in lowest terms or to mixed numbers:

- | | | | |
|----------|----------|------------|-------------|
| 1. 0.8. | 5. 0.35. | 9. 0.125. | 13. 1.25. |
| 2. 0.60. | 6. 0.65. | 10. 0.375. | 14. 2.75. |
| 3. 0.20. | 7. 0.45. | 11. 0.625. | 15. 5.125. |
| 4. 0.40. | 8. 0.55. | 12. 0.875. | 16. 10.375. |
17. Express as hundredths: 0.3, 0.5, 0.1, 0.8, 4, 63.
 18. Express as thousandths: 0.5, 0.25, 0.37, 0.09, 31.
 19. Express as hundredths: 0.470, 0.680, 0.100, 0.040, 7.8.
 20. Express 0.125 as eighths. Find 0.125 of \$1600.
 21. Express $0.33\frac{1}{3}$ as thirds. Find $0.33\frac{1}{3}$ of 621.
 22. Express $0.37\frac{1}{2}$ as eighths. Find $0.37\frac{1}{2}$ of 1768.

113. Changing Common Fractions to Decimals. Since a common fraction indicates division (pages 36, 46), we may divide the numerator by the denominator and thus change the fraction to the decimal form. For example, consider the fraction $\frac{3}{8}$.

We have found that we may annex zeros after the decimal point without changing the value; that is, $3 = 3.000$. We may now divide exactly as in United States money, and the result is 0.375.

That this is allowable may be seen from the following: 3.0 means 30 tenths, and $30 \text{ tenths} \div 8 = 3 \text{ tenths}$ with remainder 6 tenths. But 6 tenths = 60 hundredths ($\frac{6}{10} = \frac{60}{100}$), and $60 \text{ hundredths} \div 8 = 7 \text{ hundredths}$ with remainder 4 hundredths. But 4 hundredths = 40 thousandths, and $40 \text{ thousandths} \div 8 = 5 \text{ thousandths}$. Therefore $\frac{3}{8} = 0.375$.

$$\begin{array}{r} 8 \overline{) 3.000} \\ \underline{0.375} \end{array}$$

Therefore, *to reduce a common fraction to a decimal, place a decimal point after the numerator and divide by the denominator, as in United States money.*

A common fraction cannot always be reduced to an exact decimal; thus $\frac{2}{3} = 0.6\frac{2}{3} = 0.66\frac{2}{3} = 0.666\frac{2}{3}$. It is customary to write a common fraction or the plus sign to indicate that the work is not complete; thus $\frac{2}{3} = 0.666\frac{2}{3}$, 0.666..., or 0.666+. This is often written simply 0.666, using as many places as necessary.

WRITTEN EXERCISE

Change to decimals, carrying the reduction only to thousandths in the case of inexact decimals:

- | | | | | |
|--------------------|----------------------|-----------------------|----------------------|------------------------|
| 1. $\frac{1}{2}$. | 7. $\frac{7}{8}$. | 13. $\frac{5}{16}$. | 19. $\frac{1}{3}$. | 25. $2\frac{11}{25}$. |
| 2. $\frac{1}{4}$. | 8. $\frac{1}{5}$. | 14. $\frac{7}{16}$. | 20. $\frac{2}{3}$. | 26. $4\frac{9}{75}$. |
| 3. $\frac{1}{8}$. | 9. $\frac{2}{5}$. | 15. $\frac{9}{16}$. | 21. $\frac{1}{6}$. | 27. $0.1\frac{5}{8}$. |
| 4. $\frac{3}{4}$. | 10. $\frac{3}{5}$. | 16. $\frac{13}{16}$. | 22. $\frac{5}{6}$. | 28. $8.2\frac{5}{8}$. |
| 5. $\frac{3}{8}$. | 11. $\frac{4}{5}$. | 17. $\frac{1}{32}$. | 23. $\frac{7}{25}$. | 29. $4.3\frac{3}{5}$. |
| 6. $\frac{5}{8}$. | 12. $\frac{1}{16}$. | 18. $\frac{5}{32}$. | 24. $\frac{9}{25}$. | 30. $8.1\frac{3}{4}$. |

114. Adding and Subtracting Decimals. We have learned how to add and subtract such numbers as \$1.25 and \$2.10. We proceed in the same way with decimal fractions.

To add or subtract decimal fractions, write like orders under one another and add or subtract as with whole numbers.

1. Add 3.06, 9.475, and 0.1.

If the decimal points are in a column, units will come under units, tenths under tenths, and so on. Adding as with numbers representing dollars and cents or with whole numbers the sum is 12.635.

$$\begin{array}{r} 3.06 \\ 9.475 \\ 0.1 \\ \hline 12.635 \end{array}$$

2. To subtract 0.26 from 4.1.

Arranging the decimal points in a column and writing 4.1 as 4.10, the subtraction is performed in the same way as with United States money. The remainder is easily found to be 3.84.

$$\begin{array}{r} 4.10 \\ 0.26 \\ \hline 3.84 \end{array}$$

WRITTEN EXERCISE

Add the following:

1.	2.	3.	4.	5.
\$2.47	\$15.25 $\frac{1}{2}$	28.92	68.23	32.301
3.62	7.42 $\frac{1}{2}$	6.87	.09 $\frac{1}{4}$	41.025
<u>4.81</u>	<u>19.00</u>	<u>9.09</u>	<u>7.40</u>	<u>7.875</u>

Add, and also subtract:

6.	7.	8.	9.	10.
80.25 $\frac{1}{2}$	67.25	49.63	35.70	67.02
<u>40.07$\frac{1}{2}$</u>	<u>31.50</u>	<u>22.78</u>	<u>12.95</u>	<u>9.80</u>
11.	12.	13.	14.	15.
42.57	48.05	540.00	128.42	142.00
<u>6.64</u>	<u>6.97</u>	<u>.67</u>	<u>37.06</u>	<u>64.92</u>

ORAL EXERCISE

1. A small geography cost \$0.60 and a music book \$0.35. How much did the two cost together?

2. An arithmetic cost \$0.65 and a paint box \$0.35. How much did the two cost together?

3. A history cost \$0.60. I gave the dealer a 50-cent piece and a quarter of a dollar. How much change did I receive?

4. I bought a large geography for \$1.25 and gave the dealer a dollar and a half. How much change did I receive?

5. How much more did the large geography cost than the small geography? How much more than the arithmetic?

WRITTEN EXERCISE

1. The three sides of a triangle are 2.25 in., 3.8 in., and 2.95 in. What is the perimeter?

2. Three pieces of land have areas as follows: 14.85 A., 26.35 A., and 49.8 A. What is the total area?

3. A man has a field of 38.75 A. He takes 1.2 A. for a garden and 9.8 A. for a pasture. How much is left?

Add the following:

4. ✓	5.	6.	7.	8.
72.86	22.37	3.725	412.5	7.283
32.96	43.96	4.683	328.7	.796
41.83	77.63	5.928	587.5	.584
82.09	56.04	7.635	671.3	.963
73.57	29.37	4.009	428.9	4.020
61.23	92.08	5.700	571.1	5.000
4.72	70.63	6.302	266.6	.778
<u>16.8</u>	<u>7.92</u>	<u>5.777</u>	<u>733.4</u>	<u>.610</u>

Add, and also subtract :

9. 487.3 <u>209.6</u>	10. 52.37 <u>36.99</u>	11. 4.003 <u>2.876</u>	12. 2.803 <u>.196</u>	13. 1.093 <u>.027</u>
14. 22.05 <u>13.75</u>	15. 39.26 <u>13.54</u>	16. 43.01 <u>34.17</u>	17. 25.27 $\frac{1}{2}$ <u>16.42$\frac{1}{4}$</u>	18. 34.33 $\frac{1}{3}$ <u>21.66$\frac{2}{3}$</u>
19. 248.7 <u>178.8</u>	20. 32.4 <u>18.05</u>	21. 5.071 <u>2.683</u>	22. 4.638 <u>.989</u>	23. 2.708 <u>1.819</u>
24. 801.3 <u>628.4</u>	25. 51.08 <u>28.72</u>	26. 9.011 <u>6.232</u>	27. 5.117 <u>2.308</u>	28. 5.620 <u>.875</u>
29. 912.13 <u>426.48</u>	30. 22.38 <u>19.79</u>	31. 8.112 <u>4.348</u>	32. 6.213 <u>.487</u>	33. 8.112 <u>6.023</u>

34. What number must be added to 7.96 to make 19.8 ?

35. What number must be taken from 17.5 to leave 6.75 ?

36. What is the result of adding 4.93 to 17.89 ?

37. What is the result of subtracting 5.98 from 11.13 ?

38. What is the result of adding 2.69 to 3.968 ?

39. The distance from New York to Rahway is 20.8 mi., and from New York to New Brunswick 32.7 mi. How far is it, by this route, from Rahway to New Brunswick ?

40. From a barrel of oil containing 31.5 gal. there is drawn off 17.75 gal. How much oil is left ?

115. Multiplying by Tenths. If we multiply 7 by $\frac{1}{2}$ we have $\frac{7}{2}$ as the product, and this we reduce to $3\frac{1}{2}$. In the same way,

$$\frac{1}{10} \text{ of } 7 = \frac{7}{10},$$

or, in decimal form, $0.1 \times 7 = 0.7$.

In the same way, $\frac{3}{10}$ of $250 = 3 \times \frac{250}{10} = 3 \times 25 = 75$,

or $0.3 \times 250 = 75.0 = 75$.

To multiply a whole number by tenths, multiply as by a whole number, and place a decimal point before the right-hand figure.

ORAL EXERCISE

Multiply by 0.1, or .1 :

- | | | | | |
|--------|---------|---------|---------|-----------|
| 1. 25. | 3. 426. | 5. 270. | 7. 400. | 9. 2175. |
| 2. 37. | 4. 384. | 6. 360. | 8. 408. | 10. 3286. |

Multiply by 0.2, or .2 :

- | | | | | |
|---------|---------|---------|---------|----------|
| 11. 11. | 13. 20. | 15. 25. | 17. 33. | 19. 111. |
| 12. 12. | 14. 70. | 16. 50. | 18. 24. | 20. 120. |

Multiply by 0.3 and 0.5 :

- | | | | | |
|---------|---------|---------|----------|----------|
| 21. 10. | 23. 30. | 25. 11. | 27. 100. | 29. 111. |
| 22. 20. | 24. 40. | 26. 12. | 28. 200. | 30. 101. |

WRITTEN EXERCISE

- How much is 0.7 of 5280 ft. ? 0.8 of 1760 yd. ?
- How many feet in 0.6 mi. ? in 0.9 mi. ?
- How many yards in 0.7 mi. ? in 0.4 mi. ? in 0.5 mi. ?
- The distance from Philadelphia to Burlington is 18.7 mi. This is 18 mi. and how many feet ?

116. Multiplying by Decimal Fractions. We multiply by other decimal fractions in much the same way that we multiply by tenths. For example,

$$\frac{1}{100} \text{ of } 275 = \frac{275}{100} = 2\frac{75}{100},$$

or, in decimal form, $0.01 \times 275 = 2.75.$

In the same way, $\frac{1}{1000} \text{ of } 325 = \frac{325}{1000},$

or $0.001 \times 325 = 0.325.$

Furthermore, $\frac{21}{100} \times 123 = \frac{2583}{100} = 25\frac{83}{100},$

or $0.21 \times 123 = 25.83.$

Therefore we see the truth of this rule:

To multiply a whole number by a decimal, multiply as by a whole number and then, beginning at the right, point off as many decimal places in the product as there are decimal places in the multiplier.

For example, $1.32 \times 3426 = 4522.32$, as shown in the annexed work.

3426
<u>1.32</u>
6852
10278
<u>3426</u>
4522.32

WRITTEN EXERCISE

Multiply the following:

- | | | |
|-----------------------|------------------------|-------------------------|
| 1. $0.9 \times 273.$ | 11. $0.25 \times 448.$ | 21. $0.124 \times 82.$ |
| 2. $0.6 \times 483.$ | 12. $0.68 \times 382.$ | 22. $0.274 \times 64.$ |
| 3. $1.7 \times 296.$ | 13. $0.73 \times 442.$ | 23. $0.382 \times 148.$ |
| 4. $4.3 \times 381.$ | 14. $1.23 \times 668.$ | 24. $0.296 \times 234.$ |
| 5. $5.2 \times 208.$ | 15. $1.09 \times 346.$ | 25. $1.123 \times 288.$ |
| 6. $4.7 \times 306.$ | 16. $2.81 \times 342.$ | 26. $2.075 \times 840.$ |
| 7. $7.2 \times 448.$ | 17. $6.72 \times 423.$ | 27. $3.009 \times 906.$ |
| 8. $8.5 \times 596.$ | 18. $7.46 \times 828.$ | 28. $5.206 \times 744.$ |
| 9. $9.3 \times 684.$ | 19. $8.32 \times 773.$ | 29. $6.309 \times 824.$ |
| 10. $9.8 \times 709.$ | 20. $9.80 \times 662.$ | 30. $9.876 \times 792.$ |

WRITTEN EXERCISE

1. The distance from Philadelphia to Bordentown is 27.9 mi. This is how many feet more than 27 mi.?

2. The distance from New York to Newark is 10.1 mi. Express this distance as feet.

3. The area of the top of a desk is 2.8 sq. ft. Express this area as square inches.

4. A field is 32 rd. long. Express this length as feet.

Remember that $32 \times 16.5 = 16.5 \times 32$.

5. At 38 mi. an hour, how far will a train travel in 2.2 hr.?

Although the time would be stated as 2 hr. 12 min., this is more easily expressed as 2.2 hr. for purposes of multiplying.

Multiply by 0.124:

6. 426. 7. 398. 8. 507. 9. 6824. 10. 3989.

Multiply by 3.428:

11. 462. 12. 292. 13. 773. 14. 2176. 15. 2875.

Multiply by 4.009:

16. 272. 17. 341. 18. 636. 19. 4261. 20. 7892.

21. At \$125 an acre, how much will 3.42 acres of land cost?

22. At \$6.50 a ton, how much will 12.3 tons of coal cost?

23. At 27 mi. an hour, how far will an automobile travel in 1.5 hr.?

24. Which takes the less time, to multiply 4284 by $2\frac{3}{4}$ or by 2.75? Time yourself in each case.

25. Which takes the less time, to multiply 2964 by $3\frac{3}{8}$ or by 3.375? Time yourself in each case.

117. Multiplying Money. We have already studied, in the fourth grade, the multiplication of money by a whole number. For example, if we have to multiply \$12.36 by 41 we multiply as if we had 41×1236 , and then we place the decimal point between dollars and dimes in the product.

$ \begin{array}{r} \$12.36 \\ \times 41 \\ \hline 4944 \\ 50676 \\ \hline \end{array} $
--

The New Jersey course of study does not include the multiplication of a decimal fraction by an integer in this half year, but it recommends frequent reviews, and the case here considered has already been studied. It is, of course, essentially the multiplication of a decimal fraction by an integer.

WRITTEN EXERCISE

1. If one pair of club skates costs \$1.85, how much will 3 pairs cost? How much will 4 pairs cost?
2. If one tennis racket costs \$2.50, how much will 5 rackets cost? How much will 3 rackets cost?
3. If one dog collar costs \$1.35, how much will 2 dog collars cost? How much will 3 dog collars cost?
4. If 1 doz. tennis balls cost \$4.85, how much will 3 doz. tennis balls cost?
5. If one basket ball costs \$4.75, how much will 2 basket balls cost?
6. If one football costs \$2.25, how much will 3 footballs cost?
7. If one baseman's mitt costs \$1.75, how much will 5 such mitts cost?
8. If one baseball mask costs \$2.45, how much will 3 such masks cost?
9. At \$16.85 each, how much will 17 girls' bicycles cost?
10. At \$2.85 each, how much will 33 croquet sets cost?

Multiply the following:

- | | | |
|------------------------|-------------------------|------------------------------------|
| 11. $2 \times \$4.26.$ | 26. $12 \times \$3.26.$ | 41. $62 \times \$12.48.$ |
| 12. $3 \times \$5.08.$ | 27. $56 \times \$8.22.$ | 42. $28 \times \$20.09.$ |
| 13. $7 \times \$6.30.$ | 28. $94 \times \$3.28.$ | 43. $94 \times \$66.82.$ |
| 14. $9 \times \$4.77.$ | 29. $81 \times \$4.40.$ | 44. $73 \times \$81.62.$ |
| 15. $6 \times \$5.32.$ | 30. $23 \times \$4.46.$ | 45. $55 \times \$70.00.$ |
| 16. $8 \times \$6.41.$ | 31. $67 \times \$3.41.$ | 46. $39 \times \$32.01.$ |
| 17. $5 \times \$4.43.$ | 32. $85 \times \$8.23.$ | 47. $61 \times \$83.40.$ |
| 18. $9 \times \$9.96.$ | 33. $92 \times \$3.32.$ | 48. $25 \times \$31.12.$ |
| 19. $5 \times \$3.40.$ | 34. $34 \times \$5.81.$ | 49. $84 \times \$42.38.$ |
| 20. $2 \times \$6.66.$ | 35. $78 \times \$8.22.$ | 50. $30 \times \$80.09.$ |
| 21. $8 \times \$7.43.$ | 36. $66 \times \$4.92.$ | 51. $4\frac{1}{2} \times \$36.62.$ |
| 22. $4 \times \$8.84.$ | 37. $73 \times \$7.20.$ | 52. $2\frac{1}{2} \times \$41.14.$ |
| 23. $7 \times \$5.07.$ | 38. $45 \times \$6.96.$ | 53. $6\frac{1}{2} \times \$51.10.$ |
| 24. $3 \times \$2.99.$ | 39. $89 \times \$9.90.$ | 54. $2\frac{1}{4} \times \$88.60.$ |
| 25. $6 \times \$9.89.$ | 40. $57 \times \$8.09.$ | 55. $4\frac{3}{4} \times \$70.08.$ |

56. Rob buys 3 fountain pens for presents. He pays \$1.35 for each. How much does he pay for all the pens?

57. A jeweler sells 4 wrist watches at \$6.75 each. How much does he receive for all the watches?

58. A book dealer sells 6 sets of Scott's novels at \$2.43 a set. How much does he receive for all the sets?

59. A dealer buys 9 rifles at \$14.65 each. How much does he pay for the lot?

60. Fred buys 2 minnow nets at \$1.55 each. How much does he pay for both nets?

61. A dealer buys 3 pianos at \$285.50 each. How much does he pay for all the pianos?

118. Dividing by a Whole Number. We have seen that *we write United States money in decimal fractions*, \$2.50 meaning merely $\$2 + \$\frac{50}{100}$, or $\$2 + \0.50 . And since we have learned how to divide a number representing United States money by a whole number, we know how to divide a decimal fraction by a whole number. For example, to divide 843.5 by 35 we proceed as here shown.

$$\begin{array}{r}
 24.1 \\
 35 \overline{)843.5} \\
 \underline{70} \\
 143 \\
 \underline{140} \\
 35 \\
 \underline{35} \\
 0
 \end{array}$$

ORAL EXERCISE

Divide, in turn, by 3, 4, and 5:

1. 360	$\frac{360}{100}$	0.60	0.6	1.2	0.12
2. 240	$\frac{240}{100}$	0.24	2.4	24	0.024

Divide, in turn, by 6 and 7:

3. 42	4.2	0.42	420	0.042	$\frac{42}{10000}$
4. 84	8.4	0.84	840	8400	0.084

WRITTEN EXERCISE

Divide the following:

- | | | |
|----------------------|------------------------|---------------------------|
| 1. $17.28 \div 12$. | 5. $1795.5 \div 42$. | 9. $48.255 \div 15$. |
| 2. $403.2 \div 32$. | 6. $1583.25 \div 75$. | 10. $691.53 \div 89$. |
| 3. $7.011 \div 41$. | 7. $1961.96 \div 98$. | 11. $1693.44 \div 36$. |
| 4. $8.109 \div 53$. | 8. $4416.93 \div 63$. | 12. $23,340.75 \div 75$. |

13. The perimeter of a triangle whose sides are all equal is 1237.8 in. What is the length of each side?

14. The perimeter of a six-sided figure, with equal sides, is 7.62 in. What is the length of each side?

ORAL EXERCISE

State the following results:

- | | | | |
|-----------------------|------------------|-------------------|---------------|
| 1. 10×0.4 | $4 \div 10$ | 10×1.4 | $14 \div 10$ |
| 2. $10 \times \$0.04$ | $\$0.40 \div 10$ | 10×0.04 | $0.4 \div 10$ |
| 3. $10 \times \$0.25$ | $\$2.50 \div 10$ | 10×0.25 | $2.5 \div 10$ |
| 4. 10×12.5 | $125 \div 10$ | 100×0.04 | $4 \div 100$ |

119. Dividing by Powers of 10. From the above exercise and from the work on page 26, we find the following:

To divide a number by 10, move the decimal point one place to the left.

Any whole number may be thought of as having a decimal point after units. That is, 6 is the same as 6. or 6.0 or 6.00.

To divide a number by 100, move the decimal point two places to the left.

Thus $7 \div 100 = 0.07$ or $.07$.

To divide a number by 1000, move the decimal point three places to the left.

$\begin{array}{r} 600 \overline{) 2.16} \\ \underline{0.36} \end{array}$
--

To divide 216 by 600 is the same as to divide 2.16 by 6.

WRITTEN EXERCISE

Divide the following:

- | | | |
|-----------------------|-------------------------|-------------------------|
| 1. $133 \div 70$. | 8. $2984 \div 4000$. | 15. $872 \div 22,000$. |
| 2. $10.8 \div 60$. | 9. $7344 \div 9000$. | 16. $7260 \div 2500$. |
| 3. $32.9 \div 70$. | 10. $625 \div 25,000$. | 17. $154 \div 12,000$. |
| 4. $3745 \div 500$. | 11. $961 \div 31,000$. | 18. $145.7 \div 4700$. |
| 5. $2291 \div 290$. | 12. $882 \div 21,000$. | 19. $57.19 \div 190$. |
| 6. $296.1 \div 470$. | 13. $1728 \div 1200$. | 20. $571.9 \div 1900$. |
| 7. $273.6 \div 120$. | 14. $144 \div 60,000$. | 21. $162.5 \div 250$. |

120. Approximate Results. In practical work the division usually does not come out exact. In such cases we carry the quotient to as many decimal places as we please. For example, in the case of $2.863 \div 24$, if told to carry the quotient to one decimal place, the result is 0.1; if to two decimal places, the result is 0.12, *this being nearer the correct result* than 0.11; if to three decimal places, 0.119; and so on.

$$\begin{array}{r}
 0.11929 \\
 24 \overline{) 2.863} \\
 \underline{24} \\
 46 \\
 \underline{48} \\
 223 \\
 \underline{216} \\
 70 \\
 \underline{48} \\
 220 \\
 \underline{216}
 \end{array}$$

If we wish to show that the result is more than 0.119 but less than 0.1195, we may write $0.119 +$. If the result is between 0.1195 and 0.120, we may write $0.120 -$. To find a quotient to two decimal places, we should see what the third decimal place is, and write in the second decimal place the figure that gives the closest approximation.

WRITTEN EXERCISE

Divide, carrying each quotient to two decimal places:

- | | | |
|----------------------|-----------------------|------------------------|
| 1. $14.16 \div 17$. | 6. $723.4 \div 48$. | 11. $14.23 \div 82$. |
| 2. $26.32 \div 21$. | 7. $920.7 \div 36$. | 12. $40.68 \div 73$. |
| 3. $434.6 \div 32$. | 8. $528.8 \div 67$. | 13. $50.83 \div 96$. |
| 4. $726.4 \div 41$. | 9. $400.3 \div 76$. | 14. $60.27 \div 123$. |
| 5. $328.7 \div 64$. | 10. $520.8 \div 79$. | 15. $42.03 \div 247$. |

Divide, carrying each quotient to three decimal places:

- | | | |
|-----------------------|-----------------------|------------------------|
| 16. $42.7 \div 64$. | 20. $423.4 \div 82$. | 24. $4.023 \div 342$. |
| 17. $36.9 \div 77$. | 21. $64.07 \div 88$. | 25. $60.29 \div 421$. |
| 18. $40.3 \div 28$. | 22. $4.302 \div 96$. | 26. $5.277 \div 322$. |
| 19. $7.146 \div 92$. | 23. $0.528 \div 79$. | 27. $0.576 \div 201$. |

Divide, carrying each quotient to one decimal place:

- | | | |
|-----------------------|------------------------|------------------------|
| 28. $2087.2 \div 5$. | 33. $843.26 \div 72$. | 38. $52.392 \div 48$. |
| 29. $6039.3 \div 7$. | 34. $342.81 \div 66$. | 39. $62.977 \div 69$. |
| 30. $4276.8 \div 6$. | 35. $802.77 \div 69$. | 40. $34.14 \div 128$. |
| 31. $3748.1 \div 9$. | 36. $534.81 \div 92$. | 41. $46.07 \div 293$. |
| 32. $5263.7 \div 8$. | 37. $417.66 \div 87$. | 42. $99.84 \div 487$. |

43. A man has a field containing 62.1 acres. He takes $\frac{1}{3}$ of it for wheat. How many acres does he take for wheat?

44. Four boys are going camping. They pay \$17.64 for the outfit. How much is each boy's share?

45. The distance from Burlington to Bordentown is 9.2 mi. How many miles in $\frac{1}{4}$ of the distance?

46. The distance from Bordentown to South Amboy is 33.9 mi. After a person has gone $\frac{1}{4}$ of the distance, how many miles must he still go to cover the whole distance?

Divide, carrying each quotient to two decimal places:

- | | | |
|-----------------------|-----------------------|-----------------------|
| 47. $274.1 \div 36$. | 58. $42.38 \div 46$. | 69. $4287 \div 693$. |
| 48. $428.3 \div 41$. | 59. $73.42 \div 68$. | 70. $2909 \div 426$. |
| 49. $720.6 \div 84$. | 60. $82.37 \div 49$. | 71. $4702 \div 328$. |
| 50. $293.4 \div 73$. | 61. $64.86 \div 38$. | 72. $5309 \div 772$. |
| 51. $820.9 \div 29$. | 62. $42.73 \div 19$. | 73. $6027 \div 428$. |
| 52. $873.4 \div 37$. | 63. $68.42 \div 37$. | 74. $5009 \div 762$. |
| 53. $607.4 \div 49$. | 64. $72.26 \div 35$. | 75. $8121 \div 406$. |
| 54. $700.6 \div 58$. | 65. $90.71 \div 54$. | 76. $9303 \div 500$. |
| 55. $801.1 \div 63$. | 66. $82.32 \div 94$. | 77. $7000 \div 228$. |
| 56. $803.7 \div 96$. | 67. $72.08 \div 33$. | 78. $6280 \div 600$. |
| 57. $712.4 \div 45$. | 68. $73.45 \div 67$. | 79. $9000 \div 387$. |

121. Estimating Results. In the following exercise pupils should state the approximate answers orally, and these should be written down. The problems should then be solved on paper and the real result compared with the approximate one. This is one of the best checks against absurd results, and should frequently be made a subject of class drill with other written exercises.

Requiring pupils to estimate results carries out the policy already referred to on page 22. Teachers should not expect to receive the same approximations from all pupils. $3\frac{1}{2}$ times 22 ¢ is a natural approximation for Ex. 1.

WRITTEN EXERCISE

1. At 22 ¢ a pound, what will $3\frac{5}{8}$ lb. of steak cost?
2. At 18 ¢ a gallon, what will 6.25 gal. of oil cost?
3. At 26 ¢ a pound, what will $6\frac{7}{8}$ lb. of butter cost?
4. At $12\frac{1}{2}$ ¢ a yard, what will $8\frac{3}{4}$ yd. of ribbon cost?
5. At \$2 a yard, what will 2.8 yd. of velvet cost?
6. A dealer buys 200 lb. of raisins for \$19. How much do they cost a pound?
7. A dealer pays \$18.83 for 7 rocking-chairs. How much does he pay for each?
8. A dealer sells 17 pails of mackerel for \$40.80. What is the price per pail?
9. A man pays \$1560 a year for the rent of a city house. How much does he pay a month?
10. Traveling at the rate of 47 mi. an hour, how far will a train go between 8.45 A.M. and 11.30 A.M.?
11. A grocer buys a 50-gallon barrel of Porto Rico molasses for \$11.50. How much does it cost per gallon?
12. A grocer sells a half chest of tea, 140 lb. to the chest, for \$19.60. What is the price per pound?

IV. PROBLEMS RELATING TO COMMON AFFAIRS

122. Cash Accounts. In keeping a cash account it is customary to write the receipts on the left side (called the *debit* side), and the expenditures on the right side (called the *credit* side), thus:

RECEIPTS					PAYMENTS				
	19—					19—			
a	May	4	Cash on hand	\$1 50	May	5	Paid for ball	20	g
b		5	From father	20		6	Ball game	25	h
c		6	From mother	10		6	Marbles	15	i
d		7	Earned, errands	25		8	Balance	1 45	j
e				2 05				\$2 05	k
f	May	8	Balance	\$1 45					

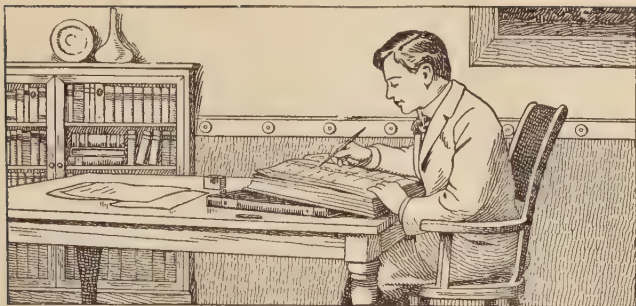
Here *a* shows cash on hand when this page of the account is begun; *b*, *c*, and *d* are receipts from father and mother, and for errands; *e* is the sum of these items on the day that the account is added to see how it stands. On the right side *g*, *h*, and *i* are bills paid. To find how much is now on hand we subtract the sum of 20¢, 25¢, and 15¢, or 60¢, from \$2.05, and we find that the remainder (in accounts called the *balance*) is \$1.45, which is written on the left, at *f*, and on the right, at *j*. To check the work, *g*, *h*, *i*, and *j* are added, and the sums, at *e* and *k*, must agree.

WRITTEN EXERCISE

Make out cash accounts and balance them as above:

1. Receipts: April 3, cash on hand, \$2.10; April 4, caring for furnace, \$0.75; April 7, selling papers, \$0.15. Expenses: April 6, book, \$0.45. Balanced on April 8.

2. Receipts: May 2, cash on hand, \$1.75; May 3, earned by errands, 20¢; May 5, from father, 25¢. Expenses, May 6, ball mitt, \$1.65. Balanced on May 7.



3. This boy is comparing his father's cash account with his own. He finds that he received \$1.00 from his father on Oct. 15, 20¢ for work on Oct. 16, 35¢ for picking apples on Oct. 17, 15¢ for work on Oct. 18, 25¢ for work on Oct. 19, and 50¢ for some chestnuts that he gathered on Oct. 20. He paid 35¢ for a book on Oct. 16, 5¢ for some marbles on Oct. 17, 20¢ for paper and ink on Oct. 18, 40¢ for a knife on Oct. 20, and on Oct. 21 he put the balance that he had on hand in the bank, where he already had \$17.50. Make out the cash account and find how much he then had in the bank.

4. For his father he made out a statement of the farm property, which showed the following items: value of the land and buildings, \$7250; standing wood and timber, \$1600; wagons and machinery, \$750; cows and horses, \$1975; oats, corn, and wheat on hand, \$490; hay and straw on hand, \$230; poultry, \$35; sewing machine and household furniture, \$325; due from creamery for milk furnished, \$72.75; due from Mr. Young for hay, \$28.50. The father still owed \$2250 on the farm, and there was a grocery bill of \$12.80 due. Make a statement like a cash account, showing the value of the property after deducting what was owed on the farm and at the grocery.

Insert dates and items as in Exs. 1-4, and balance the following accounts :

5. Receipts: \$125, \$32.60, \$28.90, \$50; expenses: \$35.50, \$15.30, \$28.40.

6. Receipts: \$16.30, \$15.50, \$10.10, \$3.50; expenses: \$4.80, \$3.45, \$20, \$4.75.

7. Receipts: \$10.30, \$5.25, \$6.32, \$7.20; expenses: \$5.30, \$2.40, \$3.80, \$0.50.

8. Receipts: \$34.30, \$15.30, \$10.75, \$6.32; expenses: \$32.50, \$8.20, \$4.30, \$3.

9. Receipts: \$42.60, \$18.50, \$13.60, \$2.80, \$4; expenses: \$15.40, \$15, \$32.75, \$4.60.

10. Receipts: \$62.30, \$15.40, \$14.75, \$21.32, \$5; expenses: \$35.75, \$10.42, \$29.60, \$28.40.

11. Receipts: \$75.50, \$62.30, \$4.72, \$16, \$25.30; expenses: \$3.25, \$80, \$14.20, \$15, \$3.25.

12. Receipts: \$1250.50, \$620, \$300, \$175, \$16.25, \$120.50, \$32.75, \$68.50; expenses: \$600, \$360, \$166.67, \$33.33, \$240.

13. Receipts: \$625.75, \$130.50, \$200, \$75.45, \$50, \$15.20, \$14.60, \$33.80; expenses: \$430, \$250, \$100, \$32.75, \$28.50.

14. Receipts: \$128.50, \$32.75, \$68.75, \$43.50, \$28.90; expenses: \$62.30, \$28.50, \$75.50, \$38.40, \$25.25, \$20.30, \$2, \$18.05, \$1.75, \$1.60, \$2.84.

15. Receipts: \$875.80, \$260.50, \$359.85, \$400, \$80, \$25.50, \$380, \$125; expenses: \$250, \$375, \$480.70, \$200, \$75, \$30.25, \$150, \$2.34, \$4.60, \$1.95, \$2.75.

16. Receipts: \$1025.30, \$535.40, \$287.50, \$126.75, \$324.80, \$125.45, \$135.42, \$12.87; expenses: \$225.35, \$430.75, \$223.50, \$121.60, \$123.75, \$2.35, \$6.27, \$4.68.

Make out cash accounts and balance them:

17. Receipts: May 1, cash on hand, \$1.25; May 3, earned by errands, 35¢. Balanced May 4.

18. Receipts: Sept. 15, cash on hand, 35¢; Sept. 16, earned by delivering papers, 16¢; Sept. 17, earned by cleaning automobile, 25¢; Sept. 18, earned by errands, 15¢. Expenses: Sept. 16, ball, 18¢; Sept. 18, dog collar, 15¢. Balanced Sept. 19.

19. Receipts: Oct. 7, cash on hand, \$2.10; Oct. 7, earned by caring for horse, 30¢; Oct. 8, from Uncle John, \$1; Oct. 9, earned by errands, 20¢. Expenses: Oct. 8, football pants, \$1.50; Oct. 9, share in football, 50¢; Oct. 10, football shoulder pads, 30¢. Balanced Oct. 11.

20. Receipts: Dec. 4, cash on hand, 62¢; Dec. 4, earned by caring for furnace, 15¢; Dec. 5, earned by cleaning walks, 20¢; Dec. 6, earned by caring for furnace, 15¢. Expenses: Dec. 6, pair of skates, 94¢. Balanced Dec. 7.

21. Receipts: May 1, cash on hand, \$36.50; May 2, R. S. Jacobs, \$20; May 4, M. R. Randall, \$32.60. Expenses: May 3, groceries, \$7.86; meat, \$3.50. Balanced May 5.

22. Receipts: April 3, cash on hand, \$25.30; April 4, J. B. Fuller, \$30; April 7, R. H. Bates, \$4.30. Expenses: April 4, groceries, \$9.32; April 6, meat, \$7.28. Balanced April 8.

23. Receipts: Feb. 6, cash on hand, \$135.60; Feb. 6, John Roberts, \$75.50; Feb. 7, M. L. Richards, \$87.90; Feb. 8, R. P. Jasper, \$42.25. Expenses: Feb. 6, rent, \$75; Feb. 7, salaries, \$60; Feb. 8, coal, \$15.30; Feb. 8, account book, \$2.25; Feb. 8, telephone rent, \$6.50; Feb. 8, gas bill, \$3.75. Balanced Feb. 9.

Problems of this kind should be made by the pupils, and they should be urged to keep their personal accounts, although these should not, as a rule, be submitted to general class inspection.

123. Cash Checks. In large stores the clerks are required to fill out cash checks like the following:

Sold by No. 127			Amount rec'd, \$15.00	
$\frac{1}{2}$ yd.	<i>Silk</i>	.57		29
$4\frac{3}{8}$ "	<i>Summer silk</i>	.38	1	67
$11\frac{1}{2}$ "	<i>Silk</i>	.99	11	39
Total			13	35
Change due			1	65
			\$15	00

WRITTEN EXERCISE

Make out cash checks for the following sales:

1. $14\frac{1}{2}$ yd. gingham @ 25¢, 9 yd. cotton @ $12\frac{1}{2}$ ¢, $10\frac{1}{2}$ yd. madras @ 35¢. Amount received, \$10.
2. $6\frac{1}{2}$ yd. cheviot @ \$1.10, $9\frac{1}{2}$ yd. velveteen @ 98¢, $5\frac{3}{8}$ yd. silk @ \$1.25. Amount received, \$25.
3. $11\frac{1}{2}$ yd. India linen @ 35¢, $6\frac{5}{8}$ yd. dimity @ 25¢, $12\frac{1}{8}$ yd. linen suiting @ 75¢. Amount received, \$20.
4. $9\frac{1}{2}$ yd. India linen @ 38¢, $5\frac{1}{8}$ yd. cheviot @ \$1.25, 15 yd. cotton @ $12\frac{1}{2}$ ¢. Amount received, \$15.
5. 7 yd. muslin @ 25¢, $12\frac{1}{2}$ yd. lining @ 11¢, $6\frac{5}{8}$ yd. India linen @ 45¢, $\frac{1}{2}$ yd. velvet @ \$3. Amount received, \$10.
6. $14\frac{1}{2}$ yd. muslin @ 24¢, $4\frac{1}{8}$ yd. velvet @ \$2.40, $16\frac{3}{4}$ yd. lining @ 12¢, $7\frac{3}{8}$ yd. silk @ \$1.60, $5\frac{3}{4}$ yd. suiting @ 80¢, $6\frac{1}{2}$ yd. ribbon @ 30¢. Amount received, \$50.

The teacher should ask the pupils to make problems similar to those given above, using the local prices of common materials.

124. Bills. If goods are not paid for by cash, which usually means that they are not paid for when they are purchased but at some later date, the dealer sends a *bill* showing the purchases, the prices, and the amount due. We studied such bills in the fourth grade. The following is a model bill, receipted by the dealers to show that it has been paid:

<i>Trenton, N.J., April 6, 1918</i>					
<i>Mrs. M. A. Olds, 179 Cairo Street</i>					
<i>Bought of</i> DOE & ROE, Grocers, 8296 MAIN ST.					
<i>Apr.</i>	<i>3</i>	<i>2 tins sliced smoked beef</i>	<i>.25</i>		<i>50</i>
	<i>4</i>	<i>3 cans corned beef</i>	<i>.13</i>		<i>39</i>
	<i>5</i>	<i>6 cans tomato soup</i>	<i>.22</i>	<i>1</i>	<i>32</i>
				<i>2</i>	<i>21</i>
<i>Received payment,</i> <i>Doe & Roe</i>					

WRITTEN EXERCISE

Make out bills for the following items, inserting the names of the purchaser and the dealer, and receipting the bills:

1. 1 baseball cap, 40¢; 1 catcher's mitt, \$1.50; 1 baseball mask, \$1.40; 1 catcher's body protector, \$1.20.

2. 1 league ball, 85¢; 3 bats @ 60¢; 2 fielder's mitts @ 75¢; 3 canvas bases @ \$1.10; 1 pitcher's toe plate, 13¢.

3. 4 hunting caps @ 30¢; 1 hunting vest, 69¢; 2 pr. corduroy hunting pants @ \$1.45; 2 hunting coats @ \$1.50.

Make out bills for the following, as on page 131 :

4. 3 qt. Lima beans @ 15¢, 4 cans tomatoes @ 14¢, 6 cans French peas @ 22¢, 5 cans California asparagus @ 42¢, 2 cans imported asparagus @ 85¢, 4 cans sardines @ 18¢.

5. 5 boxes Smyrna figs @ 60¢, 4 boxes stuffed dates @ 40¢, 6 lb. California prunes @ 20¢, 3 lb. Malaga raisins @ 22¢, 4 tins crystallized ginger @ 40¢, 6 jars marmalade @ 28¢.

6. 6 heads lettuce @ 6¢, 9½ lb. butter @ 32¢, 7½ gal. oil @ 18¢, 12 lb. raisins @ 12½¢, 6 lb. coffee @ 32¢, 4½ doz. eggs @ 36¢, ½ doz. oranges @ 60¢, 6 lb. powdered sugar @ 7½¢.

7. 14 yd. flannel @ 45¢, 32 yd. sheeting @ 30¢, 4½ yd. velvet @ \$1.60, 2½ yd. ribbon @ 40¢, 2 pr. gloves @ \$1.75.

8. 5 gal. molasses @ 45¢, 10 lb. coffee @ 18¢, 15 lb. sugar @ 7¢, 6 boxes crackers @ 8¢, 6½ doz. eggs @ 34¢, 9 lb. butter @ 33¢, 6½ lb. cheese @ 12¢.

9. 36 yd. carpet @ \$1.12½, 15 yd. border @ \$1.25, 15½ yd. cotton @ 38¢, 22 yd. cloth @ 37½¢, 7½ yd. silk @ \$1.40, 2½ doz. buttons @ 48¢, 12½ yd. calico @ 8¢.

10. 75 gro. (1 gross = 12 doz.) bone buttons @ 22¢, 675 yd. cashmere @ 85¢, 840 yd. cambric @ 17½¢, 560 yd. chambray @ 22½¢, 620 yd. flannel @ 37½¢, 950 yd. sheeting @ 24½¢.

11. 7 gro. toothbrushes @ \$9.65, 8 doz. brushes @ \$17.75, 2½ gro. nailbrushes @ \$26.80, ½ doz. clothes brushes @ \$2.92, 6 doz. boxes tooth powder @ \$2.04, 4½ doz. atomizers @ \$19.60.

12. 3 rifles @ \$12.50, 4 shot guns @ \$17.50, 400 loaded paper shells @ \$2.69 per 100, 3 hunting knives @ \$1.85, 4 redwood cleaning rods @ 75¢, 2 trap targets @ \$6.50.

13. 3 gun cases @ \$4.25, 4 hunting coats @ \$3.50, 3 hunting caps @ 33¢, 5 fishing rods @ \$2.50, 7 silk casting lines @ 80¢, 4 reels @ \$1.60.

WRITTEN REVIEW

1. How much will a man earn in 36 weeks at \$42.50 a week ?
2. At \$5.75 an acre, what is a tract of 14,264 acres of mountain land worth ?
3. How much will 615 men earn in a week if each man earns \$12.50 ? if each earns \$16.75 ? if each earns \$25.50 ?
4. How much must a dealer pay for 336 easy-chairs at \$12.50 each ? at \$16.75 each ? at \$6.25 each ?
5. How much will 75 cows cost at \$48.50 each ?
6. How much will 72 horses cost at \$87.50 each ? +
7. How much will 288 whalebones cost at 25¢ each ? †
8. How much will 484 yd. of silk cost at 75¢ a yard ? †
9. How much will 364 yd. of cloth cost at 75¢ a yard ? +
10. How much will 624 yd. of jute burlap cost at $37\frac{1}{2}$ ¢ a yard ? †
11. How much must a school pay for 25 manual-training benches at \$8.40 each ?
12. How much must a school pay for 25 jack planes at \$1.60 each ? at \$1.68 each ? at \$1.72 each ?

Find the cost of :

13. 25 books at \$1.20 each ; at \$1.45 each.
14. 25 wagons at \$116 each ; at \$125 each.
15. 50 cows at \$54 a head ; at \$65.50 a head.
16. 140 books at $62\frac{1}{2}$ ¢ each ; at 75¢ each.
17. 25 tons of hay at \$16.40 a ton ; at \$14.50 a ton.
18. $12\frac{1}{2}$ yards of cloth at 35¢ a yard ; at 50¢ a yard.
19. 25 dozen pencils at 48¢ a dozen ; at 52¢ a dozen.
20. 48 office desks at \$25 each ; at \$16.50 each.

21. A dealer pays \$124 for some gingham at 25¢ a yard. How many yards of gingham does he buy?

Think of \$124 as 12,400¢, and then divide 12,400 by 25.

22. A dealer pays \$160 for some percaline at 25¢ a yard. How many yards of percaline does he buy?

23. A dealer pays \$180 for some spools of darning cotton at 25¢ a dozen. How many dozen spools does he buy?

24. At $33\frac{1}{3}$ ¢ a yard, how many yards of canvas can be bought for \$135?

25. At $33\frac{1}{3}$ ¢ a yard, how many yards of jute burlap can be bought for \$165?

26. At $\$3.33\frac{1}{3}$ a day, how many days must a man work in order to earn \$130?

27. At $3\frac{1}{3}$ ¢ a yard, how many yards of cheesecloth can be bought for \$115?

28. At $2\frac{1}{2}$ ¢ a skein, how many skeins of cotton does a manufacturer sell if his bill is \$125.75?

29. At \$1.25 a box, how many boxes of fancy candies can be bought for \$180?

30. At \$125 each, how many horses can be bought for \$1750?

31. At \$5 each, how many rocking-chairs does a factory sell if the bill is \$3375?

32. A manufacturer sells \$3750 worth of typewriters at \$50 each. How many does he sell?

33. How long will it take an express train to go 362.5 mi., at the rate of 50 mi. an hour?

34. How long will it take a freight train to go 167.5 mi., at the rate of 25 mi. an hour?

35. A man has three adjacent lots with a frontage of 148.5 ft. One of the lots has a frontage of 52.75 ft. and another of 49.5 ft. What is the frontage of the third lot?

36. A man who had a farm of 215.8 acres bought an adjacent farm of $192\frac{3}{4}$ acres. He sold a field of $72\frac{1}{3}$ acres and another of 68.25 acres. How much land had he left?

37. The perimeter of a triangle is 27.25 ft., and two of the sides are $8.33\frac{1}{3}$ ft. and $9.75\frac{2}{3}$ ft. What is the third side?

38. The distance from New Orleans to Baton Rouge is 89.4 mi., and from New Orleans to Natchez by way of Baton Rouge, 214.1 mi. How far is it from Baton Rouge to Natchez?

39. The distance from Chicago to Lincoln is 561.4 mi., and from Chicago to Colorado Springs by way of Lincoln, 1072.2 mi. How far is it from Lincoln to Colorado Springs?

40. If the four sides of a field are 22.75 rd., 17.65 rd., 28.6 rd., and 18.35 rd., what is the perimeter of the field?

41. From the sum of 218.7 lb. and 306.4 lb. subtract the sum of 175.4 lb. and 163.7 lb.

42. How many bushels of potatoes will 24 acres produce if an average acre produces 125 bu.?

43. If a dealer buys three barrels of sugar containing respectively 313.6 lb., 314.8 lb., and 315.5 lb., how many pounds does he buy in all?

44. A man bought three barrels of sugar containing respectively 312.75 lb., 314.75 lb., and 316.5 lb. How many pounds did he buy in all?

45. A man has to make a railway journey of 254.7 miles. After he has gone $\frac{1}{3}$ of the distance he receives a telegram calling him halfway back to his starting point. After he gets there, how many miles is it to the end of his journey?

125. Price List. The following is a price list of articles interesting to boys and girls. It is used in the problems on pages 136 and 137.

Boys' storm-proof boots, \$1.50	Small typewriter, \$5
Girls' school shoes, \$1.45	School drawing set, 30¢
Girls' skating cap, 60¢	School-book satchel, 50¢
Boys' sweater, \$1.50	Postcard projecting lantern, \$2.50
Boys' silver watch, \$3.50	Postcard album, 65¢
Girls' watch, \$3.75	Single-barreled shotgun, \$6.25
Rubber-stamp outfit, 40¢	Rifle, \$13.50
Cartridge belt, \$1.35	Catcher's mitt, \$1.50
Baseball bat, 60¢	Catcher's mask, \$2
Football pants, \$1.50	Basket ball, \$3.25
Bicycle, \$16.75	Banjo, \$9.50
Scout camera, \$1.75	Guitar, \$4.75
Hammock, \$1.75	

WRITTEN EXERCISE

Find the cost of the following:

1. 2 sweaters, 3 skating caps, a boys' watch, and 2 hammocks.
2. A bicycle, a guitar, and 2 catcher's mitts.
3. A pair of boys' storm-proof boots, 2 pairs of girls' shoes, and a school drawing set.
4. A rubber-stamp outfit, a shotgun, and a cartridge belt.
5. 4 baseball bats, a catcher's mitt, a basket ball, 2 catcher's masks, and a rifle.
6. 4 pairs of football pants, 2 basket balls, and a catcher's mask.
7. Make out three different lists of things you would like to buy, six items in each, and find the cost of each list.

Make out bills for the following:

8. 2 pairs of girls' shoes, 2 pairs of boys' boots, 1 school drawing set, 1 rubber-stamp outfit, 2 postcard albums.

9. 3 girls' skating caps, 2 boys' sweaters, 2 pairs of football pants, 1 catcher's mitt, 1 catcher's mask.

10. 1 boys' watch, 1 girls' watch, 1 basket ball, 3 school drawing sets, 3 postcard albums.

11. 2 cartridge belts, 1 shotgun, 1 rifle, 2 scout cameras, 3 baseball bats, 2 hammocks.

12. 2 bicycles, 3 typewriters, 2 projecting lanterns, 3 basket balls, 2 guitars, 3 banjos.

13. 6 hammocks, 2 typewriters, 4 girls' watches, 5 boys' watches, $\frac{1}{2}$ doz. girls' skating caps.

14. $\frac{1}{2}$ doz. boys' sweaters, 8 pairs of boys' boots, 4 cartridge belts, $\frac{1}{2}$ doz. baseball bats, 4 pairs of football pants.

15. $\frac{1}{2}$ doz. scout cameras, $\frac{1}{2}$ doz. catcher's masks, 8 catcher's mitts, 4 rifles, 2 shotguns.

16. 8 projecting lanterns, 9 postcard albums, 25 school drawing sets, 7 typewriters, 3 school satchels.

17. 5 basket balls, 4 banjos, 4 guitars, 8 hammocks, 9 baseball bats, 7 pairs of football pants, 8 rubber-stamp outfits.

18. 15 rifles, 16 single-barreled shotguns, 9 cartridge belts, 9 baseball bats, 6 pairs of football pants, 2 bicycles, 8 basket balls, 4 scout cameras, 6 hammocks, 2 postcard projecting lanterns.

Teachers should have the pupils obtain price lists, either at the local stores or from catalogues, and make bills of goods in which they are interested. The pupils should see that some of the bills in this exercise are such as might represent goods purchased by them or by their parents, while some of the bills include such numbers of articles as would be purchased only by dealers. An effort should always be made to relate a topic of this kind to the personal and home interests of the pupil.

126. Marketing. The following is a price list of household supplies. It may be used in the examples on pages 138 and 139, or the current prices in the various places may be used.

Oranges, per dozen, 40¢	Celery, per bunch, 10¢
Butter, per pound, 38¢	Apples, per peck, 40¢
Eggs, per dozen, 44¢	Cheese, per pound, 14¢
Sugar, 25-pound bag, \$1.30	Cranberries, per quart, 14¢
Oysters, per dozen, 25¢	Turkey, per pound, 26¢
Coffee, per pound, 30¢	Halibut, per pound, 15¢

WRITTEN EXERCISE

Find the cost of the following :

1. $2\frac{1}{2}$ doz. oranges and $3\frac{1}{2}$ lb. of butter.
2. $3\frac{1}{2}$ doz. eggs and a 25-pound bag of sugar.
3. 2 doz. oysters and 5 lb. of coffee.
4. 3 bunches of celery and $\frac{1}{2}$ bu. of apples.
5. $2\frac{1}{2}$ lb. of cheese and 4 lb. of coffee.
6. An 8-pound turkey and a quart of cranberries.
7. 4 lb. of halibut and $2\frac{1}{2}$ doz. eggs.
8. $\frac{1}{2}$ doz. oranges, $2\frac{1}{2}$ lb. of butter, 2 doz. eggs, and a 25-pound bag of sugar.
9. 3 doz. oysters, a 9-pound turkey, 2 qt. of cranberries, and a peck of apples.
10. 6 lb. of coffee, 4 doz. eggs, $6\frac{1}{2}$ lb. of butter, 4 bunches of celery, and 6 lb. of halibut.
11. 3 doz. oranges, $1\frac{1}{2}$ lb. of butter, $2\frac{1}{2}$ doz. eggs, 25 lb. of sugar, 1 pk. of apples, 3 lb. of cheese.
12. 2 doz. oysters, 7 lb. of coffee, 4 doz. eggs, $2\frac{1}{2}$ pk. of apples, 3 qt. of cranberries, and a 12-pound turkey.

Make out bills for the following, inserting the names of purchaser and dealer and receipting the bills :

13. 4 bunches of celery, 2 pk. of apples, 2 lb. of butter, 2 doz. oysters, 4 lb. of halibut.

14. 4 qt. of cranberries, a 14-pound turkey, 2 lb. of cheese, 25 lb. of sugar, 3 doz. oranges.

15. A 15-pound turkey, 3 qt. of cranberries, 1 pk. of apples, 4 bunches of celery, 3 doz. eggs, 1 doz. oranges.

16. 2 lb. of butter @ 38¢, 2 lb. of domino sugar @ 8½¢, 2 packages of puffed rice @ 14¢, 3 doz. eggs @ 34¢, 2 lb. of powdered sugar @ 6¢, 2 lb. of coffee @ 30¢.

17. 2 doz. oranges @ 60¢, 3 doz. eggs @ 33¢, 2 lb. of butter @ 44¢, 7 lb. of granulated sugar @ 5¢, ½ lb. of tea @ 90¢, 2 grapefruits @ 15¢, 4 tins of sugar wafers @ 25¢.

18. 2 bottles of malt vinegar @ 22¢, 3 packages of corn flakes @ 10¢, 1½ doz. oranges @ 60¢, 3 doz. eggs @ 44¢, 2½ lb. of butter @ 42¢, 4 cans of pea soup @ 29¢.

19. 1 pt. of olive oil, 50¢, 2 boxes of domino sugar @ 17¢, 1½ lb. of tea @ 90¢, ½ doz. cakes of soap @ \$1.08 per doz., 3 doz. oranges @ 40¢, ½ lb. of milk chocolate @ 50¢.

20. 2 baskets of figs @ 30¢, 3 doz. cakes of soap @ \$1.06 a dozen, 2 bottles of vinegar @ 22¢, 2 lb. of chocolate mints @ 80¢, 2 boxes of layer figs @ 55¢.

21. 4 bottles of ammonia @ 24¢, 3 boxes of salt @ 10¢, ½ gal. of olive oil @ \$3 per gallon, 3 cans of mustard @ 25¢, 2 bags of flour @ 85¢, 4 lb. of confectioners' sugar @ 6½¢.

22. 1½ lb. of pork tenderloin @ 38¢, 1¾ lb. of French lamb chops @ 28¢, 2½ lb. of steak @ 26¢, 8 lb. of roast beef @ 34¢, 1½ lb. of fish @ 22¢, 2 packages of sausage @ 25¢.

127. Solving Problems. In solving a problem there is one thing that the teacher will ask you to do, and two other things may be required. These three things are (1) to work rapidly and accurately; that is, to take the shortest road to the answer, and to be certain that the answer is correct; (2) to put neatly on paper not merely the operation but a brief explanation; (3) to give a brief analysis or oral explanation.

For example, if 5 yd. of cloth cost \$2.10, how much will 12 yd. cost?

(1) <i>The number work:</i>	\$0.42
$12 \times \$2.10$	$\begin{array}{r} 12 \\ \hline 84 \\ 42 \\ \hline \$5.04 \end{array}$
$\frac{12}{5}$	

This is the form in which the number work of this example should appear on paper or on the blackboard, whether the written work is required or not.

(2) *The written explanation:*

5 yd. cost \$2.10.

1 yd. costs $\frac{1}{5}$ of \$2.10.

12 yd. cost $12 \times \frac{1}{5}$ of \$2.10, or \$5.04.

This is the form in which the written explanation of this example should appear, in case the teacher requires a written explanation.

(3) *The oral analysis:*

Since 5 yd. cost \$2.10, 1 yd. will cost $\frac{1}{5}$ of \$2.10, and 12 yd. will cost $12 \times \frac{1}{5}$ of \$2.10, or \$5.04.

Teachers will not need to call for all this work with every example. Sometimes it will be necessary to emphasize (2) and sometimes (3), but the important thing is that (1) should be quickly and neatly, and, above all, *accurately*, done.

128. Estimating the Answer. One of the best ways to avoid absurd answers is to *estimate the result* in advance. Write down this estimate and compare it with the answer, and if there is a great difference, look over your work again.

For example, if 5 yd. cost \$2.10, we know that 12 yd. will cost nearly $2\frac{1}{2}$ times as much, or about \$5. When we solve, if we find such a result as \$50.40, we see at once that there is a mistake, probably in the position of the decimal point. The correct result is \$5.04.

Always estimate the answer in advance, write down the estimate, and compare it with the result. If there is a great difference, the work is probably wrong.

129. Accuracy. In any single operation the work is *right* or it is *wrong*. A business man will not excuse a bookkeeper who writes \$9250.75 instead of \$90,250.75. Only a zero is missing, but it means a difference of over \$80,000.

A pupil may be excused if he does not understand the wording or the meaning of a problem, but for inaccurate numerical work he must be judged as he will be judged in business — his work is right or else it is wrong.

Always check the number work in some way. The first consideration in arithmetic is accuracy.

130. Businesslike Work. Work as business men work, putting the additions and subtractions in columns except when a written analysis is required.

Although the problems in this book are varied in content and relate to the ordinary affairs of life, teachers are advised to supplement them as they would those of any other arithmetic. The pupils should be encouraged to tell of their purchases and allowances and to inquire as to prices in their localities. In rural schools the pupils should learn about the cost of farm products, the keeping of farm books, their opportunities for earning money, and the problems of agriculture. If they take trips to the city they should be encouraged to report the distances traveled by them and their expenditures.

WRITTEN REVIEW

1. The distance from Philadelphia to South Amboy is 61.8 mi., but the railway charges a fraction of a mile as a full mile. At $2\frac{1}{2}\phi$ a mile, how much will a ticket cost from Philadelphia to South Amboy?

2. The distance from New York to Rahway is 20.8 mi., and from New York to Perth Amboy 27.8 mi. How much will a ticket from Rahway to Perth Amboy cost at $2\frac{1}{2}\phi$ a mile?

3. Ralph buys a ticket to New York and return for \$1.10. He takes two rides in the subway in New York, and four trolley rides, the subway and trolley rides costing 5ϕ each. He pays 35ϕ for his luncheon. How much does the trip cost?

4. Mr. Roberts takes John and Mary to Philadelphia. He pays \$1.50 for his return ticket, and half price for each of the children. They take three trolley rides in Philadelphia at 5ϕ per ride for each, and spend \$1.45 for luncheon. How much does the whole trip cost?

5. The distance from Chicago to Jackson is 209.31 mi., and from Jackson to Detroit 75.43 mi. How far is it from Chicago to Detroit by way of Jackson?

6. The distance from Detroit to St. Thomas is 112.01 mi., and from St. Thomas to Buffalo 139.13 mi. How far is it from Detroit to Buffalo by way of St. Thomas?

7. Taking the distances as given in Exs. 5 and 6, find how far it is from Chicago to Buffalo by this route.

8. The distance from Buffalo to Albany is 296.64 mi., and from Albany to Boston 202 mi. How far is it from Buffalo to Boston by way of Albany?

9. Taking the distances as given in Exs. 5, 6, and 8, find how far it is from Chicago to Boston by this route.

10. A farmer sowed 60 acres of land, using 780 qt. of timothy seed. How much seed did he use per acre?

11. A farmer sowed 70 acres, using 1050 qt. of timothy seed and clover seed mixed. How much seed did he use per acre?

12. A farmer sowed 90 acres, using 2520 qt. of timothy seed and redtop seed mixed equally. How many quarts of each kind did he use per acre?

13. A Maryland farmer experimented with Italian rye grass, using 3250 lb. to sow 50 acres. How many pounds did he use per acre?

14. A farmer found that 30 sheep ate 11,550 lb. of dry fodder in a year. What was the average amount per sheep?

15. A farmer found that 20 horses consumed 162,400 lb. of hay and grain in a year. What was the average per horse?

16. A dealer found that 60 bu. of white-clover seed weighed 3780 lb. How much did it weigh per bushel?

17. A dealer told one of his customers that 1080 lb. of clover would seed 90 acres. How much is that per acre? If he mixed other seed with the clover so as to use this amount to cover 4 acres, how much clover seed would he use per acre?

18. A farmer found that it took 980 lb. of red clover seed for 70 acres, and 1200 lb. of crimson clover for 80 acres. What was the amount of each used per acre?

19. How many pounds of Paris green will be needed to spray 300 trees, allowing 5 gal. of the mixture to a tree, if every 150 gal. contains 1 lb. of Paris green?

20. If a man receives a salary of \$1728 per year, how much does he receive per month?

21. If a man receives a salary of \$1092 for the 52 weeks of a year, how much does he receive per week?

22. If a man pays \$8569 for 209 acres of land, how much does it cost per acre?

23. If a book jobber pays \$3402 for 42 sets of an encyclopedia, how much does he pay per set?

24. If a stock dealer buys some cattle for \$71,961, paying \$51 per head, how many head does he buy?

25. If a jobber sells during the year \$82,350 worth of shoes at \$61 a case, how many cases does he sell?

26. If a dealer sells \$142,560 worth of linseed oil during a year at \$22 per barrel, how many barrels does he sell?

27. If a factory sells 3684 ready-made suits of a certain grade for \$92,100, what is the price per suit?

28. If the product of two numbers is 2,137,512, and one of them is 372, what is the other number?

29. If a state pays \$2,325,798 for constructing 471 mi. of model road, what is the average cost per mile?

30. If it costs \$5,236,602 to construct 287 mi. of railroad, what is the average cost per mile?

31. The product of two numbers is 6,251,616, and one number is 2016. Find the other number.

32. What is the smallest number that must be subtracted from 615,476 so that the result divided by 1875 gives a quotient with no remainder?

33. If a company employs 8009 men on the same schedule of wages, and the pay roll for these men is \$6,247,020 a year, what is the annual wages of each?

34. If a company employs 250 men on the same schedule of wages, and the pay roll for these men is \$270,000 a year, what is the monthly wages of each?

35. This girl bought for her mother 7 yd. of silk at \$1.35 a yard. How much did it cost?

36. This girl's mother sent her to buy 5 yd. of serge at \$1.75 a yard. How much did it cost? How much change should she receive from a \$10 bill?



37. If the girl's purchases at another time consist of toweling at 80¢, muslin at \$1, needles at 45¢, thread at 16¢, beads at 25¢, and braid at 24¢, how much change should she receive from \$5?

38. She saw a lady buy 13 yd. of broadcloth and pay \$45.50 for it. How much did it cost a yard?

39. If the merchant charges \$34.65 for 21 yd. of velvet, how much does he charge a yard?

40. If the merchant sells tweeds at \$3.25 a yard, how much will he receive for 18 yd.? how much for 37 yd.?

41. If a lady buys 27 yd. of silk at \$1.45, 3 yd. of velvet at \$1.75, and 22 yd. of serge at \$1.60, how much is her bill?

42. A man pays \$2.52 for 12 lb. of meat. How much does it cost him a pound?

43. If a man pays 28¢ a pound for 12 lb. of meat, how much does it cost him?

44. How many inches are there in 1 ft.? How many feet are there in 432 in.?

45. If the average length of a man's step is 27 in., how many steps will he take in going 837 in.?

46. If the average length of a man's step is 27 in., how many inches will he go in taking 275 steps?

47. If a man earns \$2.65 in each of the 27 working days of a certain month, how much does he earn in all?

48. A dealer sold 75 barrels of sweet potatoes at \$2.25 a barrel. How much did he receive?

49. A dealer sold 64 barrels of sweet potatoes at \$1.80 a barrel, and 36 barrels at \$2.40 a barrel. How much did he receive for them all?

50. If three farmers, having equal amounts of oats to thresh, pay \$38.10 for the use of a threshing machine, how much should each pay?

51. If the altitude of Mt. McKinley is 20,464 ft., and that of Mt. Everest 29,002 ft., how much higher is Mt. Everest than Mt. McKinley?

52. If the total railway mileage in the United States was 193,346 miles at one time and 229,951 miles a few years later, how much was the increase in mileage?

53. If a school buys 3 doz. blackboard pointers at \$1.15 a dozen, a dictionary holder at \$6.45, a dictionary at \$6.75, and a teacher's desk at \$13.50, what is the total cost?

54. If the greatest known depth of the Atlantic Ocean is 27,366 ft., and Mt. Washington is 6279 ft. high, how high does Mt. Washington stand above the bottom of the Atlantic at its greatest known depth?

55. A workman receiving \$0.31 an hour works 48 hr. one week and 43 hr. the next week. How much does he receive for the two weeks' work?

56. A village lot was sold for \$745, which was \$130 less than it cost. For how much should it have been sold in order that the owner might have gained \$130?

57. A train that while running makes 45 mi. an hour requires 2 hr. 6 min. to go from New York to Philadelphia. It makes two stops of 3 min. each. How far is it from New York to Philadelphia?

58. A clerk in a store receives \$60 a month, and works 12 hr. a day. If he worked only 8 hr. a day, how much would he receive if he were paid according to the number of hours?

59. A grocer has to pay the following bills on the first of the month: \$175.25, \$32.47, \$58.65, \$82.75, \$29.60, and \$59.80. What is the total amount? How much does the total amount lack of being \$500?

60. A man asked \$5825 for his farm, but was forced to sell it for \$3925. He thereby lost as much on what it cost him as he would have gained if he had sold it at the price first asked. What did it cost him?

61. A man buys a house for \$3750. He pays \$1950 down and agrees to pay the balance in equal monthly payments of \$50 each. How many months will it take him to pay for the house? How many years will it take?

62. At an ordinary rate, 124 words a minute, how long will it take a man to deliver a speech of 12 pages, if each page contains 31 lines and the lines average 11 words each? How long would it take a slow speaker to deliver the same speech, if he averaged 93 words a minute?

63. A certain city pays 320 street cleaners \$4512 for 6 days' work. What is the daily wages of each man?

64. The area of Texas is 265,780 sq. mi. and that of the District of Columbia is 70 sq. mi. How many pieces the size of the District of Columbia could be cut from Texas, and how many square miles would then remain?

65. The area of Rhode Island is 1250 sq. mi. How many pieces the size of Rhode Island could be cut from Texas (see Ex. 64), and how many square miles would then remain?

66. At the time when the population of New York City was 4,318,035 the area was 327 sq. mi. What was the average population per square mile?

67. In 1820 Illinois had a population of only 55,162. When the population had reached 5,074,904 it was how many times that of 1820?

68. In 1820 Indiana had a population of 147,178. When the population had reached 2,502,026 it was how many times that of 1820?

69. When the population of the United States was 91,972,266, and the number of children 12 years of age was 1,532,700, the total population was how many times the number of children 12 years of age?

70. The area of Iowa is 56,025 sq. mi., and that of Indiana is 36,350 sq. mi. Taking the area of the United States (including Alaska) as 3,616,484 sq. mi., how many times is the combined area of these two states exactly contained in it, and how many square miles remain over?

71. In 1800 the ordinary expenses of our national government were \$7,411,370 a year. When these expenses had reached \$548,441,380, this was how many times the sum in 1800?

V. MEASURING AND DRAWING TO SCALE

WRITTEN EXERCISE

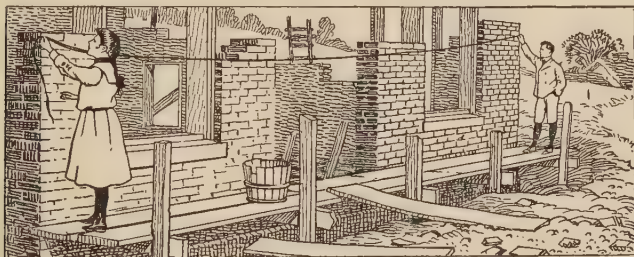
1. A baseball "diamond," or infield, is a square 90 ft. on a side. Draw this to the scale 1 in. to 20 ft.

As we found on page 81, this means that we represent 20 ft. by 1 in. Therefore 90 ft., which is $4\frac{1}{2} \times 20$ ft., will be represented by $4\frac{1}{2}$ in.

2. On the infield the pitcher's plate is 60.5 ft. from the point where the two sides of the diamond meet at the home base. Draw another plan of the infield, using the scale 1 in. to 30 ft., and mark the position of the pitcher's plate.

3. The first, second, and third bases are placed in squares 15 in. on a side. Draw one of these squares to the scale $\frac{1}{5}$.

This means that 5 in. is to be represented by 1 in.



4. Fred and Rose measure a house that is being built near the school. They find it to be 30 ft. by 20 ft. Draw a ground plan of the house to the scale 1 in. to 5 ft.

5. They find that the front door is 4 ft. by 7 ft. 6 in. Draw a plan of the door to the scale $\frac{1}{12}$.

6. A brick is 8 in. long and 4 in. wide. Draw a plan of the top to the scale $\frac{1}{2}$.

Teachers should give much work in measuring and in drawing to scale.

7. Rob and Mary are measuring a cellar that is being dug for a house. They use a tape that is 100 ft. long. It is divided into feet, inches, half inches, and quarter inches. How many inches in the tape? How many half inches? How many quarter inches?



8. They find that the workmen have dug only part of the cellar, this part being 6 ft. wide. Express the width in yards. Express the width in inches.

9. They find that the excavation is 22 ft. 6 in. long. Express this in feet and a fraction; in yards and a fraction.

10. The distance from where Rob is to the house is 412 ft. 6 in. A rod being $16\frac{1}{2}$ ft., express the distance in rods.

11. If a field is 17 rd. long, what is the length in feet?

12. If a field is 19 rd. long and 12 rd. wide, what length of fence is needed to inclose it?

13. If Jack steps 27 in. each time, and takes 450 steps in coming from his home to the school, how many feet does he come?

In each half year the teacher should give plenty of work in practical measuring. The pupils should measure the lengths of their ordinary steps, and should learn to pace such distances as the length of the room and the width of the playground. They should become perfectly familiar with the divisions of the ruler, and should have practice in estimating the height of the room, of trees, and of the schoolhouse; the width of the street and of pieces of land; and the distances they travel in coming to school. In order to be real, this work must apply to local conditions and must therefore be given by the teacher.

REVIEW AND DRILL

131. Purpose of this Review and Drill. You have now studied the four operations with whole numbers and with common and decimal fractions. This review is a condensed arithmetic, in few pages.

Teachers should read the note on page 83.

I. DRILL SECTION

WRITTEN EXERCISE

1. Write in words the numbers 2,304,507 and 100.047.
2. Write in figures two hundred forty-seven thousandths.

Show that the following curious results are correct :

3.

$$\begin{aligned} 9 \times 9 + 7 &= 88 \\ 98 \times 9 + 6 &= 888 \\ 987 \times 9 + 5 &= 8888 \\ 9876 \times 9 + 4 &= 88888 \\ 98765 \times 9 + 3 &= 888888 \\ 987654 \times 9 + 2 &= 8888888 \\ 9876543 \times 9 + 1 &= 88888888 \\ 98765432 \times 9 + 0 &= 888888888 \end{aligned}$$

4.

$$\begin{aligned} 1 \times 1 &= 1 \\ 11 \times 11 &= 121 \\ 111 \times 111 &= 12321 \\ 1111 \times 1111 &= 1234321 \\ 11111 \times 11111 &= 123454321 \\ 111111 \times 111111 &= 12345654321 \\ 1111111 \times 1111111 &= 1234567654321 \\ 11111111 \times 11111111 &= 123456787654321 \end{aligned}$$

Write as rapidly as you can the results of the following :

- | | | | |
|---------------------------------------|--|--|--|
| 5. $\frac{1}{2} + \frac{1}{4}$. | 10. $\frac{1}{2} + \frac{3}{4}$. | 15. $\frac{1}{2} + \frac{1}{8}$. | 20. $\frac{1}{2} + \frac{7}{8}$. |
| 6. $\frac{1}{2} - \frac{1}{4}$. | 11. $\frac{3}{4} - \frac{1}{2}$. | 16. $\frac{1}{2} - \frac{1}{8}$. | 21. $\frac{7}{8} - \frac{1}{4}$. |
| 7. $\frac{1}{2} \times \frac{1}{4}$. | 12. $\frac{1}{2} \times \frac{3}{4}$. | 17. $\frac{1}{2} \times \frac{1}{8}$. | 22. $\frac{5}{8} \times \frac{3}{4}$. |
| 8. $\frac{1}{2} \div \frac{1}{4}$. | 13. $\frac{1}{2} \div \frac{3}{4}$. | 18. $\frac{1}{2} \div \frac{1}{8}$. | 23. $\frac{3}{4} \div \frac{7}{8}$. |
| 9. $\frac{1}{4} \div \frac{1}{2}$. | 14. $\frac{3}{4} \div \frac{1}{2}$. | 19. $\frac{1}{8} \div \frac{1}{2}$. | 24. $\frac{7}{8} \div \frac{3}{4}$. |

	I	II	III	IV	V	VI	VII
<i>a</i> 124	$\frac{1}{8}$	$1\frac{1}{8}$	$2\frac{1}{3}$	$3\frac{1}{3}$	$4\frac{2}{3}$	$3\frac{3}{2}$	
<i>b</i> 236	$\frac{1}{4}$	$1\frac{3}{4}$	$2\frac{1}{2}$	$4\frac{1}{16}$	$5\frac{5}{16}$	$3\frac{4}{8}$	
<i>c</i> 307	$1\frac{5}{8}$	$1\frac{7}{8}$	$2\frac{2}{3}$	$4\frac{3}{32}$	$5\frac{1}{2}$	$4\frac{1}{16}$	
<i>d</i> 570	$\frac{3}{8}$	$2\frac{1}{16}$	$2\frac{5}{6}$	$5\frac{1}{8}$	$6\frac{3}{8}$	$5\frac{3}{4}$	
<i>e</i> 692	$\frac{1}{2}$	$2\frac{1}{4}$	$3\frac{1}{3}$	$5\frac{5}{32}$	$7\frac{1}{2}$	$6\frac{1}{16}$	
<i>f</i> 801	$1\frac{9}{16}$	$3\frac{3}{8}$	$3\frac{1}{2}$	$5\frac{7}{32}$	$7\frac{3}{2}$	$7\frac{3}{2}$	
<i>g</i> 998	$\frac{7}{8}$	$3\frac{7}{16}$	$4\frac{2}{3}$	$6\frac{1}{4}$	$8\frac{5}{8}$	$8\frac{6}{4}$	

In each of the above columns perform the following operations:

- | | |
|----------------------|--------------------------|
| 25. $a + b + c.$ | 41. $a + b + c + d + e.$ |
| 26. $b + c + d.$ | 42. $b + c + d + e + f.$ |
| 27. $c + d + e.$ | 43. $c + d + e + f + g.$ |
| 28. $d + e + f.$ | 44. $a + b + d + e + f.$ |
| 29. $e + f + g.$ | 45. $a + b + e + f + g.$ |
| 30. $a + c + d + e.$ | 46. $a + c + d + e + g.$ |
| 31. $b + d + e + f.$ | 47. $b + d + e + f - a.$ |
| 32. $c + e + f + g.$ | 48. $c + d + e + g - b.$ |
| 33. $b + c + d - a.$ | 49. $d + e + f + g - a.$ |
| 34. $c + e + f - b.$ | 50. $c + e + f + g - b.$ |
| 35. $d + e + f - c.$ | 51. $b + c + d + e - a.$ |
| 36. $c + d + f - b.$ | 52. $c + d + f + g - b.$ |
| 37. $a + c + d - b.$ | 53. $a + b + c + d - e.$ |
| 38. $a + d + e - g.$ | 54. $a + b + c + e - d.$ |
| 39. $a + e + f - d.$ | 55. $a + b + c + f - e.$ |
| 40. $b + d + e - c.$ | 56. $a + b + d + g - c.$ |

Teachers should read and be guided by the suggestions on page 83. These efficiency tests should be used only as far as necessary, and the pupils should time themselves, keeping and comparing their records. The work may be reviewed in Grade VI if the necessity arises.

II. PROBLEMS WITHOUT NUMBERS

1. If you have a price list of baseball supplies, and wish to find the cost of a certain number of bats and a certain number of suits, how do you proceed?

2. If you know the cost of a certain number of tennis balls, how do you find the cost of a certain other number?

3. If you go marketing and buy a certain number of pounds of sugar at a certain price per pound, and give the dealer a bill, how do you find, mentally, the change that is due you?

4. If you know the distance from Philadelphia to the place where you live, and the cost of a railway ticket per mile, how do you find the cost of a ticket to Philadelphia?

5. If you know the number of bushels of oats eaten by a horse in a certain number of days, how do you find the number of quarts eaten per day?

6. If you know a man's salary per year, how do you find how much he earns per month? per week? per working day?

7. If you find from a time table the distance from New York to Philadelphia, and the time of leaving and arriving, how do you find the rate of a train per hour?

8. If you are told to draw a plan of this room on a given scale, what measurements do you take and how do you proceed to draw to this scale?

9. If a house is being erected near the school and you wish to find the number of square feet to be covered by it, how do you proceed to find out?

10. How do you tell whether a given number is divisible by 2?

11. How do you tell whether a given number is divisible by 3?

12. How do you tell whether a given number is divisible by 5?

13. If you know the number of minutes you are in school each day, how do you find the number of hours you are in school a week?

III. INDUSTRIAL PROBLEMS

DOES IT PAY TO HAVE GOOD ROADS?



1. Do good roads pay? That is the practical question for you to answer, because your parents help pay for the roads. Here is a picture of a bad road. A teamster had to haul $7\frac{1}{2}$ tons of barbed wire from the railway, a distance of 23 mi. He found that he could haul only 1500 lb. to a load and that it took him $1\frac{1}{2}$ da. to do it. How long did it take him to haul the $7\frac{1}{2}$ tons of wire, and what did it cost at \$4 per day for man and team?

2. In Ex. 1, what was the cost of hauling 1 ton 1 mi., or the cost of one ton-mile, as it is ordinarily called?

3. On the next page is a picture of the same road after improvement. The teamster finds that with the same team he can now easily haul $2\frac{1}{2}$ tons to the load and do the work in 1 da. What does it now cost to haul a ton of wire? What is the cost per ton-mile?

4. Comparing the results in Exs. 2 and 3, what common fraction less is the cost per ton-mile, owing to good roads?

5. If a teamster has to haul 30 tons of merchandise 23 mi. over this road, what is the cost if the road is in the bad condition referred to in Ex. 1? What is the cost if the road is in the good condition referred to in Ex. 3 and shown in the following picture?



6. A farmer lives 10 mi. from the railway. The road was formerly so bad that with a two-horse team he could haul only 40 bu. of wheat, and it took a day to make the round trip. At \$4 a day for man and team, how much did it cost per bushel to haul the wheat?

7. The county recently borrowed money to improve its roads. The farmer can now haul 75 bu. with his team. How much does it cost per bushel to haul the wheat now?

8. Comparing Exs. 6 and 7, how much more money does the farmer really get per bushel, now that he has good roads?

9. Taking 60 lb. as the weight of a bushel of wheat, and comparing Exs. 6 and 7, what has been the reduction in cost per ton-mile?

10. The average cost of hauling cotton to market in the Southern states is 80¢ per bale. If they market 16,250,000 bales a year, what is the cost of hauling it?

11. States often borrow the money to build good roads. New York state has borrowed \$100,000,000 for this purpose, and pays each year as interest 0.04 of this sum. How much interest does it pay per year? New York City pays $\frac{7}{10}$ of this, because of its great wealth. How much is left for the rest of the state to pay?

12. Sometimes a state neglects its roads after they are improved. By this neglect a certain road went to pieces in a single year, and it cost \$1400 a mile to restore it. If it had been kept in repair, the total cost would have been only \$608 a mile. On a road 78 mi. long, how much did the state waste by its carelessness?

13. Automobiles help pay for the roads they use. In a recent year there were 1,800,000 motor vehicles of all kinds in this country, and they paid on an average \$10 a year for registration and licenses. How much did they pay to the states to offset the damage they did to the roads?

14. Suppose a county produces a large amount of wheat, say an average of 30 bu. to the acre, and this wheat has to be hauled an average of 5 mi. If good roads will reduce the hauling cost 5¢ per ton-mile, let us see how much it will increase the value of the wheat per acre. First, let us see what part of a ton of wheat is produced per acre. At 60 lb. to the bushel, what part of a ton is produced?

15. Having solved Ex. 14, let us find the saving on this part of a ton for 5 mi. at 5¢ per ton-mile.

16. Following the work in Ex. 15, let us now consider how much this means per square mile. There being 640 acres to a square mile, how much is the saving per square mile?

17. If the county mentioned in Exs. 14-16 contains 300 sq. mi., and 0.6 of the land is used for growing wheat, how much does the county gain per year by having good roads, considering only the increase in the value of the wheat?

18. If a county borrows \$125,750 for building good roads, and pays each year as interest 0.05 of this sum, how much are the annual interest charges on the county?

GRADE VI. FIRST HALF

I. REVIEW

ORAL EXERCISE

Add rapidly:

1.	$\begin{array}{r} 2 \\ 3 \\ \hline \end{array}$	$\begin{array}{r} 12 \\ 3 \\ \hline \end{array}$	$\begin{array}{r} 32 \\ 3 \\ \hline \end{array}$	$\begin{array}{r} 32 \\ 13 \\ \hline \end{array}$	$\begin{array}{r} 42 \\ 13 \\ \hline \end{array}$	$\begin{array}{r} 52 \\ 13 \\ \hline \end{array}$	$\begin{array}{r} 52 \\ 23 \\ \hline \end{array}$	$\begin{array}{r} 52 \\ 33 \\ \hline \end{array}$
2.	$\begin{array}{r} 3 \\ 4 \\ \hline \end{array}$	$\begin{array}{r} 13 \\ 4 \\ \hline \end{array}$	$\begin{array}{r} 3 \\ 14 \\ \hline \end{array}$	$\begin{array}{r} 33 \\ 14 \\ \hline \end{array}$	$\begin{array}{r} 33 \\ 24 \\ \hline \end{array}$	$\begin{array}{r} 53 \\ 14 \\ \hline \end{array}$	$\begin{array}{r} 53 \\ 24 \\ \hline \end{array}$	$\begin{array}{r} 53 \\ 34 \\ \hline \end{array}$
3.	$\begin{array}{r} 4 \\ 5 \\ \hline \end{array}$	$\begin{array}{r} 14 \\ 5 \\ \hline \end{array}$	$\begin{array}{r} 4 \\ 15 \\ \hline \end{array}$	$\begin{array}{r} 14 \\ 15 \\ \hline \end{array}$	$\begin{array}{r} 34 \\ 15 \\ \hline \end{array}$	$\begin{array}{r} 54 \\ 15 \\ \hline \end{array}$	$\begin{array}{r} 54 \\ 25 \\ \hline \end{array}$	$\begin{array}{r} 74 \\ 25 \\ \hline \end{array}$
4.	$\begin{array}{r} 5 \\ 6 \\ \hline \end{array}$	$\begin{array}{r} 15 \\ 6 \\ \hline \end{array}$	$\begin{array}{r} 25 \\ 6 \\ \hline \end{array}$	$\begin{array}{r} 35 \\ 6 \\ \hline \end{array}$	$\begin{array}{r} 75 \\ 6 \\ \hline \end{array}$	$\begin{array}{r} 15 \\ 16 \\ \hline \end{array}$	$\begin{array}{r} 25 \\ 16 \\ \hline \end{array}$	$\begin{array}{r} 35 \\ 16 \\ \hline \end{array}$
5.	$\begin{array}{r} 6 \\ 7 \\ \hline \end{array}$	$\begin{array}{r} 16 \\ 7 \\ \hline \end{array}$	$\begin{array}{r} 36 \\ 7 \\ \hline \end{array}$	$\begin{array}{r} 76 \\ 7 \\ \hline \end{array}$	$\begin{array}{r} 7 \\ 8 \\ \hline \end{array}$	$\begin{array}{r} 17 \\ 8 \\ \hline \end{array}$	$\begin{array}{r} 37 \\ 8 \\ \hline \end{array}$	$\begin{array}{r} 67 \\ 8 \\ \hline \end{array}$
6.	$\begin{array}{r} 8 \\ 8 \\ \hline \end{array}$	$\begin{array}{r} 18 \\ 8 \\ \hline \end{array}$	$\begin{array}{r} 28 \\ 8 \\ \hline \end{array}$	$\begin{array}{r} 48 \\ 8 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ 9 \\ \hline \end{array}$	$\begin{array}{r} 18 \\ 9 \\ \hline \end{array}$	$\begin{array}{r} 28 \\ 9 \\ \hline \end{array}$	$\begin{array}{r} 68 \\ 9 \\ \hline \end{array}$

Teachers of this grade should not expect pupils at the beginning of the term to be proficient in the work of the preceding grade. Two or three weeks, or even a longer time, should be spent in a careful and systematic review of the work in fractions. There should also be a brief review of the work in whole numbers, particularly with respect to "endings," as in the above exercise. Teachers will find additional material in the preceding Review and Drill section.

WRITTEN EXERCISE

Add, timing yourself:

1.	2.	3.	4.	5.
426	206	328	304	868
348	473	477	472	747
<u>239</u>	<u>489</u>	<u>806</u>	<u>283</u>	<u>523</u>
6.	7.	8.	9.	10.
482	445	300	567	819
690	567	968	742	977
<u>307</u>	<u>782</u>	<u>867</u>	<u>382</u>	<u>642</u>
11.	12.	13.	14.	15.
544	686	772	362	682
288	692	283	844	884
389	908	476	928	788
<u>977</u>	<u>703</u>	<u>299</u>	<u>377</u>	<u>645</u>
16.	17.	18.	19.	20.
\$2.93	\$6.68	\$3.79	\$8.29	\$3.90
6.88	4.92	4.26	9.36	4.00
4.02	3.98	5.08	4.88	.66
3.77	.72	8.66	.72	7.88
<u>4.29</u>	<u>6.88</u>	<u>.77</u>	<u>4.86</u>	<u>9.21</u>
21.	22.	23.	24.	25.
42.83	83.47	67.00	37.82	72.83
31.77	92.68	4.29	42.86	64.41
62.82	71.88	34.06	98.77	8.78
91.84	92.36	78.08	73.40	90.26
<u>72.68</u>	<u>48.52</u>	<u>55.72</u>	<u>90.07</u>	<u>44.83</u>

Add, then subtract, timing yourself:

26.	27.	28.	29.	30.
\$37.42	\$50.17	\$62.23	\$70.03	\$81.10
29.68	36.29	46.72	42.35	63.35
31.	32.	33.	34.	35.
\$60.31	\$71.13	\$82.24	\$80.06	\$91.19
46.72	38.02	67.79	53.17	72.21

Multiply, and then divide, the first of the following numbers by the second:

36. 3796 by 26.	41. 5929 by 11.	46. 90,090 by 91.
37. 8464 by 46.	42. 2451 by 19.	47. 98,901 by 99.
38. 6510 by 31.	43. 8514 by 33.	48. 55,821 by 69.
39. 4599 by 73.	44. 3317 by 31.	49. 54,439 by 77.
40. 8772 by 43.	45. 1530 by 51.	50. 74,994 by 87.

Add, timing yourself:

51.	52.	53.	54.	55.
2746	2096	\$279.62	\$34.72	63,428
3142	3708	348.70	68.93	41,092
4828	5527	52.97	82.78	64,087
9643	6802	681.73	4.36	23,441
5172	3942	309.42	52.91	62,887
6838	8261	78.65	.86	91,980
4939	7838	421.88	42.82	75,576
7282	4029	900.09	70.07	42,179
2736	7128	271.41	42.88	27,142
7264	6344	362.33	31.66	36,875
2968	3462	426.30	28.42	48,625

ORAL EXERCISE

1. For the bottom of a flower box we use wood $\frac{7}{8}$ in. thick, but that for the sides is half as thick. How thick is that for the sides?

2. In drawing a plan for a flower box we use the scale $\frac{1}{8}$. The plan is 5 in. long. How long is the box?

3. If a box is $5\frac{1}{4}$ in. wide and 8 times as long, how long is the box?

4. If a box is 36 in. long and $\frac{1}{8}$ as wide, how wide is it?

5. A drawing of a kite is made on the scale $\frac{1}{6}$. The length of the kite in the drawing is $4\frac{1}{2}$ ". How long is the kite?

6. A kite is 27" long and two thirds as wide. What is the width of the kite?

7. A kite is 24" long and 14" wide. The width is what fraction of the length?

8. A kite is 27" long and $13\frac{1}{2}$ " wide. The length is how many times the width? The width is what fraction of the length?

WRITTEN EXERCISE

Perform the operations indicated:

1. $\frac{1}{2} + \frac{3}{5}$.

9. $\frac{3}{4} - \frac{1}{3}$.

17. $\frac{1}{2}$ of $\frac{3}{4}$.

25. $\frac{1}{5} \div \frac{1}{3}$.

2. $\frac{1}{2} + \frac{1}{3}$.

10. $\frac{1}{2} - \frac{1}{3}$.

18. $\frac{1}{3}$ of $\frac{1}{2}$.

26. $\frac{1}{3} \div \frac{1}{5}$.

3. $\frac{1}{2} + \frac{2}{3}$.

11. $\frac{2}{3} - \frac{1}{2}$.

19. $\frac{1}{2}$ of $\frac{2}{3}$.

27. $\frac{2}{5} \div \frac{1}{3}$.

4. $\frac{1}{3} + \frac{3}{8}$.

12. $\frac{1}{2} - \frac{3}{8}$.

20. $\frac{3}{8}$ of $\frac{1}{2}$.

28. $\frac{1}{3} \div \frac{2}{5}$.

5. $\frac{1}{3} + \frac{5}{8}$.

13. $\frac{5}{8} - \frac{1}{2}$.

21. $\frac{1}{2}$ of $\frac{5}{8}$.

29. $\frac{3}{5} \div \frac{2}{3}$.

6. $\frac{1}{4} + \frac{7}{8}$.

14. $\frac{7}{8} - \frac{1}{4}$.

22. $\frac{1}{4}$ of $\frac{7}{8}$.

30. $\frac{2}{3} \div \frac{3}{5}$.

7. $\frac{3}{4} + \frac{5}{8}$.

15. $\frac{3}{4} - \frac{5}{8}$.

23. $\frac{3}{4}$ of $\frac{5}{8}$.

31. $\frac{2}{3} \div \frac{5}{8}$.

8. $\frac{3}{5} + \frac{7}{8}$.

16. $\frac{7}{8} - \frac{3}{4}$.

24. $\frac{3}{4}$ of $\frac{7}{8}$.

32. $\frac{5}{8} \div \frac{2}{3}$.

ORAL EXERCISE

1. How much is $\frac{1}{2}$ of $\frac{1}{2}$? $\frac{1}{2}$ of $\frac{1}{4}$? $\frac{1}{2}$ of $\frac{1}{8}$? $\frac{1}{2}$ of $\frac{1}{16}$?
2. In working in the shop we need smaller fractions than $\frac{1}{16}$.
If we have three distances, each $\frac{1}{32}$ in., what is their sum?
3. How much is $\frac{1}{3}$ of $\frac{3}{4}$? $\frac{1}{3}$ of $\frac{3}{8}$? $\frac{1}{3}$ of $\frac{3}{16}$? $\frac{1}{3}$ of $\frac{3}{32}$?
4. In adding $\frac{3}{8}$ and $\frac{5}{16}$ what common denominator do we use?
In adding $\frac{3}{8}$ and $\frac{5}{32}$ what common denominator do we use?
5. A piece of blotting paper $\frac{1}{16}$ in. thick is laid upon a piece of cardboard $\frac{1}{32}$ in. thick. How thick are the two together?

WRITTEN EXERCISE

Express as thirty-seconds:

- | | | | | | |
|---------------------|--------------------|--------------------|--------------------|--------------------|---------------------|
| 1. $\frac{1}{16}$. | 2. $\frac{1}{8}$. | 3. $\frac{3}{8}$. | 4. $\frac{7}{8}$. | 5. $\frac{3}{4}$. | 6. $\frac{3}{16}$. |
|---------------------|--------------------|--------------------|--------------------|--------------------|---------------------|

Express as sixteenths:

- | | | | | | |
|---------------------|---------------------|---------------------|-----------------------|-----------------------|----------------------|
| 7. $\frac{2}{32}$. | 8. $\frac{4}{32}$. | 9. $\frac{8}{32}$. | 10. $\frac{16}{32}$. | 11. $\frac{30}{32}$. | 12. $\frac{4}{64}$. |
|---------------------|---------------------|---------------------|-----------------------|-----------------------|----------------------|

Express as twenty-fourths:

- | | | | | | |
|----------------------|----------------------|----------------------|---------------------|---------------------|---------------------|
| 13. $\frac{1}{12}$. | 14. $\frac{5}{12}$. | 15. $\frac{7}{12}$. | 16. $\frac{3}{8}$. | 17. $\frac{5}{6}$. | 18. $\frac{7}{6}$. |
|----------------------|----------------------|----------------------|---------------------|---------------------|---------------------|

Express as twelfths:

- | | | | | | |
|----------------------|----------------------|-----------------------|----------------------|-----------------------|----------------------|
| 19. $\frac{2}{24}$. | 20. $\frac{6}{24}$. | 21. $\frac{22}{24}$. | 22. $\frac{3}{36}$. | 23. $\frac{15}{36}$. | 24. $\frac{4}{48}$. |
|----------------------|----------------------|-----------------------|----------------------|-----------------------|----------------------|

Find the value of:

- | | | | |
|---------------------------------------|---------------------------------------|---------------------------------------|---------------------------------------|
| 25. $\frac{1}{2}$ of $\frac{1}{16}$. | 29. $\frac{1}{2}$ of $\frac{1}{9}$. | 33. $\frac{1}{2}$ of $\frac{1}{12}$. | 37. $\frac{1}{2}$ of $\frac{1}{18}$. |
| 26. $\frac{1}{4}$ of $\frac{1}{8}$. | 30. $\frac{1}{3}$ of $\frac{1}{6}$. | 34. $\frac{1}{3}$ of $\frac{1}{8}$. | 38. $\frac{1}{4}$ of $\frac{1}{9}$. |
| 27. $\frac{1}{2}$ of $\frac{1}{10}$. | 31. $\frac{1}{2}$ of $\frac{1}{11}$. | 35. $\frac{1}{4}$ of $\frac{1}{6}$. | 39. $\frac{1}{3}$ of $\frac{1}{12}$. |
| 28. $\frac{1}{4}$ of $\frac{1}{5}$. | 32. $\frac{1}{11}$ of $\frac{1}{2}$. | 36. $\frac{1}{6}$ of $\frac{1}{4}$. | 40. $\frac{1}{6}$ of $\frac{1}{6}$. |

ORAL EXERCISE

1. If you are making a careful measurement and find the thickness of a board to be $\frac{4}{32}$ in., how will you express this in lowest terms? Express $\frac{8}{32}$ in lowest terms.

2. A piece of plate glass is found to be $\frac{1}{6}\frac{6}{4}$ in. thick. Express this in lowest terms.

3. A distance is given as $\frac{75}{100}$ mi. Express this in lowest terms. Express $\frac{80}{100}$ in lowest terms.

4. In measuring a piece of steel a workman uses an instrument that shows the thickness to be $\frac{6}{48}$ in. Express this in lowest terms.

5. The width of sheeting is expressed in fourths of a yard. The width of a certain piece is $\frac{1}{4}$. Express this in yards.

WRITTEN EXERCISE

Perform the operations indicated:

1. $\frac{7}{8} + \frac{1}{2}$.

7. $\frac{1}{2} + \frac{1}{3}$.

13. $\frac{3}{4} + \frac{1}{3}$.

19. $\frac{3}{5} + \frac{1}{2}$.

2. $\frac{7}{8} - \frac{1}{2}$.

8. $\frac{1}{2} - \frac{1}{3}$.

14. $\frac{3}{4} - \frac{1}{3}$.

20. $\frac{3}{5} - \frac{1}{2}$.

3. $\frac{7}{8}$ of $\frac{1}{2}$.

9. $\frac{1}{2}$ of $\frac{1}{3}$.

15. $\frac{3}{4}$ of $\frac{1}{3}$.

21. $\frac{3}{5}$ of $\frac{1}{2}$.

4. $\frac{1}{2}$ of $\frac{7}{8}$.

10. $\frac{1}{3}$ of $\frac{1}{2}$.

16. $\frac{1}{3}$ of $\frac{3}{4}$.

22. $\frac{1}{2}$ of $\frac{3}{5}$.

5. $\frac{7}{8} \div \frac{1}{2}$.

11. $\frac{1}{2} \div \frac{1}{3}$.

17. $\frac{3}{4} \div \frac{1}{3}$.

23. $\frac{3}{5} \div \frac{1}{2}$.

6. $\frac{1}{2} \div \frac{7}{8}$.

12. $\frac{1}{3} \div \frac{1}{2}$.

18. $\frac{1}{3} \div \frac{3}{4}$.

24. $\frac{1}{2} \div \frac{3}{5}$.

25. Each side of a window box 27" long is nailed to the bottom, the nails being placed $1\frac{1}{2}$ " from each end, and 6" apart. How many nails will be needed for the two sides?

26. If a box with vertical sides is 4" wide and $24\frac{1}{4}$ " long, inside measure, and is filled with earth to a depth of $4\frac{1}{2}$ ", how many cubic inches of earth will be used?

27. A desk is 4 ft. long and $2\frac{1}{4}$ ft. wide. What is the perimeter of the desk?

28. A field is $10\frac{3}{4}$ rd. long and $6\frac{1}{4}$ rd. wide. What is the perimeter of the field?

29. A playground is $30\frac{2}{3}$ yd. long and $19\frac{1}{3}$ yd. wide. What is the perimeter of the playground?

30. In making a toy engine a workman fastened a plate of iron $\frac{1}{16}$ in. thick on a plate $\frac{1}{32}$ in. thick. What was the total thickness of the double plate?

31. In making a paper box a boy pasted some paper $\frac{1}{64}$ in. thick on some strawboard $\frac{3}{32}$ in. thick. How thick was the material for the box then?

32. A man in a machine shop winds some wire $\frac{1}{64}$ in. in diameter closely around a piece of steel that is $\frac{5}{8}$ in. in diameter. What is then the diameter of the wound piece of steel?

33. In making a trellis for some flowers the class used wood that was $\frac{3}{8}$ in. thick. The wood was then planed down $\frac{1}{32}$ in. How thick was the wood after planing?

34. In making a box the wood was $\frac{5}{16}$ in. thick. It was then carefully sandpapered and lost $\frac{1}{64}$ in. in thickness. How thick was it after being sandpapered?

35. From a pad of paper $\frac{7}{16}$ in. thick, paper was torn off, reducing its thickness $\frac{3}{64}$ in. How thick was the pad then?

36. From a piece of ribbon $4\frac{3}{4}$ yd. long there is cut $2\frac{3}{8}$ yd. How much ribbon is left?

37. A triangular flower bed is $12\frac{1}{3}$ ft. on each side. What is the perimeter of the flower bed?

38. In making a box, three pieces of cardboard, each $\frac{1}{64}$ in. thick, are glued together. What is the entire thickness?

ORAL EXERCISE

1. If you have $\frac{3}{8}$ yd. of cloth and need $\frac{1}{3}$ of it for a piece of work, what part of a yard do you need?

2. If the class has $\frac{7}{8}$ gal. of milk and $\frac{4}{7}$ of it is used in one lesson, what part of a gallon is used?

Think of $\frac{4}{7} \times \frac{7}{8}$, and cancel 4 and 7 mentally.

$\frac{1}{3}$ of $\frac{3}{8}$ is often written $\frac{1}{3} \times \frac{3}{8}$, $\times$ being read "of" after a fraction, but "times" after a whole number or a mixed number.

3. If the distance to a certain house is $\frac{5}{16}$ mi., how far has a boy gone when he has covered $\frac{4}{5}$ of this distance?

Find the value of:

- | | | | |
|-------------------------------------|-------------------------------------|-------------------------------------|--------------------------------------|
| 4. $\frac{1}{2}$ of $\frac{1}{2}$. | 6. $\frac{1}{4}$ of $\frac{1}{2}$. | 8. $\frac{1}{2}$ of $\frac{2}{3}$. | 10. $\frac{1}{3}$ of $\frac{1}{4}$. |
| 5. $\frac{1}{2}$ of $\frac{1}{4}$. | 7. $\frac{1}{2}$ of $\frac{1}{3}$. | 9. $\frac{2}{3}$ of $\frac{1}{2}$. | 11. $\frac{1}{3}$ of $\frac{3}{4}$. |

WRITTEN EXERCISE

Add the following:

- | | | |
|---|--|--|
| 1. $4\frac{1}{3}$, $3\frac{2}{3}$, $5\frac{1}{3}$. | 3. $4\frac{7}{8}$, $3\frac{2}{3}$, $5\frac{1}{2}\frac{1}{4}$. | 5. $2\frac{1}{10}$, $16\frac{3}{20}$, $27\frac{7}{30}$. |
| 2. $2\frac{1}{8}$, $1\frac{1}{8}$, $6\frac{3}{8}$. | 4. $3\frac{1}{16}$, $5\frac{3}{32}$, $7\frac{11}{64}$. | 6. $8\frac{7}{16}$, $15\frac{1}{2}\frac{5}{8}$, $73\frac{5}{6}\frac{1}{4}$. |
7. The sides of a triangle are $3\frac{3}{4}$ in., $2\frac{7}{8}$ in., and $3\frac{1}{2}$ in., respectively. What is the perimeter?

Subtract the following:

- | | | |
|--|---------------------------------------|---------------------------------------|
| 8. $28\frac{5}{8} - 7\frac{2}{3}\frac{5}{2}$. | 10. $12\frac{21}{3} - 8\frac{1}{4}$. | 12. $31\frac{5}{6} - 7\frac{7}{12}$. |
| 9. $32\frac{7}{8} - 9\frac{3}{32}$. | 11. $15\frac{1}{4} - 7\frac{1}{3}$. | 13. $42\frac{5}{6} - 8\frac{5}{12}$. |
14. From the sum of $16\frac{1}{2}$ ft. and $4\frac{1}{2}$ ft. subtract $12\frac{3}{4}$ ft.

Find the value of:

- | | | | |
|--------------------------------------|---------------------------------------|---------------------------------------|---|
| 15. $\frac{1}{2}$ of $\frac{2}{3}$. | 17. $\frac{5}{8}$ of $\frac{4}{5}$. | 19. $\frac{1}{16}$ of $\frac{2}{3}$. | 21. $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$. |
| 16. $\frac{2}{3}$ of $\frac{1}{2}$. | 18. $\frac{3}{8}$ of $\frac{8}{21}$. | 20. $\frac{1}{16}$ of $\frac{2}{5}$. | 22. $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$. |

23. A piece of tin $\frac{1}{40}$ in. thick is folded once. How thick are both parts together?

24. A piece of paper $\frac{1}{60}$ in. thick is folded once, and then folded again, making four leaves. How thick are the four leaves together?

25. If the height of the upright piece in a sundial is $\frac{5}{32}$ of the diameter of the dial and the diameter of the dial is 8 in., what is the height of the upright piece?

26. If the length of a paddle of a water wheel is $\frac{3}{7}$ of the diameter and the diameter is 21 in., what is the length of the paddle?

27. A certain recipe calls for 8 oz. of sugar. To make $2\frac{1}{2}$ times the recipe, how many ounces of sugar must we use?

28. A certain recipe calls for 2 cups of cream. To make $3\frac{1}{2}$ times the recipe, how much cream must we use?

29. If an automobile goes a mile in 59 sec. (or $\frac{59}{60}$ min.) in a certain race, how long will it take to go $\frac{3}{4}$ mi. at the same rate?

30. A man has a city lot that contains $\frac{1}{2}\frac{2}{5}$ A. If he sells $\frac{5}{8}$ of it, how much does he sell? How much has he left?

31. If it takes $2\frac{7}{8}$ yd. of cloth to make a boy's suit, how many yards will it take to make a dozen suits?

32. A tin cup is found to hold $\frac{1}{16}\frac{5}{6}$ pt. When it is $\frac{2}{3}$ full the cup contains what part of a pint?

33. If a glass jar holds $\frac{1}{16}\frac{5}{6}$ qt., what part of a quart will it contain when $\frac{4}{5}$ full? What part of a quart will it contain when $\frac{2}{5}$ full? when $\frac{3}{5}$ full?

34. In making the plans for a house an architect wishes to draw a line that shall be $\frac{2}{5}$ as long as one that is $\frac{1}{16}\frac{5}{6}$ in. long. How long shall he draw it?

ORAL EXERCISE

1. A fence $\frac{1}{3}$ mi. long incloses a square lot. What is the length of each side?
2. If we use $\frac{1}{4}$ of a cup of cream in making a dozen cookies, what part of a cup do we use for each cooky?
3. If a man divides a garden that contains $2\frac{5}{8}$ of an acre into five equal parts, what part of an acre is in each part?
4. If we cut $2\frac{1}{2}$ yd. of cloth into two equal parts, how long is each part?
5. If we have a spelling slip $10\frac{1}{2}$ in. long and fold it cross-wise into two equal parts, how long is each part?

WRITTEN EXERCISE

1. If a sheet of veneer is $\frac{1}{16}$ in. thick, how many sheets pressed together will have a thickness of $1\frac{1}{4}$ in.?
2. If a sheet of blotting paper is $\frac{1}{32}$ in. thick, how many sheets are there in a pile that is $1\frac{3}{4}$ in. thick?
3. If a bicycle is going at the rate of a mile in $\frac{1}{8}$ hr., how many miles will it go in $\frac{3}{4}$ hr.?
4. If a wagon wheel makes $\frac{1}{18}$ of a revolution in going 1 ft., how many feet will it go in making 7 revolutions?

Divide the following :

5. $5 \div \frac{3}{4}$.

6. $75 \div \frac{3}{8}$.

7. $12\frac{1}{2} \div \frac{5}{8}$.

8. $24\frac{1}{2} \div \frac{1}{2}$.

Multiply the following :

9. $2\frac{1}{2} \times 3$.

10. $3\frac{2}{3} \times 9$.

11. $5\frac{1}{8} \times 2$.

12. $3\frac{7}{8} \times 2\frac{3}{4}$.

Divide the following :

13. $\frac{3}{4} \div \frac{4}{5}$.

14. $\frac{4}{5} \div \frac{3}{4}$.

15. $\frac{2}{3} \div \frac{4}{5}$.

16. $\frac{4}{5} \div \frac{2}{3}$.

17. If 2 lb. of flour are produced from $2\frac{1}{2}$ lb. of wheat, the weight of the flour is what part the weight of the wheat?

18. If a cubic inch of copper weighs $5\frac{3}{10}$ oz., how many cubic inches in a copper wire weighing $10\frac{3}{10}$ oz.?

19. If a sheet of paper and an envelope together weigh $\frac{5}{16}$ oz., how much will 32 sheets of paper and 32 envelopes weigh? how much will 48 sheets of paper and 48 envelopes weigh?

20. A dealer bought 2500 bu. of wheat at $83\frac{1}{2}$ ¢ a bushel and sold it at a profit of \$37.50. What was the selling price per bushel?

21. The diameter of a bicycle wheel is $2\frac{1}{3}$ ft. and the circumference is $3\frac{1}{7}$ times the diameter. How many times will the wheel turn in going 1 mi. (5280 ft.)?

22. A grocer bought a barrel of sugar weighing 432 lb. for $4\frac{7}{8}$ ¢ a pound and sold it for $5\frac{1}{4}$ ¢ a pound. How much did he gain by the transaction?

23. Two men are working on the same job for the same wages. One works 4 da. and the other 5 da. What part of the total wages should each receive?

24. If a certain quality of wheat produces $\frac{1}{2}\frac{7}{10}$ of its weight in flour, how many pounds of flour will be produced from 320 lb. of wheat?

25. If a certain quality of wheat produces $\frac{1}{2}\frac{7}{10}$ of its weight in flour, how many pounds of wheat are necessary to produce 340 lb. of flour?

26. From a bin containing $32\frac{5}{8}$ bu. of wheat there were taken out at various times $5\frac{1}{4}$ bu., $3\frac{7}{8}$ bu., and $6\frac{1}{2}$ bu. There were added during this time $3\frac{3}{4}$ bu. and $7\frac{1}{4}$ bu. How many bushels were there then in the bin?

ORAL EXERCISE

Read aloud the following :

1. 0.2.	9. 1.6.	17. 6.125.	25. 0.122.
2. 0.7.	10. 2.9.	18. 7.335.	26. 0.302.
3. 0.9.	11. 3.5.	19. 0.475.	27. 2.002.
4. 0.12.	12. 4.75.	20. 5.500.	28. 0.007.
5. 0.65.	13. 5.35.	21. 5.005.	29. 9.009.
6. 0.70.	14. 6.40.	22. 6.075.	30. 0.225.
7. 0.85.	15. 7.08.	23. 6.750.	31. 200.025.
8. 0.09.	16. 7.10.	24. 0.875.	32. 220.005.

Read aloud as dollars and decimals of a dollar :

33. \$1.25.	34. \$25.05.	35. \$26.70.	36. \$125.50.
-------------	--------------	--------------	---------------

WRITTEN EXERCISE

Write in figures :

- Seven tenths; one tenth; five tenths; nine tenths.
- Twelve hundredths; fifty-five hundredths; nine hundredths; one hundredth; twenty hundredths.
- One hundred twenty-five thousandths; forty-seven thousandths; ten thousandths; five thousandths.
- One hundred fifty thousandths; seven hundred sixty-five thousandths; seventy-five thousandths; eight thousandths.
- Seventeen thousandths; two thousandths; two hundred six thousandths; thirty-five thousandths; seven thousandths; nineteen thousandths; twenty-seven thousandths.
- One hundred twenty-five and sixty-two thousandths; one hundred and seventy-eight thousandths; one hundred seventy-eight thousandths; one hundred and one thousandth.

Write as decimal fractions :

- | | | | | |
|----------------------|------------------------|--------------------------|--------------------------|--------------------------|
| 7. $\frac{1}{10}$. | 11. $\frac{4}{10}$. | 15. $\frac{232}{1000}$. | 19. $\frac{723}{1000}$. | 23. $\frac{3}{10}$. |
| 8. $\frac{6}{10}$. | 12. $\frac{5}{10}$. | 16. $\frac{348}{1000}$. | 20. $\frac{418}{1000}$. | 24. $\frac{30}{100}$. |
| 9. $\frac{2}{10}$. | 13. $\frac{7}{10}$. | 17. $\frac{29}{1000}$. | 21. $\frac{328}{1000}$. | 25. $\frac{300}{1000}$. |
| 10. $\frac{8}{10}$. | 14. $\frac{62}{100}$. | 18. $\frac{37}{1000}$. | 22. $\frac{25}{1000}$. | 26. $\frac{350}{1000}$. |

Write as whole numbers or mixed decimals :

- | | | | | |
|-----------------------|-----------------------|-------------------------|---------------------------|---------------------------|
| 27. $\frac{11}{10}$. | 31. $\frac{12}{10}$. | 35. $\frac{123}{100}$. | 39. $\frac{6225}{1000}$. | 43. $\frac{7225}{1000}$. |
| 28. $\frac{13}{10}$. | 32. $\frac{15}{10}$. | 36. $\frac{225}{100}$. | 40. $\frac{3007}{1000}$. | 44. $\frac{2800}{1000}$. |
| 29. $\frac{16}{10}$. | 33. $\frac{27}{10}$. | 37. $\frac{625}{100}$. | 41. $\frac{4040}{1000}$. | 45. $\frac{7805}{1000}$. |
| 30. $\frac{20}{10}$. | 34. $\frac{38}{10}$. | 38. $\frac{450}{100}$. | 42. $\frac{4500}{1000}$. | 46. $\frac{6200}{1000}$. |

Add, and also subtract :

- | 47. | 48. | 49. | 50. | 51. |
|---------------|---------------|---------------|---------------|---------------|
| 127.42 | 403.61 | 512.71 | 623.42 | 803.40 |
| <u>109.64</u> | <u>362.83</u> | <u>329.86</u> | <u>435.64</u> | <u>637.68</u> |

Multiply the following :

- | | | |
|-----------------------|------------------------|-------------------------|
| 52. 1.3×48 . | 57. 4.25×64 . | 62. 3.42×286 . |
| 53. 2.9×62 . | 58. 6.75×84 . | 63. 4.25×664 . |
| 54. 4.7×45 . | 59. 5.24×72 . | 64. 7.75×384 . |
| 55. 8.2×68 . | 60. 8.36×48 . | 65. 5.33×296 . |
| 56. 7.7×88 . | 61. 9.82×67 . | 66. 4.56×821 . |

Divide, carrying the quotients to two decimal places :

- | | | |
|----------------------|----------------------|-----------------------|
| 67. $6.25 \div 25$. | 71. $3.49 \div 26$. | 75. $8.34 \div 26$. |
| 68. $6.75 \div 75$. | 72. $47.2 \div 34$. | 76. $342.8 \div 16$. |
| 69. $8.48 \div 16$. | 73. $39.4 \div 28$. | 77. $426.9 \div 31$. |
| 70. $7.72 \div 16$. | 74. $6.42 \div 77$. | 78. $54.68 \div 62$. |

132. Aliquot Parts of a Dollar. You have already learned that:

$$50\text{¢} = \$\frac{1}{2}$$

$$12\frac{1}{2}\text{¢} = \$\frac{1}{8}$$

$$16\frac{2}{3}\text{¢} = \$\frac{1}{6}$$

$$25\text{¢} = \$\frac{1}{4}$$

$$33\frac{1}{3}\text{¢} = \$\frac{1}{3}$$

$$20\text{¢} = \$\frac{1}{5}$$

It is convenient to use these parts of a dollar in solving problems.

For example, if 1 yd. of cloth costs $33\frac{1}{3}\text{¢}$, how much will 27 yd. cost?

Instead of taking $27 \times 33\frac{1}{3}\text{¢}$, it is easier to take $27 \times \$\frac{1}{3} = \9 .

WRITTEN EXERCISE

Find the cost of:

1. 36 baseball bats @ 50¢.

2. 48 skating caps @ 50¢.

3. 36 postcard albums @ $66\frac{2}{3}\text{¢}$.

Notice that $66\frac{2}{3}\text{¢} = 2 \times 33\frac{1}{3}\text{¢} = \$\frac{2}{3}$.

4. 66 dog collars @ $33\frac{1}{3}\text{¢}$, and 36 @ $66\frac{2}{3}\text{¢}$.

5. 24 football inflaters @ 10¢; @ $12\frac{1}{2}\text{¢}$.

6. 28 pairs of dumb-bells @ 20¢; @ 25¢.

7. 48 pairs of Indian clubs @ 25¢; @ $33\frac{1}{3}\text{¢}$; @ 50¢.

8. 56 dog muzzles @ 25¢; @ 50¢; $37\frac{1}{2}\text{¢}$.

Notice that $37\frac{1}{2}\text{¢} = 3 \times 12\frac{1}{2}\text{¢} = 3 \times \$\frac{1}{8} = \$\frac{3}{8}$.

9. 16 tennis-racket covers @ \$1; @ \$1.25; @ 75¢.

75¢ is how many times 25¢? Then 75¢ is what part of \$1?

10. If 3 croquet sets cost \$4.35, how much will 6 croquet sets cost? Solve by one multiplication.

11. If 8 pairs of skates cost \$10, how much will 2 pairs of skates cost? Solve by one division.

12. If 9 fishing rods cost \$11.25, how much will 3 fishing rods cost?

13. If 4 tennis rackets cost \$8, how much will 2 rackets cost?
14. If 12 fishing reels cost \$7.80, how much will 3 fishing reels cost? how much will 4 fishing reels cost?
15. If 4 spoon hooks cost \$1.76, find how much 2 spoon hooks will cost. Then find how much 6 spoon hooks will cost.
16. If 6 trout baskets cost \$5.40, find how much 3 trout baskets will cost. Then find how much 9 trout baskets will cost.
17. If 2 folding camp cots cost \$4.50, find how much one will cost. Then find how much three will cost.
18. If 3 cameras cost \$14.25, how much will 4 cameras cost? Solve by one division and one addition, or else by one multiplication by a mixed number.
19. If 6 telegraph outfits for learners cost \$17.10, how much will 2 outfits cost? how much will 4 outfits cost?
20. If 3 music bags cost \$2.94, how much will 9 music bags cost? how much will 11 music bags cost?
21. If 2 German accordions cost \$7.30, how much will 3 German accordions cost?
22. If 2 violins cost \$9.50, how much will 5 violins cost? Solve by one multiplication.
23. If 3 banjos cost \$6.75, how much will 7 banjos cost? Solve by one multiplication.
24. If 4 mandolins cost \$11.40, how much will 6 mandolins cost? Solve by one multiplication.
25. If 3 shotguns cost \$29.25, how much will 15 shotguns cost?
26. If 6 boxes of writing paper cost \$1.80, how much will 9 boxes of writing paper cost? how much will 12 boxes cost? how much will 15 boxes cost?

II. DECIMAL FRACTIONS

133. Meaning of Decimal Fractions. We have already learned the meaning of decimal fractions, but before continuing our work we shall again consider this meaning. We know that \$2.25 means \$2 + 25 cents, which is the same as $\$2\frac{25}{100}$, or $\$2\frac{1}{4}$.

In the same way, 2.25 miles means $2\frac{25}{100}$ miles, or $2\frac{1}{4}$ miles; 1.50 means $1\frac{50}{100}$, or $1\frac{1}{2}$; 1.75 means $1\frac{75}{100}$, or $1\frac{3}{4}$; and 2.125 means $2\frac{125}{1000}$, or $2\frac{1}{8}$.

In the same way we see that

$$\begin{array}{lll} \frac{6}{10} = 0.6 & \frac{125}{1000} = 0.125 & 4\frac{7}{100} = 4.07 \\ \frac{125}{100} = 1.25 & \frac{1275}{1000} = 1.275 & 4\frac{7}{1000} = 4.007 \end{array}$$

Since we know that $\frac{5}{10} = \frac{50}{100}$, we see that $0.5 = 0.50$.

To change $\frac{4}{3}$ to a decimal fraction, we have $4 \div 3 = 1.333 +$.

It was not necessary to write a zero at the left of the decimal point in the above cases, for 0.5 means the same as .5. The zero is often written there to call attention more quickly to the decimal point.

Pronounce the word *and* at the decimal point only. Thus 100.023 is read "one hundred and twenty-three thousandths," while 0.123 is read "one hundred twenty-three thousandths."

The names of the first six decimal places, beginning at the decimal point, are tenths, hundredths, thousandths, ten-thousandths, hundred-thousandths, millionths. The number 3.423579 is read "3 and 423 thousand 579 millionths."

WRITTEN EXERCISE

Write as decimal fractions:

- | | | | | |
|----------------------|-----------------------|---------------------|---------------------|---------------------|
| 1. $\frac{5}{100}$. | 5. $\frac{25}{100}$. | 9. $\frac{1}{2}$. | 13. $\frac{2}{5}$. | 17. $\frac{3}{8}$. |
| 2. $\frac{7}{100}$. | 6. $\frac{50}{100}$. | 10. $\frac{1}{4}$. | 14. $\frac{3}{4}$. | 18. $\frac{5}{8}$. |
| 3. $\frac{1}{100}$. | 7. $\frac{75}{100}$. | 11. $\frac{1}{5}$. | 15. $\frac{2}{3}$. | 19. $\frac{1}{9}$. |
| 4. $\frac{3}{100}$. | 8. $\frac{37}{100}$. | 12. $\frac{3}{5}$. | 16. $\frac{4}{5}$. | 20. $\frac{7}{8}$. |

Reduce to common fractions in lowest terms or to mixed numbers :

21. 0.2.	31. 0.125.	41. 0.0125.	51. 1.25.
22. 0.4.	32. 0.625.	42. 0.0250.	52. 2.50.
23. 0.6.	33. 0.250.	43. 0.0375.	53. 3.75.
24. 0.8.	34. 0.500.	44. 0.0500.	54. 6.25.
25. 0.10.	35. 0.375.	45. 0.0625.	55. 1.875.
26. 0.15.	36. 0.735.	46. 0.7500.	56. 18.75.
27. 0.25.	37. 0.875.	47. 0.1875.	57. 187.5.
28. $0.12\frac{1}{2}$.	38. $0.333\frac{1}{3}$.	48. $0.3333\frac{1}{3}$.	58. $3.37\frac{1}{2}$.
29. $0.16\frac{2}{3}$.	39. $0.166\frac{2}{3}$.	49. $0.6666\frac{2}{3}$.	59. 3.375.
30. $0.66\frac{2}{3}$.	40. $0.666\frac{2}{3}$.	50. $0.1666\frac{2}{3}$.	60. $3.3\frac{3}{4}$.

Reduce to decimal fractions, carrying the results to three decimal places :

61. $\frac{1}{8}$.	63. $\frac{5}{9}$.	65. $\frac{3}{16}$.	67. $\frac{5}{12}$.	69. $\frac{11}{12}$.
62. $\frac{5}{6}$.	64. $\frac{7}{9}$.	66. $\frac{5}{16}$.	68. $\frac{7}{12}$.	70. $\frac{11}{16}$.

Write in figures :

71. Seven tenths ; one hundredth ; five ten-thousandths.
72. Twelve hundredths ; fifty-five hundredths ; nine hundredths ; one hundred twenty-seven millionths.
73. Two hundred thirty-five thousandths ; ninety-eight thousandths ; six hundred forty-seven thousand five hundred seventy-five millionths.
74. One hundred twenty-five and sixty-two ten-thousandths ; seven hundred and seventy-eight hundred-thousandths ; seven hundred seventy-eight millionths ; one hundred and one ten-thousandth ; one thousand and two ten-thousandths ; one thousand two millionths.

134. Multiplication of a Decimal by a Whole Number. In multiplying a decimal by an integer we proceed exactly as in multiplying United States money.

For example, required to multiply 24.125 by 35.

Since 5 thousandths multiplied by 5 is 25 thousandths, we know that the 5 will go in the thousandths' place.

Proceeding with the multiplication in the ordinary way, we finally place the decimal point in the product before the 375 so as to have the 5 come in the thousandths' place as above stated.

$$\begin{array}{r} 24.125 \\ \quad 35 \\ \hline 120\ 625 \\ 723\ 75 \\ \hline 844.375 \end{array}$$

135. Multiplication of a Decimal by a Mixed Number. Multiplication of a decimal by a mixed number may be illustrated by the case of $2\frac{1}{2} \times 1.48$.

First taking $\frac{1}{2}$ of 1.48, we know that $\frac{1}{2}$ of $1\frac{48}{100} = \frac{48}{200}$, or 0.24. Then 2×8 hundredths = 16 hundredths, so we write the 6 in the hundredths' place. Proceeding in the ordinary way, the sum of the partial products is 3.70.

$$\begin{array}{r} 1.48 \\ \quad 2\frac{1}{2} \\ \hline 74 \\ \hline 2\ 96 \\ \hline 3.70 \end{array}$$

WRITTEN EXERCISE

Multiply the following:

- | | | |
|-----------------|------------------|-------------------|
| 1. 27.25 by 25. | 5. 65.232 by 32. | 9. 313.45 by 72. |
| 2. 61.82 by 37. | 6. 34.375 by 81. | 10. 46.925 by 93. |
| 3. 42.12 by 45. | 7. 68.325 by 50. | 11. 20.005 by 75. |
| 4. 62.09 by 35. | 8. 42.175 by 62. | 12. 29.407 by 87. |

13. If a twelve-story city building averages 10.62 ft. to a story, how tall is the building?

14. If one side of a square is 21.33 in., how long is the perimeter of the square?

15. The measure called the meter, studied later, is 39.37 in. long. How many inches in 62 meters?

ORAL EXERCISE

Multiply the following:

1. 2 in.	\$2	$\frac{2}{10}$	$\frac{2}{5}$	$\frac{2}{100}$	0.2	0.02
<u>3</u>	<u>3</u>	<u>3</u>	<u>3</u>	<u>3</u>	<u>3</u>	<u>3</u>
2. 4 ft.	\$4	$\frac{4}{10}$	$\frac{4}{5}$	$\frac{4}{100}$	0.4	0.04
<u>2</u>	<u>2</u>	<u>2</u>	<u>2</u>	<u>2</u>	<u>2</u>	<u>2</u>

State rapidly the following products:

3. $\frac{1}{10} \times \frac{3}{10}$	$\frac{2}{10} \times \frac{3}{10}$	$\frac{2}{10} \times \frac{5}{10}$	$\frac{5}{10} \times \frac{9}{10}$
4. 0.1×0.3	0.2×0.3	0.2×0.5	0.5×0.9

136. Multiplying by a Decimal Fraction. This is best understood from the following examples:

1. Multiply 6.2 by 3.5.

If we were to multiply 62 by 35 the result would be 2170. And since tenths multiplied by tenths equals hundredths, the result is 21.70; that is, $2\frac{17}{100}$ or $21\frac{7}{100}$ or 21.7.

So we see that we multiply as with whole numbers, and point off as many decimal places in the product, beginning at the right, as there are decimal places in both factors together.

$$\begin{array}{r} 6.2 \\ 3.5 \\ \hline 310 \\ 186 \\ \hline 21.70 \\ 21.7 \end{array}$$

2. Multiply 0.42 by 0.35.

If we were to multiply 42 by 35 the result would be 1470. And since hundredths multiplied by hundredths equals ten-thousandths, the result is 0.1470; that is, $\frac{42}{100} \times \frac{35}{100} = \frac{1470}{10000} = 0.1470 = 0.147$.

$$\begin{array}{r} 0.42 \\ 0.35 \\ \hline 210 \\ 126 \\ \hline 0.1470 \end{array}$$

137. Decimal Places in the Product. In multiplying decimals,

Multiply as with whole numbers, and then, beginning at the right, point off as many decimal places in the product as there are decimal places in all the factors.

ORAL EXERCISE

Multiply the following :

- | | | | |
|---------------------|------------------|------------------|------------------|
| 1. 0.7×3 | 0.9×4 | 0.8×5 | 0.3×7 |
| 2. 0.2×4 | 0.4×8 | 0.9×9 | 0.8×7 |
| 3. 0.1×0.1 | 0.1×0.2 | 0.2×0.3 | 0.2×0.4 |
| 4. 0.3×0.4 | 0.3×0.5 | 0.4×0.4 | 0.4×0.5 |
| 5. 0.6×0.7 | 0.6×0.8 | 0.7×0.8 | 0.8×0.9 |

6. Why will there be three decimal places in the product of 0.75 multiplied by 0.3 ? of 2.325 multiplied by 7 ?

7. How do you know the number of decimal places in the product of 0.62 and 2.79 ? of 2.756 and 3.123 ?

Without multiplying, tell the order of the decimal part of :

- | | | |
|----------------------|---------------------|----------------------|
| 8. 2.8×7.9 | 2.04×6.09 | 1.28×2.67 |
| 9. 1.7×1.44 | 1.23×3.456 | 3.421×0.713 |

WRITTEN EXERCISE

Multiply the following :

- | | | |
|----------------------|-----------------------|-----------------------|
| 1. $2 \times 0.79.$ | 11. $0.27 \times 7.$ | 21. $1.5 \times 67.$ |
| 2. $0.82 \times 8.$ | 12. $8 \times 9.23.$ | 22. $2.7 \times 3.4.$ |
| 3. $3 \times 1.75.$ | 13. $0.9 \times 84.$ | 23. $0.8 \times 74.$ |
| 4. $3 \times 0.96.$ | 14. $0.3 \times 9.9.$ | 24. $0.7 \times 4.9.$ |
| 5. $0.67 \times 9.$ | 15. $4.7 \times 35.$ | 25. $1.4 \times 75.$ |
| 6. $5 \times 2.42.$ | 16. $8.1 \times 8.1.$ | 26. 1.9×2.6 |
| 7. $4 \times 0.78.$ | 17. $48 \times 0.8.$ | 27. $0.6 \times 48.$ |
| 8. $0.83 \times 4.$ | 18. $6.7 \times 1.8.$ | 28. 0.9×4.7 |
| 9. $6 \times 7.23.$ | 19. $0.7 \times 96.$ | 29. $3.8 \times 46.$ |
| 10. $7 \times 0.93.$ | 20. $0.8 \times 9.8.$ | 30. 4.7×3.6 |

Multiply the following :

- | | | |
|---------------------------|----------------------------|--|
| 31. 7.3×4.8 . | 36. 3.49×80.23 . | 41. 0.600×0.500 . |
| 32. 96×9.6 . | 37. 2.99×124.63 . | 42. $6.50 \times \$28.43$. |
| 33. 5.4×3.5 . | 38. 89.1×49.3 . | 43. $0.3 \times \$333.33\frac{1}{3}$. |
| 34. 43.2×98.6 . | 39. 72.9×87.09 . | 44. 8.001×234.56 . |
| 35. 0.823×2946 . | 40. 36.4×49.87 . | 45. $0.25 \times \$648.36$. |

46. At \$17.50 per front foot (that is, per foot fronting the street), what will a piece of land with a frontage of 28.3 ft. cost?

47. A certain tank when full holds 42.8 gal. How much does it contain when 0.37 full?

48. A certain rectangle is 0.78 as wide as it is long. It is 4.9 in. long. What is the perimeter?

49. A certain rectangle is 3.42 times as long as it is wide. It is 4.8 ft. wide. What is the perimeter?

50. A city lot is 45.8 ft. wide and 76.3 ft. deep. What is the area of the lot?

51. The circumference of a circle is 3.1416 times the diameter. If the diameter is 42.27 ft., what is the circumference?

52. How many square rods of land are there in a piece of land 80 rd. wide and 114 rd. long? How many acres? What is it worth at \$87.50 an acre?

53. There are three towns, A, B, C, forming the three corners of a triangle. From A to B is 27.32 mi., from B to C is 1.4 times as far, and the perimeter of the triangle is 92.8 mi. How far is it from C to A?

In all applied problems of this kind, as well as in all abstract work, teachers should urge their pupils to estimate the results in advance, for the reason given on page 22. Every business man does this so as to avoid any absurd result due to an error in computation. In Ex. 53 a fair estimate would be 20 mi.

ORAL EXERCISE

State the values of:

- | | | | |
|---------------------------------|------------------------------|--------------------------------|---------------------|
| 1. $12 \times \frac{1}{12}$ | $3 \times \frac{1}{8}$ | $10 \times \frac{1}{10}$ | 10×0.1 |
| 2. 10×0.2 | 10×0.6 | $10 \times \frac{1}{100}$ | 10×0.01 |
| 3. $100 \times \frac{1}{100}$ | 100×0.01 | 100×0.02 | 100×0.09 |
| 4. $100 \times \frac{2.5}{100}$ | $100 \times \frac{7.5}{100}$ | 100×0.25 | 100×0.75 |
| 5. 100×0.38 | 100×0.75 | $1000 \times \frac{2.5}{1000}$ | 1000×0.025 |

6. We have found that to multiply a decimal fraction by 10, we need only move the decimal point one place in which direction? How is it when we multiply by 100? by 1000?

138. Multiplying by Multiples of 10. On page 19 we found how to multiply by 10, 100, 1000, and so on for other powers of 10. We also found how to multiply by such numbers as 40, 700, and so on. To multiply decimal fractions by such numbers is substantially the same as to multiply dollars and cents. Therefore, to multiply 487.7 by 600 is the same as to multiply by 6, moving the decimal point two places to the right.

To multiply 89.07 by 740 is the same as to multiply by 74, moving the decimal point one place to the right.

WRITTEN EXERCISE

Multiply the following:

- | | | |
|------------------------|---------------------------|----------------------------|
| 1. 40×6.73 . | 8. 900×37.08 . | 15. 6700×495.4 . |
| 2. 30×4.87 . | 9. 400×61.8 . | 16. 2500×374.65 . |
| 3. 60×9.81 . | 10. 1000×1.25 . | 17. 3500×408.08 . |
| 4. 90×86.4 . | 11. 2000×6.725 . | 18. $35,000 \times 1.25$. |
| 5. 80×36.5 . | 12. 6970×43.2 . | 19. 75×12.48 . |
| 6. 70×625.3 . | 13. 9250×67.25 . | 20. 750×1.248 . |
| 7. 80×65.42 . | 14. 8220×350.4 . | 21. 7500×0.1248 . |

WRITTEN EXERCISE

Multiply the following:

1. 1.5×32 .

4. $3\frac{4}{5} \times 6.5$.

7. $9\frac{2}{3} \times 7.2$.

2. $\frac{1}{2} \times 27.3\frac{1}{2}$.

5. $\frac{2}{3} \times 1.6\frac{3}{4}$.

8. $\frac{4}{5} \times 1.9\frac{1}{2}$.

3. $2\frac{1}{2} \times 4.3\frac{2}{3}$.

6. $4\frac{2}{5} \times 1.6\frac{7}{8}$.

9. $5\frac{3}{8} \times 4.6\frac{1}{5}$.

10. State a brief rule for multiplying by a decimal fraction.

11. State a brief rule for multiplying one common fraction by another. Illustrate by $\frac{2}{3}$ of $\frac{5}{7}$.*Multiply as indicated, timing yourself:*

12.

0.735

6.25

13.

1.326

4.32

14.

2.307

53.6

15.

298.7

0.634

16.

812.8

0.437

17.

296.4

0.0124

18.

82.32

0.063

19.

487.6

2.083

20.

\$14.92

2.35

21.

\$15.75

3.42

22.

\$14.87

2.91

23.

\$65.04

9.09

24.

1484

3.2\frac{1}{2}

25.

2636

4.1\frac{1}{4}

26.

3792

6.23\frac{1}{3}

27.

178.32

1.24\frac{2}{3}

28. A cubic foot of water weighs 62.5 lb. Ice is 0.92 as heavy as water, and cork is 0.25 as heavy as water. What does a cubic foot of ice weigh? a cubic foot of cork?

29. Taking the weight of water as in Ex. 28 and knowing that a certain kind of granite is 2.78 times as heavy as water, find the weight of a cubic foot of the granite.

ORAL EXERCISE

1. Divide 10 by 2; 100 by 20. Compare the results.
2. Divide \$2.50 by 5; \$25 by 50. Compare the results.
3. If multiplying both dividend and divisor by the same number does not change the quotient, how does the quotient of $360 \div 4$ compare with that of $36 \div 0.4$?

139. Division by Decimals. Since both dividend and divisor may be multiplied by the same number without changing the quotient,

In the division of decimal fractions, multiply both dividend and divisor by 10 enough times to make the divisor an integer, and divide as in United States money.

Divide 6.25 by 2.5.

Multiplying 6.25 and 2.5 by 10, we have 62.5 and 25.

The result is the same if we divide 62.5 by 25.

Dividing as in United States money, $62.5 \div 25 = 2$ with 12.5 still to be divided. $12.5 \div 25 = 0.5$. Therefore the quotient is 2.5.

We might, if we desired, multiply both 6.25 and 2.5 by 100, giving 625 to be divided by 250. The result would be the same.

2.5	
25	62.5
	50
	12 5
	12 5

WRITTEN EXERCISE

Divide the following:

- | | | |
|------------------------|-------------------------|------------------------|
| 1. $125 \div 25$. | 7. $462 \div 22$. | 13. $625 \div 0.25$. |
| 2. $12.5 \div 2.5$. | 8. $46.2 \div 2.2$. | 14. $600 \div 0.06$. |
| 3. $62.9 \div 3.7$. | 9. $4.62 \div 2.2$. | 15. $629 \div 0.37$. |
| 4. $8.05 \div 3.5$. | 10. $0.462 \div 0.21$. | 16. $7.14 \div 4.2$. |
| 5. $305 \div 0.61$. | 11. $11.56 \div 3.4$. | 17. $30.5 \div 0.61$. |
| 6. $6.232 \div 0.82$. | 12. $17.466 \div 7.1$. | 18. $62.32 \div 8.2$. |

140. Illustrative Problems. 1. Divide 16.35 by 500.

Here it is better to divide both dividend and divisor by 100, canceling the zeros and moving the decimal point two places to the left.

$$\begin{array}{r} 500 \overline{)0.1635} \\ \underline{0.0327} \end{array}$$

2. Divide 3.82 by 2.5, carrying the quotient to one decimal place.

Multiplying both numbers by 10, we have $38.2 \div 25 = 1.5$ with 7 remainder. If we do not write the fraction $\frac{7}{25}$ in the quotient, the quotient is written 1.5 +, meaning 1.5 and a fraction less than $\frac{1}{2}$.

3. Divide 0.049 by 0.17, carrying the quotient to two decimal places.

Multiplying both numbers by 100, we have $4.9 \div 17 = 0.28$ with 14 remainder. If we do not write the fraction $\frac{14}{17}$ in the quotient, the quotient is written 0.29 -, meaning 0.28 and a fraction more than $\frac{1}{2}$.

WRITTEN EXERCISE

Divide the following :

1. $17.25 \div 500.$

7. $15.720 \div 12,000.$

2. $3.248 \div 200.$

8. $133.10 \div 11,000.$

3. $817.6 \div 700.$

9. $404.30 \div 13,000.$

4. $82.48 \div 8000.$

10. $\$6844.50 \div 900.$

5. $0.7364 \div 7000.$

11. $\$7337.75 \div 500.$

6. $74.34 \div 9000.$

12. $\$1247.01 \div 300.$

In the following divisions, carry the quotient to three decimal places :

13. $17.78 \div 1.5.$

17. $423.6 \div 0.27.$

21. $97.634 \div 0.125.$

14. $62.23 \div 0.23.$

18. $167.8 \div 3.7.$

22. $483.62 \div 42.7.$

15. $19.26 \div 3.1.$

19. $8.348 \div 2.68.$

23. $826.3 \div 12.5.$

16. $21.43 \div 3.7.$

20. $41.73 \div 2.18.$

24. $42.78 \div 3.162.$

WRITTEN EXERCISE

Divide the following:

- | | | |
|------------------------|--------------------------|--------------------------|
| 1. $5.12 \div 0.4$. | 19. $14.04 \div 0.9$. | 37. $0.051 \div 1.7$. |
| 2. $6.75 \div 0.5$. | 20. $16.02 \div 0.6$. | 38. $4.06 \div 0.029$. |
| 3. $5.74 \div 0.7$. | 21. $17.01 \div 0.3$. | 39. $97.02 \div 0.9$. |
| 4. $6.64 \div 0.8$. | 22. $15.04 \div 0.4$. | 40. $39.12 \div 0.3$. |
| 5. $7.92 \div 0.9$. | 23. $27.63 \div 0.03$. | 41. $28.64 \div 0.8$. |
| 6. $7.86 \div 0.6$. | 24. $231.2 \div 0.17$. | 42. $32.16 \div 0.8$. |
| 7. $8.64 \div 200$. | 25. $0.057 \div 1.9$. | 43. $41.06 \div 0.2$. |
| 8. $96.8 \div 400$. | 26. $8.12 \div 0.029$. | 44. $15.47 \div 0.7$. |
| 9. $73.2 \div 300$. | 27. $36.08 \div 3.28$. | 45. $52.02 \div 0.03$. |
| 10. $5.85 \div 500$. | 28. $1.036 \div 0.28$. | 46. $4.875 \div 0.05$. |
| 11. $4.72 \div 0.02$. | 29. $62.32 \div 0.41$. | 47. $49.6 \div 0.004$. |
| 12. $5.64 \div 0.03$. | 30. $115.6 \div 0.17$. | 48. $37.2 \div 0.006$. |
| 13. $7.64 \div 0.04$. | 31. $3.509 \div 12.1$. | 49. $28.8 \div 0.012$. |
| 14. $7.65 \div 0.05$. | 32. $1.9044 \div 4.6$. | 50. $0.812 \div 0.29$. |
| 15. $87.6 \div 0.06$. | 33. $1.0989 \div 3.3$. | 51. $8.118 \div 0.615$. |
| 16. $37.1 \div 0.07$. | 34. $0.8118 \div 1.23$. | 52. $55.2 \div 0.06$. |
| 17. $0.475 \div 2.5$. | 35. $19.536 \div 22.2$. | 53. $0.057 \div 3.8$. |
| 18. $1.488 \div 1.2$. | 36. $17.139 \div 59.1$. | 54. $195.36 \div 2.22$. |

55. How many times will a steam roller 12.86 ft. in circumference turn in going 141.46 ft.?

56. A merchant paid \$61.20 for some wheat at \$0.68 per bushel. How many bushels of wheat did he buy?

57. A wheel is 33 in. in diameter, and the circumference is 3.1416 times as much. How many times will the wheel turn in going 10,367.28 in.?

141. Cancellation with Decimals. What is the quotient of 17.8×45.39 divided by 0.267 ?

Multiplying both dividend and divisor by 1000 to make the divisor integral, we have for the dividend 178×4539 (that is, 10 times 17.8 and 100 times 45.39, which is the same as 1000 times the product of 17.8 and 45.39). Factoring and canceling, we have $2 \times 1513 = 3026$.

$ \begin{array}{r} 2 \times \cancel{89} \quad 1513 \\ \underline{17\cancel{8} \times \cancel{45}39} \\ 267 \\ \cancel{3} \times \cancel{89} \end{array} $
--

WRITTEN EXERCISE

Find the value of:

1. $\frac{3.9 \times 0.84}{15.6}$.

4. $\frac{5.5 \times 0.42}{0.77}$.

7. $\frac{2.1 \times 0.4}{0.015}$.

2. $\frac{1.4 \times 0.6}{0.63}$.

5. $\frac{4.5 \times 6.3}{8.1}$.

8. $\frac{0.21 \times 0.6}{0.126}$.

3. $\frac{2.2 \times 2.4}{4 \times 7.7}$.

6. $\frac{5.6 \times 0.3}{0.63 \times 0.8}$.

9. $\frac{1.8 \times 5.5}{9.9 \times 0.1}$.

10. The product of 6.3, 1.4, and 5.7 is how many times as great as the product of 17.1, 0.06, and 7?

11. The product of 0.2, 0.21, and 7.5 is how many times as great as the product of 0.14, 0.3, and 1.5?

12. How many times is the product of 1.6, 0.01, and 1.5 contained in the product of 0.032, 2.25, and 3?

Find the value of:

13. $\frac{0.8 \times 1.5 \times 0.6}{3.6 \times 0.2 \times 10}$.

15. $\frac{0.2 \times 3.6 \times 1.43}{9.9 \times 0.4 \times 2.6}$.

14. $\frac{1.2 \times 0.39 \times 1.4}{2.6 \times 0.42 \times 2}$.

16. $\frac{6.3 \times 4.2 \times 0.17}{2.7 \times 3.4 \times 0.49}$.

17. A block of marble 3.5 ft. by $4\frac{1}{4}$ ft. by 2.2 ft. weighs how many times as much as one that is $2\frac{1}{2}$ ft. by 7.7 ft. by 0.85 ft.?

WRITTEN REVIEW

Add the following:

1.	2.	3.	4.
240.4	64.28	33.21	169.28
802.2	85.	150.5	3.7
<u>388.7</u>	<u>108.51</u>	<u>46.48</u>	<u>2.33</u>
5.	6.	7.	8.
4.028	0.387	1.043	0.2909
.396	.0426	.427	.0628
.285	.0539	6.091	.0703
<u>3.942</u>	<u>.0708</u>	<u>.709</u>	<u>.0289</u>

Add, and also subtract:

9.	10.	11.	12.
90.32	209.8	6.273	81.04
<u>4.86</u>	<u>49.2</u>	<u>.992</u>	<u>6.98</u>
13.	14.	15.	16.
1.035	0.2809	82.481	64.035
<u>.876</u>	<u>.1589</u>	<u>2.909</u>	<u>37.029</u>
17.	18.	19.	20.
86.431	102.98	62.035	80.289
<u>28.902</u>	<u>72.89</u>	<u>51.975</u>	<u>79.939</u>

Multiply or divide, as indicated:

- | | | |
|---------------------------|----------------------------|---------------------------|
| 21. 1.4×6.64 . | 26. 5.8×439.002 . | 31. $20.72 \div 0.28$. |
| 22. 3.6×72.08 . | 27. $825.468 \div 4.2$. | 32. $38.8692 \div 5.4$. |
| 23. $888.16 \div 2.8$. | 28. $1131.264 \div 4.8$. | 33. $388.692 \div 5.4$. |
| 24. $103.60 \div 0.56$. | 29. 1.6×140.64 . | 34. $3.88692 \div 5.4$. |
| 25. 1.5×553.35 . | 30. 6.2×53.134 . | 35. $38869.2 \div 0.54$. |

Divide the following:

- | | | |
|-----------------------|------------------------|-------------------------|
| 36. $1.6 \div 0.4$. | 39. $2.22 \div 0.02$. | 42. $36.08 \div 3.28$. |
| 37. $0.25 \div 0.5$. | 40. $3.33 \div 0.03$. | 43. $1.036 \div 0.56$. |
| 38. $4.9 \div 0.7$. | 41. $3.64 \div 0.04$. | 44. $62.32 \div 0.82$. |

45. A man pays \$113 for coal at \$5.65 a ton. How many tons does he buy?

46. The product of two numbers is 156.25, and one of the numbers is 5000. What is the other number?

47. The product of two numbers is 6.51021, and one of the numbers is 3.207. What is the other number?

48. If wind blows in a hurricane at the rate of 8796.6 ft. per minute, what is the rate per second?

49. A knot is 1.152 mi. When a steamship is traveling 23.04 mi. an hour, what is its rate in knots?

50. A contractor bought a shipment of bricks for \$176.40, paying \$7.20 a thousand. How many bricks did he buy?

51. A dealer paid \$54.40 freight on some oil at \$0.68 per thousand pounds. How many pounds were there?

52. A contractor bought a lot of shingles for \$717.75, paying \$4.12 $\frac{1}{2}$ a thousand. How many thousand shingles did he buy?

53. Wheat that cost 98¢ a bushel is sold for \$1.04 a bushel. How many bushels must be sold to realize a profit of \$132?

54. How many times will a steam roller 4.25 ft. in circumference turn in rolling the length of a lawn 136 ft. long?

55. How many lengths of fence, each 12.75 ft. long, will be required to inclose a square field 522.75 ft. on a side?

56. A cubic foot of a certain kind of granite weighs 173.75 lb., and a cubic foot of water weighs 62.5 lb. The granite is how many times as heavy as water?

III. PER CENTS

142. Per Cent. Another name for *hundredths* is *per cent*. For example, instead of saying "five hundredths" we may say "five per cent." The two mean the same.

There is a special sign for per cent, $\%$. Thus we write 5% for 5 per cent, and it means the same as $\frac{5}{100}$.

Any fraction with 100 for its denominator may be written with the sign $\%$. Thus,

$$\frac{7}{100} = 7\%$$

$$\frac{1}{100} = 1\%$$

$$\frac{35}{100} = 35\%$$

143. Writing Fractions as Per Cents. We may write any fraction as some per cent by reducing it to hundredths. Thus,

$$\frac{1}{2} = \frac{50}{100} = 50 \text{ per cent} = 50\%$$

$$\frac{1}{5} = \frac{20}{100} = 20 \text{ per cent} = 20\%$$

$$\frac{1}{4} = \frac{25}{100} = 25 \text{ per cent} = 25\%$$

$$\frac{2}{5} = \frac{40}{100} = 40 \text{ per cent} = 40\%$$

$$\frac{3}{4} = \frac{75}{100} = 75 \text{ per cent} = 75\%$$

$$\frac{3}{5} = \frac{60}{100} = 60 \text{ per cent} = 60\%$$

$$\text{For example, } 50\% \text{ of } 24 = \frac{1}{2} \text{ of } 24 = 12$$

$$25\% \text{ of } 36 = \frac{1}{4} \text{ of } 36 = 9$$

ORAL EXERCISE

Find the following :

1. 50% of 20.

10. 25% of 8.

19. 75% of 8.

2. 50% of 40.

11. 25% of 16.

20. 75% of 12.

3. 50% of 30.

12. 25% of 40.

21. 40% of 10.

4. 50% of 50.

13. 20% of 15.

22. 40% of 20.

5. 50% of 11.

14. 20% of 25.

23. 40% of 40.

6. 50% of 15.

15. 20% of 35.

24. 60% of 10.

7. 50% of 16.

16. 20% of 45.

25. 60% of 15.

8. 50% of 21.

17. 20% of 50.

26. 60% of 20.

9. 50% of 25.

18. 20% of 55.

27. 60% of 25.

144. Changing Fractions to Per Cents. We have already learned that another name for hundredths is *per cent*; that is,

$\frac{25}{100}$, or 0.25, may be written 25%, or 25 per cent;

and similarly, we see that

$$0.15 = 15\%$$

$$0.20 = 20\%$$

$$0.01 = 1\%$$

$$0.37 = 37\%$$

$$0.50 = 50\%$$

$$0.01\frac{1}{2} = 1\frac{1}{2}\%$$

If we have a decimal fraction like 0.3, we may annex a zero, because $\frac{3}{10} = \frac{30}{100}$, and write it 0.30, or 30%.

If we have a common fraction like $\frac{1}{2}$, we may change it to a decimal and then to per cent. Thus,

$$\frac{1}{2} = \frac{50}{100} = 0.50 = 50\%.$$

We therefore have the following important table of fractions commonly met in business. This should be learned.

$$\frac{1}{2} = 0.5 = 50\%$$

$$\frac{1}{5} = 0.2 = 20\%$$

$$\frac{1}{4} = 0.25 = 25\%$$

$$\frac{3}{5} = 0.6 = 60\%$$

$$\frac{3}{4} = 0.75 = 75\%$$

$$\frac{1}{10} = 0.1 = 10\%$$

$$\frac{1}{8} = 0.12\frac{1}{2} = 12\frac{1}{2}\%$$

$$\frac{1}{20} = 0.05 = 5\%$$

WRITTEN EXERCISE

Write as common fractions, and also as per cents:

$$1. 0.1. \quad 6. 0.62. \quad 11. 0.03\frac{1}{8}. \quad 16. 0.2. \quad 21. 0.16\frac{2}{3}.$$

$$2. 0.12. \quad 7. 0.62\frac{1}{2}. \quad 12. 0.33. \quad 17. 0.02. \quad 22. 0.75.$$

$$3. 0.01. \quad 8. 0.06\frac{1}{4}. \quad 13. 0.33\frac{1}{3}. \quad 18. 0.87. \quad 23. 0.07\frac{1}{2}.$$

$$4. 0.37. \quad 9. 0.31. \quad 14. 0.66. \quad 19. 0.87\frac{1}{2}. \quad 24. 0.08\frac{1}{3}.$$

$$5. 0.37\frac{1}{2}. \quad 10. 0.31\frac{1}{4}. \quad 15. 0.66\frac{2}{3}. \quad 20. 0.16. \quad 25. 0.03\frac{3}{4}.$$

26. Express 0.40 as a common fraction and as per cent.

27. A man has 3.75 acres of land in a pasture. Express this as a mixed number.

WRITTEN EXERCISE

Find 50% of:

1. 64	88	44	96	124	264	368
2. 65	89	48	98	127	512	648

Find 20% of:

3. 25	50	35	65	125	250	325
4. 26	52	55	68	129	640	720

Find 40% and 60%, respectively, of:

5. 35	45	65	75	135	320	450
6. 36	47	67	78	139	420	640

Find 75% and 30%, respectively, of:

7. 16	24	28	48	272	720	168
8. 32	33	31	34	235	360	367

Express as fractions:

9. 80%	60%	10%	30%	40%	90%
10. 15%	20%	25%	35%	5%	4%

11. A class at school contains 48 pupils, and 50% of them are boys. How many are girls?

12. A boy had 64 marbles and lost 25% of them. How many marbles did he lose and how many did he have left?

13. A man had \$1640 in the bank and drew out 10% of his money. How much did he draw out? How much was left?

14. If Fred has made 40 runs in this season's games, and Frank has made 10% more, how many runs has Frank made?

15. Jack has won 35 games at tennis and has lost 20% less than this number. How many games has he lost?

16. Jackson Park has an area of 539 acres, and Washington Park has 69% as much. What is the area of Washington Park?

17. In a school of 350 pupils 8% were in the sixth grade. How many were in that grade?

18. In a school of 250 pupils 12% are in the third grade and 14% in the first. How many in each of these grades?

19. Of 320 pupils 5% have been tardy or absent this month. How many have not been tardy or absent?

20. In a school of 330 pupils, if there should be an increase of $3\frac{1}{3}\%$ next year, how many pupils would there then be?

21. In a school of 250 pupils 4% are absent to-day. How many pupils are absent?

22. If 16% of the pupils in a school of 200 pupils were not in the school last year, how many new pupils are there this year?

23. If 24% of the pupils in a school of 250 pupils are in the sixth and seventh grades, how many are in those grades?

24. If 25% of the pupils in a school of 372 pupils are in the first and second grades, how many are in those grades? in all the other grades?

25. If 55% of the pupils in a school of 280 pupils are girls, how many girls in the school? how many boys?

26. How much must be added to 250 ft. to increase it 17%? to increase it 25%? to increase it 100%?

27. How much must be added to 148 lb. to increase it 25%? to increase it 50%? to increase it 75%?

28. A column of mercury in a thermometer is 3.6 in. high. If it increases 10% in height, how high is it then?

29. A book is $\frac{4}{5}$ in. thick. The cover is $12\frac{1}{2}\%$ of this thickness. How thick is the cover?

30. A rectangle $4\frac{1}{4}$ in. long and $3\frac{1}{2}$ in. wide is increased 40% in length and $33\frac{1}{3}\%$ in width. What are the length and width now?

31. A certain water tank holds 60,000 gal. A large pump at the waterworks pumps 5% of this amount in a single stroke. How many gallons does it pump in one stroke?

Think of 5% as $\frac{1}{20}$.

32. A certain building in Jersey City cost \$1,240,000, and another building cost 50% as much. How much did the second building cost?

33. If the receipts at the Hoboken docks were 125 carloads on one day, and 20% less the next day, what were the receipts on the second day?

34. The total value of the property in Chicago at the time of the great fire of 1871 was estimated as \$600,000,000, and 31% of it was destroyed. What was the value of the property destroyed?

35. If 500 tons of ore are shipped to Hoboken, and 15% of it is loaded on a steamer, how many tons are loaded on the steamer?

36. A large electric company has certain engines of 22,000 horse power, and certain others of 66% as much horse power. Find the horse power of the latter engines.

37. If you have 25 questions in arithmetic and are marked 80%, how many questions did you answer correctly?

38. If you have 30 words in a spelling test and are marked 90%, how many words did you spell correctly?

39. A man bought a bicycle for \$40 and sold it at a loss of 35%. How much did he lose?

40. A dealer bought 83 rocking-chairs at \$6 apiece and sold them at a profit of 15%. How much did he gain?

ORAL EXERCISE

State the values of:

1. $\frac{1}{2}$ of 48	0.5 of 48	0.50 of 48	50% of 48
2. $\frac{1}{4}$ of 36	0.25 of 36	25% of 36	25% of 80
3. $\frac{3}{4}$ of 20	0.75 of 20	75% of 20	75% of 12
4. $\frac{1}{4}$ of 40	0.25 of 40	25% of 40	75% of 80

WRITTEN EXERCISE

Find the values of:

- | | | |
|-----------------|----------------|----------------------------|
| 1. 25% of 168. | 12. 5% of 34. | 23. 0.25×484 . |
| 2. 75% of 184. | 13. 5% of 48. | 24. 25% of 536. |
| 3. 20% of 125. | 14. 6% of 250. | 25. 0.75×368 . |
| 4. 30% of 140. | 15. 6% of 320. | 26. 75% of 460. |
| 5. 40% of 160. | 16. 4% of 120. | 27. $\frac{1}{5}$ of 1275. |
| 6. 60% of 180. | 17. 4% of 230. | 28. 20% of 35. |
| 7. 70% of 150. | 18. 3% of 125. | 29. $\frac{3}{4}$ of 724. |
| 8. 80% of 120. | 19. 3% of 235. | 30. 75% of 724. |
| 9. 50% of 167. | 20. 2% of 426. | 31. $\frac{3}{4}$ of 236. |
| 10. 20% of 241. | 21. 7% of 341. | 32. 75% of 236. |
| 11. 90% of 340. | 22. 8% of 468. | 33. 75% of 448. |

34. In the world's championship series in a certain year, the players' share was \$135,000, of which the Giants received 40%. How much money did they receive?

35. In the same year the Athletics received 60% of the players' share, which was \$135,000 as stated in Ex. 34. How much money did they receive?

36. The St. Louis club was at bat 4800 times in one season and the base hits were 25%. How many base hits did they make?

IV. DENOMINATE NUMBERS

1. If John picked 12 qt. of berries to-day and 50% as many yesterday, how many quarts did he pick yesterday? how many quarts did he pick in the two days?

2. If Charles gathered 21 qt. of chestnuts and Will gathered $\frac{2}{3}$ as many, how many quarts did Will gather? how many quarts did they both gather?

3. If Mary lives $\frac{1}{2}$ mi. from school and Louise lives half as far, how far from school does Louise live?

4. If school opens at 9 A.M. and closes at 30 min. past 3 P.M., and there is $1\frac{1}{4}$ hr. recess at noon, how many hours are we in school to-day?

5. There being $5\frac{1}{2}$ yd. in a rod, how many yards in $17\frac{2}{3}$ rd.? how many yards in 32 rd.?

6. How much will $8\frac{3}{4}$ doz. pencils cost at the rate of 25¢ ($\$ \frac{1}{4}$) for a half dozen?

7. At the rate of $3\frac{3}{4}$ lb. a week, how much sugar will a family use in the $4\frac{2}{7}$ weeks of January?

8. At the rate of $14\frac{7}{8}$ lb. of meat a week, how much meat will a family use in the $4\frac{2}{7}$ weeks of April?

9. How far will a commercial salesman travel in 6 da. at an average of $42\frac{1}{2}$ mi. a day?

10. An ocean steamer averaged $16\frac{7}{8}$ mi. an hour for $23\frac{1}{2}$ hr. What was the total distance?

11. Some blotting paper is $\frac{3}{64}$ in. thick. What will be the height of a pile of $2\frac{3}{4}$ doz. sheets?

12. If an arithmetic is $1\frac{1}{8}$ in. thick and a reader is $1\frac{5}{16}$ in. thick, what is the height of a pile of 7 arithmetics and 9 readers?

13. At 28¢ a pound, what will $4\frac{1}{2}$ lb. of steak cost?

In this and the following examples estimate the approximate answer and write it down. After the problem is solved compare the approximate answer with the real one. This is one of the best checks against absurd results. Thus $5 \times 28\text{¢}$ is a reasonable approximation for $4\frac{1}{2} \times 28\text{¢}$.

14. At 24¢ a pound, what will $6\frac{3}{4}$ lb. of meat cost?

15. At 16¢ a pound, what will $5\frac{1}{4}$ lb. of cheese cost?

16. At 20¢ a yard, what will $18\frac{3}{4}$ yd. of calico cost?

17. At \$6.50 a ton, what will $5\frac{3}{4}$ tons of coal cost?

18. If 6 yd. of silk cost \$6.75, what is the price per yard?

19. How many bushels of potatoes will 32 acres produce, if an average acre produces 125 bu.?

20. How many bushels of oats will 81 acres produce, if an average acre produces $33\frac{1}{3}$ bu.?

21. How much will 250 doz. yd. of millinery braid cost at $62\frac{1}{2}\text{¢}$ a dozen yards? at 75¢ a dozen yards?

22. There are 44,000 yd. in 25 mi. How many yards are there in one mile?

23. If 25 tons of coal cost \$150, what will be the cost of 11 tons? of 15 tons? of 32 tons? of 75 tons?

24. How long will it take an express train to go 362.5 mi. at the rate of 50 mi. an hour?

25. A dealer pays \$124 for some gingham at 25¢ a yard. How many yards of gingham does he buy?

26. A dealer pays \$160 for some percaline at 25¢ a yard. How many yards of percaline does he buy?

27. If you buy $3\frac{1}{2}$ doz. oranges at 40¢ a dozen, and 2 doz. bananas at 25¢ a dozen, how much change should you receive from \$2?

145. Tables for Reference. While these tables have been learned and used in the preceding work, they are here inserted for reference and for such review as may be necessary.

146. Table of Length. The following is the table of length :

12 inches (in.) = 1 foot (ft.)

3 feet = 1 yard (yd.)

$5\frac{1}{2}$ yards, or $16\frac{1}{2}$ feet = 1 rod (rd.)

320 rods, or 5280 feet = 1 mile (mi.)

The *hand* (4 in.) is used in measuring the height of horses at the shoulder. Sailors use the *fathom* (6 ft.) and the *cable length* (120 fathoms) for measuring depths, and the *knot* (nautical mile = $1\frac{3}{4}$ common, or statute, miles = 6080 $\frac{1}{4}$ ft.) for distances at sea.

1. Required to find the number of inches in 3 ft.

$$3 \text{ ft.} = 3 \times 12 \text{ in.} = 36 \text{ in.}$$

2. Required to find the number of feet in 27 rd.

$$27 \text{ rd.} = 27 \times 16\frac{1}{2} \text{ ft.} = 445\frac{1}{2} \text{ ft.}$$

ORAL EXERCISE

Find the number of inches in :

1. 5 ft.

3. 10 ft.

5. 1 yd.

7. $\frac{1}{2}$ yd.

2. 9 ft.

4. $11\frac{1}{2}$ ft.

6. 10 yd.

8. $\frac{1}{4}$ yd.

Find the number of feet in :

9. 2 yd.

11. 7 yd.

13. 1 rd.

15. $5\frac{1}{2}$ yd.

10. 4 yd.

12. 10 yd.

14. 10 rd.

16. $\frac{1}{2}$ yd.

17. If you buy $\frac{1}{4}$ yd. of velvet, how many inches do you buy?

18. If a horse is 16 hands high, how would you find his height in inches? in feet and inches?

WRITTEN EXERCISE

1. A schoolroom is 24 ft. long and 18 ft. wide. Draw a plan of the floor to the scale 1 in. to 6 ft.
2. A dining room is 16 ft. long and 14 ft. wide. Draw a plan of the floor to the scale 1 in. to 4 ft.
3. A field is 28 rd. long and 20 rd. wide. Draw a plan of the field to the scale 1 in. to 4 rd.
4. Measure the cover of your geography and draw a plan to the scale $\frac{1}{4}$.
5. A schoolhouse is 60 ft. long and 40 ft. wide. It stands on a lot that is 140 ft. long and 80 ft. wide. Draw a plan of the lot, and on the plan show the position of the schoolhouse. Use the scale 1 in. to 20 ft.
6. A road is 40 rd. long and 4 rd. wide. It is intersected at right angles by a road 3 rd. wide. Draw a plan of the first road and show the intersection. Use the scale of 1 in. to 8 rd.
7. A barn is 80 ft. long and 32 ft. wide. Draw a plan to the scale 1 in. to 16 ft. A road 48 ft. wide runs in front of the barn and 64 ft. from it. Show on the plan the position of the road.
8. At one side of a schoolroom is a blackboard 3 ft. high. The chalk rack is 3 ft. from the floor. The room is 24 ft. long and 12 ft. high. Draw a plan of the side of the room to the scale 1 in. to 1 yd.
9. A door in a schoolhouse is 3 ft. 6 in. wide, and 7 ft. 2 in. high. Draw two plans of the door, one to the scale of 1 in. to 1 ft., and the other to half this scale.

Throughout the work in denominate numbers the pupils should make actual measurements of desks, windows, corridors, school grounds, and schoolrooms, drawing these to scale. The work is particularly valuable if applied to measurements of playgrounds in which the pupils have an interest.

147. Table of Square Measure. The following is the table of square measure:

144 square inches (sq. in.) = 1 square foot (sq. ft.)

9 square feet = 1 square yard (sq. yd.)

$30\frac{1}{4}$ square yards = 1 square rod (sq. rd.)

160 square rods = 1 acre (A.)

640 acres = 1 square mile (sq. mi.)

Carpenters, architects, and mechanics often write 8" for 8 in., and 5' for 5 ft. They also use sq." and sq.' for square inches and square feet. In this book both of these forms are used.

If a square is 1 in. on a side, it is a square inch.

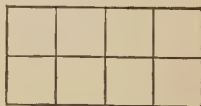
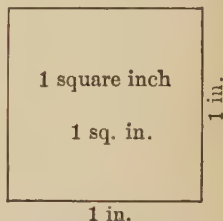
If a rectangle is 4 in. long and 2 in. wide, its area is 4×2 sq. in., or 8 sq. in.

The pupils may express this in either of these two forms:

$$4 \times 2 \text{ sq. in.} = 8 \text{ sq. in.},$$

or $2 \times 4 \text{ sq. in.} = 8 \text{ sq. in.}$

In practical work we simply multiply 4 by 2, saying: "There are *as many* square inches as 2×4 , or 8 sq. in."



WRITTEN EXERCISE

1. Find the number of square inches in the top of a desk whose area is 3 sq. ft.
2. The school platform has an area of 3 sq. yd. What is its area in square feet?
3. A flower bed has an area of 2 sq. rd. What is its area in square yards? in square feet?
4. The school grounds contain half an acre. What is the area in square rods?

148. Table of Cubic Measure. The following is the table of cubic measure:

1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.)

27 cubic feet = 1 cubic yard (cu. yd.)

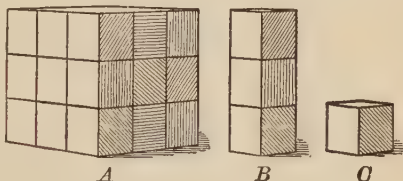
128 cubic feet = 1 cord (cd.)

A *perch* of stone or masonry is usually considered as 1 rd. long, 1 ft. high, and $1\frac{1}{2}$ ft. thick, and it contains $24\frac{3}{4}$ cu. ft. It varies, however, in different parts of the country. A cubic yard of earth is considered a load.

If a block is 1 in. on an edge, and is in the form of a cube, the volume is 1 cu. in., as in *C*.

If a block is 1 in. square at the base and 3 in. high, the volume is 3 cu. in., as in *B*.

If a block is 3 in. square at the base, the base contains 3×3 sq. in., or 9 sq. in. If the block is 3 in. high, the volume is 3×9 cu. in., or 27 cu. in., as in *A*.



If a block is 4 in. long, 3 in. wide, and 2 in. high, the volume is $2 \times 3 \times 4$ cu. in. = 24 cu. in.

WRITTEN EXERCISE

Find the volumes of boxes of these dimensions:

1. 3 in., 4 in., 5 in.
2. 2 in., 5 in., 9 in.
3. 4 in., 7 in., 12 in.
4. 8 in., 12 in., 16 in.
5. $3\frac{1}{2}$ in., 8 in., $9\frac{1}{2}$ in.
6. 4 ft., $12\frac{1}{2}$ ft., 14 ft.
7. $2\frac{1}{2}$ ft., $3\frac{1}{2}$ ft., 4 ft.
8. $2\frac{3}{4}$ ft., $3\frac{1}{2}$ ft., 8 ft.
9. 3.2 in., 2.5 in., 10 in.
10. 4.8 in., 2.7 in., 10 in.
11. How many cubic feet in 8 perches of $24\frac{3}{4}$ cu. ft. each?

149. Table of Weight. The following is the common table of weight:

$$16 \text{ ounces (oz.)} = 1 \text{ pound (lb.)}$$

$$2000 \text{ pounds} = 1 \text{ ton (T.)}$$

The ton of 2000 lb. is sometimes called the *short ton*, there being a *long ton* of 2240 lb. which is used in some transactions in mining products.

Goldsmiths still use an old table of *troy weight*, and apothecaries and physicians use the table of *apothecaries' weight* in prescriptions of medicine, but neither is of enough importance to justify teaching it.

The avoirdupois pound contains 7000 grains, the troy pound (12 troy ounces) 5760 grains. Therefore 1 lb. of iron is heavier than 1 lb. of gold.

A *carat* weight, used in weighing diamonds, is (since 1913) 200 milligrams, or 3.08647 grains. The word "carat" is also used in speaking of the purity of gold and means $\frac{1}{24}$; 16 carats fine means $\frac{16}{24}$ pure gold.

ORAL EXERCISE

Find the number of ounces in:

1. $\frac{1}{2}$ lb. 2. $\frac{3}{4}$ lb. 3. $\frac{3}{4}$ lb. 4. $1\frac{1}{2}$ lb. 5. 2 lb.

Find the number of pounds in:

6. $\frac{1}{2}$ T. 7. $\frac{1}{4}$ T. 8. $\frac{3}{4}$ T. 9. $1\frac{1}{2}$ T. 10. 2 T.

11. There is an old question as to which is the heavier, a pound of lead or a pound of feathers. Which is the heavier?

12. We might change Ex. 11 and ask which is the heavier, a pound of gold or a pound of feathers. Which is the heavier?

WRITTEN EXERCISE

- How many pounds in $2\frac{1}{2}$ T.? in $1\frac{3}{4}$ T.? in $3\frac{3}{4}$ T.?
- How many ounces in $2\frac{1}{2}$ lb.? in $2\frac{3}{4}$ lb.? in $3\frac{1}{4}$ lb.?
- How many ounces in 7 lb.? in $7\frac{1}{4}$ lb.? in $9\frac{3}{4}$ lb.?
- How many grains are there in $\frac{1}{16}$ lb., or 1 oz.?

150. Table of Liquid Measure. The following is the common table of liquid measure:

4 gills (gi.) = 1 pint (pt.)

2 pints = 1 quart (qt.)

4 quarts = 1 gallon (gal.)



A gallon contains 231 cu. in. Barrels vary in size, although in estimating the capacity of tanks and cisterns, $31\frac{1}{2}$ gal. are considered a barrel (bbl.), and 2 bbl. a hogshead.

There is also a table of *apothecaries' liquid measure*, in which 16 fluid ounces make 1 pint.

ORAL EXERCISE

1. If you need $\frac{1}{2}$ pt. of cream for a recipe, how many gills do you need? what part of a quart do you need?
2. If you need 3 pt. of milk for some cooking, how many quarts do you need?
3. A quart is what part of a gallon? Does your answer show you why this measure is called a quart?
4. How many pints are there in a gallon? How many gills?
5. Name the various measures shown in the picture.
6. Name something that is sold by the pint; something that is sold by the quart; something that is sold by the gallon.

151. Table of Dry Measure. The following is the common table of dry measure:

2 pints (pt.) = 1 quart (qt.)

8 quarts = 1 peck (pk.)

4 pecks = 1 bushel (bu.)

A bushel contains nearly $2150\frac{1}{2}$ cu. in., or about $1\frac{1}{4}$ cu. ft. The *dry quart* contains $67\frac{1}{2}$ cu. in. and the *liquid quart* contains $57\frac{3}{4}$ cu. in.



ORAL EXERCISE

1. In the picture point to the quart, the peck, and the bushel measures.
2. If you feed a horse 8 qt. of oats a day, how many days will 1 bu. of oats last?
3. If you have 2 bu. of shelled corn, how long will this last your chickens if you feed them 1 qt. of corn a day?
4. How many quarts in 2 pk.? in 4 pk.?
5. How many pecks in 2 bu.? in 4 bu.? in 8 bu.?
6. How many quarts in 1 bu.? in 2 bu.? in 10 bu.?
7. How many pints in 3 qt.? in 8 qt.? in 1 pk.? in 2 pk.?
8. Name something that is sold by the bushel; something that is sold by the peck.

152. Table of Time. The following is the table of time:

60 seconds (sec.) = 1 minute (min.)

60 minutes = 1 hour (hr.)

24 hours = 1 day (da.)

7 days = 1 week (wk.)

12 months (mo.) = 1 year (yr.)

Thirty days has September,
April, June, and November.

The other months have 31 days, except February, which has 28 days in common years and 29 days in leap years. The *common year* has 365 days, or 52 weeks and 1 day; the *leap year* has 366 days. A *century* is 100 years. 45 min. after 7 is written 7:45, 7:45, or often 7 45.

Years whose numbers are divisible by 4 but not by 100, and those divisible by 400, are leap years. Thus 1916, 2000, but not 2100, are leap years.

153. Table of Value. The table of value is as follows:

10 mills = 1 cent (ct. or ¢)

10 cents = 1 dime (d.)

10 dimes = 1 dollar (\$)

10 dollars = 1 eagle

The term "eagle" is not used, and the mill is not coined.

WRITTEN EXERCISE

1. If you go to school $9\frac{1}{2}$ mo. a year, and we allow 4 wk. to the month, how many weeks do you go to school each year?

2. If you are in school $5\frac{1}{2}$ hr. a day for 5 da. each week, you are in school how many hours a week?

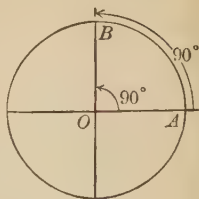
3. If you are in school $5\frac{1}{2}$ hr. a day, how many minutes a day are you in school?

4. If you are saving 5-cent pieces to buy a 75-cent box of note paper, how many 5-cent pieces will you have to save?

154. Circular Measure. The circumference of a circle is measured in *degrees*.

There are 360 degrees (written 360°) in a circumference and 90° in a right angle.

Because the angle at O , the center, increases just as fast as the part of the circumference between its arms increases, an angle is also measured in degrees.



155. Perpendicular Lines. A line that makes a right angle with another line is said to be *perpendicular* to it.

156. Arc. A portion of a circumference is called an *arc*.

157. Arc and Angle Measure. In measuring arcs,

$$360^\circ = \text{a circumference}$$

That is, we divide the circumference, as stated above, into 360° .

In measuring angles we divide four right angles into 360° .

That is,

$$360^\circ = 4 \text{ right angles}$$

In measuring both angles and arcs,

$$60 \text{ seconds } (60'') = 1 \text{ minute } (1')$$

$$60 \text{ minutes} = 1 \text{ degree } (1^\circ)$$

ORAL EXERCISE

1. How many degrees in $\frac{1}{2}$ of a circumference?
2. If one line is perpendicular to another, what angle do the two lines form? How many degrees in the angle?
3. Is an angle acute, right, or obtuse if it contains 100° ? 60° ? 90° ? 120° ? 1° ?
4. How many minutes in 5° ? in 7° ? in 11° ? in 25° ?
5. How many seconds in $3'$? in $5'$? in $20'$? in 1° ?

158. Table of Dozens. In some kinds of business this table is common:

12 units = 1 dozen (doz.)

12 dozen, or 144 = 1 gross (gr.)

12 gross, or 1728 = 1 great gross

The term "great gross" is going out of use. Years ago the word "score" was often used for "twenty."

ORAL EXERCISE

1. How many eggs are 3 doz. eggs? 4 doz. eggs?
2. How many oranges are $2\frac{1}{2}$ doz. oranges? $3\frac{1}{2}$ doz. oranges?
3. How many bananas are $1\frac{1}{4}$ doz. bananas?
4. How many pens in 2 gross? in 6 gross? in 8 gross?
5. How many dozen in $\frac{2}{3}$ of a gross? in $3\frac{1}{2}$ gross?
6. What will a gross of padlocks cost at \$5 a dozen?
7. How many years old is a man who is fourscore years? one who is threescore years? one who is fourscore years and ten?

WRITTEN EXERCISE

1. How many are 7 great gross? $9\frac{2}{3}$ gross?
2. What will 9 gr. of tablets cost at 25¢ a dozen?
3. At \$7.20 a gross what will a dozen lamp chimneys cost? what will $2\frac{1}{2}$ doz. cost?
4. A dealer bought 10 gr. of blackboard erasers of which 10% were damaged. How many were in good condition?
5. Which is cheaper, to buy a gross of articles for \$125 or to buy 12 doz. at \$11 a dozen? How much cheaper?
6. How much older is a man who is fourscore years and eight than one who is threescore years and ten?

159. Counting Sheets of Paper. Formerly, sheets of paper were counted thus:

24 sheets = 1 quire

20 quires, or 480 sheets = 1 ream

Now, for convenience in counting, 500 sheets are often called a *ream*, and the word "quire" is used only for folded note paper, other paper being usually sold by the pound.

ORAL EXERCISE

1. How many sheets of fine writing paper in 2 quires? in 5 quires? in $3\frac{1}{2}$ quires? in 10 quires? in $12\frac{1}{2}\%$ of a quire?

2. If writing paper costs 15¢ a quire, how much will 4 quires cost? How many sheets will there be?

Notice that 4 times 24 is the same as 4 times (25 - 1), or 100 - 4.

3. At 20¢ a quire, how much will 7 quires of paper, with \$1 worth of envelopes, cost?

4. If a dealer buys paper at 19¢ a quire and sells it at 25¢ a quire, how much will he make on 10 quires?

WRITTEN EXERCISE

1. How many sheets in 17 quires? in $33\frac{1}{3}\%$ of a quire?

2. In a dozen reams of 500 sheets each how many more sheets of paper are there than in a dozen reams of 480 sheets each?

3. At $16\frac{2}{3}\%$ a quire, how much will $7\frac{1}{2}$ quires of paper cost?

Remember the use of aliquot parts.

4. At $12\frac{1}{2}\%$ a quire, how much will $9\frac{1}{2}$ quires of paper cost? $6\frac{1}{2}$ quires? a dozen quires?

5. At 24¢ a quire for paper, and 1¢ each for envelopes, how much will it cost for paper, envelopes, and postage for eight letters from where you live to some other place in this country? Allow 1 sheet of paper to a letter.

160. Denominate Numbers. You have already learned the meaning of denominate numbers. You know that 3 ft. is a denominate number.

161. Compound Numbers. Denominate numbers in which two or more units are expressed are called *compound numbers*. For example, 2 ft. 3 in., 4 mi. 3 rd. 2 ft.

162. Reduction. We may express 2 ft. 3 in. as 27 in., or as $2\frac{3}{4}$ ft. This changing from one form to another without changing the value is called *reduction*.

163. Reduction Descending. Reduction to a lower denomination is called *reduction descending*. For example, 3 ft. may be reduced to 36 in.

164. Reduction Ascending. Reduction to a higher denomination is called *reduction ascending*. For example, 24 oz. may be reduced to 1 lb. 8 oz.

Simple cases of reduction, familiar from the work in primary arithmetic, have already been considered in the preceding pages.

Definitions like the above have little value, the subject of compound numbers having lost most of its former importance. We speak of "2 ft. 3 in." instead of " $2\frac{3}{4}$ ft.," but we rarely use the pound and ounce *together*, usually speaking of "2 lb. 4 oz." as "2 pounds and a quarter." Hence but little time should be given to work involving more than two denominations.

ORAL EXERCISE

1. Express 10 bu. as pecks ; as quarts ; as pints.
2. Express 64 qt. as pecks ; as bushels ; as pints.
3. Express 40 qt. as gallons ; as pints ; as gills.
4. Express 40 pt. as quarts ; as gallons ; as gills.
5. Express 20 ft. as inches ; 21 ft. as yards.
6. Express 36 in. as yards ; 36 ft. as yards.

165. Examples in Reduction Descending.**1. Reduce 160 rd. to inches.**

Because 1 rd. = $16\frac{1}{2}$ ft.,

we see that 160 rd. = $160 \times 16\frac{1}{2}$ ft. = 2640 ft.

Because 1 ft. = 12 in.,

we see that 2640 ft. = 2640×12 in. = 31,680 in.

While we think of multiplying $16\frac{1}{2}$ ft. by 160, we actually multiply 160 by $16\frac{1}{2}$, because this is shorter, and so for 2640 times 12 in.

$$\begin{array}{r} 160 \\ 16\frac{1}{2} \\ \hline 80 \\ 960 \\ 160 \\ \hline 2640 \text{ (ft.)} \\ 12 \\ \hline 31680 \text{ (in.)} \end{array}$$

2. Reduce 3 T. 42 lb. to ounces.

Because 1 T. = 2000 lb.,

we see that 3 T. = 3×2000 lb. = 6000 lb.

6000 lb. + 42 lb. = 6042 lb.

Because 1 lb. = 16 oz.,

we see that 6042 lb. = 6042×16 oz. = 96,672 oz.

$$\begin{array}{r} 2000 \\ 3 \\ \hline 6000 \\ 42 \\ \hline 6042 \text{ (lb.)} \\ 16 \\ \hline 36252 \\ 6042 \\ \hline 96672 \text{ (oz.)} \end{array}$$

It must be clearly understood that these two reductions are given simply for purposes of explanation. Practically we do not have as difficult cases as these, most reductions being merely from feet or yards to inches, or some equally simple case.

WRITTEN EXERCISE

Reduce the following :

- | | |
|----------------------------|--|
| 1. 8 bu. 2 pk. to pecks. | 7. 3 mi. 75 rd. to rods. |
| 2. 10 rd. 3 ft. to feet. | 8. 2 A. to square rods. |
| 3. 175 rd. 9 ft. to feet. | 9. 5'6" to inches. |
| 4. 2 lb. 2 oz. to ounces. | 10. 5 yd. 1 ft. to feet. |
| 5. 2 gal. 3 qt. to quarts. | 11. $2\frac{3}{4}$ A. to square rods. |
| 6. 10 cu. to cubic feet. | 12. $23\frac{1}{2}$ cu. yd. to cubic feet. |

13. A carpenter saws a length of 7 ft. 11 in. from a board. How many inches does he saw off?
14. A lady is using 2 yd. 8 in. of cloth. How many inches of cloth is she using?
15. A dealer is putting 7 lb. 12 oz. of candy in boxes. How many ounce boxes can he fill with the candy?
16. A man puts $16\frac{1}{2}$ gal. of gasoline in a tank. How many quarts of gasoline does he put in?
17. A milk dealer sells 2 gal. 2 qt. of cream. How many pints of cream does he sell?
18. A farmer has $16\frac{1}{2}$ bu. of oats in his bin. How many pecks of oats has he? How many quarts?
19. A farmer has built 64 rd. 8 ft. of fence. How many feet of fence has he built?
20. If a pile of wood contains 17 cords, how many cubic feet of wood does it contain?
21. How many square rods are there in $1\frac{1}{4}$ acres?
22. A box is 3 ft. 8 in. long and 1 ft. 7 in. wide. How many inches in the perimeter of the box?
23. A strip of carpet is 19 yd. 2 ft. long. How many feet in the length of the strip?
24. In excavating a cellar 600 cu. yd. of earth are removed. How many cubic feet of earth are removed?
25. A piece of land is 12 rd. long and 7 rd. wide. How many square rods in the piece? How many square feet?
26. A man buys a load of coal weighing 1 T. 750 lb. How many pounds of coal does he buy?

As suggested on page 22, teachers should frequently call the attention of their pupils to the necessity of estimating an answer in advance of the solution.

166. Examples in Reduction Ascending. 1. Reduce 200 pt. to gallons.

Because $1 \text{ pt.} = \frac{1}{2} \text{ qt.}$,
 we see that $200 \text{ pt.} = 200 \text{ times } \frac{1}{2} \text{ qt.} = 100 \text{ qt.}$
 Because $1 \text{ qt.} = \frac{1}{4} \text{ gal.}$,
 we see that $100 \text{ qt.} = 100 \text{ times } \frac{1}{4} \text{ gal.} = 25 \text{ gal.}$

$$\begin{array}{r} 2 \overline{) 200} \text{ (pt.)} \\ 4 \overline{) 100} \text{ (qt.)} \\ \underline{ 25} \text{ (gal.)} \end{array}$$

2. Reduce 175 in. to feet and inches.

Because $1 \text{ in.} = \frac{1}{12} \text{ ft.}$,
 we see that $175 \text{ in.} = 175 \text{ times } \frac{1}{12} \text{ ft.}$
 $= 14 \frac{7}{12} \text{ ft.} = 14 \text{ ft. } 7 \text{ in.}$

$$\begin{array}{r} 12 \overline{) 175} \text{ (in.)} \\ \underline{ 14} \text{ (ft.)} \\ 7 \text{ in. rem.} \end{array}$$

It should be understood that these reductions are rather more difficult than those usually found in practical work. Indeed, in business, the reductions are commonly such as can be made mentally.

WRITTEN EXERCISE

Reduce the following:

1. 96 in. to feet.
2. 48 oz. to pounds.
3. 176 oz. to pounds.
4. 3600 min. to hours.
5. 27 pk. to bushels.
6. 92 in. to feet.
7. 96 in. to yards.
8. 7200 sec. to hours.
9. 275 ft. to rods and feet.
10. 16,000 lb. to tons.
11. 527 pt. to bushels, quarts, and pints.
12. 500 sq. in. to square feet and square inches.
13. Reduce 475 yd. to a fraction of a mile.
14. Reduce $\frac{3}{4}$ in. to a fraction of a foot; of a yard.
15. Reduce $266\frac{2}{3}$ rd. to a fraction of a mile.
16. Reduce $6\frac{1}{2}$ in. to a fraction of a foot; of a yard.
17. A lady uses 81 in. of ribbon. Express this as yards and inches; as yards and a fraction.

18. A carpenter uses 42 in. of molding. Express this as feet and inches; as feet and a fraction; as yards and inches.

19. A milk dealer sells 302 qt. of milk. Express this as gallons and quarts; as gallons and a fraction.

20. A milk dealer sells 73 pt. of cream. Express this as quarts and pints; as quarts and a fraction; as gallons and pints; as gallons and a fraction.

21. A dealer sells 137 qt. of gasoline. Express this as gallons and quarts; as gallons and a fraction.

22. A boy is running a half-mile race. After he has run 660 yd. what fraction of the distance has he run?

23. Express 775 in. as feet; as yards.

24. Express 32 in. as feet; 32 oz. as pounds; 32 ft. as yards; 32 pt. as quarts; 32 qt. as gallons.

25. Express 96 oz. as pounds; 96 in. as yards and a fraction; 96 ft. as yards; 96 min. as hours and minutes.

26. If you need $9\frac{1}{2}$ yd. of cloth for a garment, and have 72 in. already, how many yards will you have to buy?

27. If you cut 86 sq. in. from a piece of cloth containing $2\frac{1}{2}$ sq. ft., how many square feet and square inches are left?

28. If you cut off $\frac{5}{8}$ of a piece of tin containing 3 sq. ft., how many square inches do you cut off?

29. From a sheet of tin 3 ft. long and $2\frac{1}{2}$ ft. wide a piece is cut containing 500 sq. in. How many square feet and square inches are left?

30. From an iron plate 2 ft. 6 in. by 7 ft. 6 in. a piece is cut containing 108 sq. in. How many square feet are left?

31. If from a piece of glass 4 ft. 6 in. long by 3 ft. wide a strip 1 in. wide is cut from each side, how many square inches are left?

V. BILLS AND RECEIPTS

ORAL EXERCISE

1. What is a bill for goods purchased ?
2. Why is a bill given to the purchaser ?
3. How is a bill receipted ? Why is it receipted ?
4. In making out a bill, where are the dates of the various purchases written ? where are the items of cost written ? where is the total amount written ?

The teacher should have the class review page 131. He should ask the pupils to bring to school bills that have been used in business transactions, talking them over with the pupils, and show the advantage of the conventional forms of ruling, heading, and receipting. The pupils should be asked to make out bills first on blank forms, if they can be obtained, and afterwards on paper which they themselves have ruled. Their imagination should be used to make bills which might represent, as far as possible, actual business transactions. They should imagine or represent that the teacher or one of the members of the class has sold the rest of the class a bill of goods or done a piece of work for them or rented a house to them. The resulting bill should contain the names of all the parties to the transaction.

WRITTEN EXERCISE

Make out bills for the following in full form, inserting the names of the dealers and the purchasers, and receipting each bill :

1. 2 pairs of skates @ \$1.35, 3 sweaters @ \$1.15.
2. $14\frac{1}{2}$ yd. gingham @ 25¢, 9 yd. cotton @ $12\frac{1}{2}$ ¢.
3. $6\frac{1}{2}$ yd. cheviot @ \$1.10, $5\frac{3}{8}$ yd. silk @ \$1.25.
4. $11\frac{1}{2}$ yd. Persian lawn @ 65¢, $6\frac{5}{8}$ yd. dimitry @ 25¢, $12\frac{1}{8}$ yd. linen suiting @ 75¢, $9\frac{1}{2}$ yd. velveteen @ 98¢.
5. $9\frac{1}{2}$ yd. Persian lawn @ $37\frac{1}{2}$ ¢, $5\frac{1}{8}$ yd. cheviot @ \$1.25, 15 yd. cotton @ $12\frac{1}{2}$ ¢, $10\frac{1}{2}$ yd. madras @ 35¢.
6. 7 yd. muslin @ 25¢, $12\frac{1}{2}$ yd. lining @ 11¢, $6\frac{5}{8}$ yd. linen lawn @ 50¢, 7 yd. suiting @ \$1.75, 5 doz. buttons @ 50¢.

167. Bill for Labor or Material. Not only do people make out bills for goods purchased at stores but they make out bills for labor or for materials furnished. The following is such a bill:

<i>Passaic, N.J., April 5, 1916</i>					
<i>Mr. Robert Johnson,</i>					
To B. F. BROOKS, Dr.					
Mar.	17	To 250 ft. lumber @ \$40 per M.	10		
	22	To 4 days' labor @ \$4.50	18		
		<i>Received payment,</i> <i>B. F. Brooks</i>		28	

WRITTEN EXERCISE

1. Make out a bill for labor and materials, inserting names, dates, and items. Receipt the bill.

Make out bills for the following, inserting names, dates, and amounts, and receipting each bill:

2. May 7, 400 ft. of lumber @ \$48 per M (per 1000 board feet); 24 lb. nails @ 9¢; May 9, 5 da. labor @ \$4.

3. June 1, 6 da. labor @ \$2.25; June 8, 5½ da. labor @ \$2.25; June 15, 5¼ da. labor @ \$2.25; June 15, to repair of lawn mower, \$1.30.

In all bills a fraction of a cent is counted as a cent by the one who makes out the bill. In other computations the fraction is usually dropped if it is less than half a cent.

4. Sept. 8, to repairs to mowing machine, \$4.35; Sept. 9, to repair of horse rake, \$1.25; Sept. 12, to 6 da. labor @ \$2.50.

168. Receipts. A receipt need not necessarily be on a bill for goods or services. If Robert owes George \$1.25 on a catcher's mask that he bought from him last week, and pays the money to-day, he may ask for a receipt so that the fact of the payment shall not be forgotten. Such a receipt would read as follows:

\$1.25	Camden, N. J., April 6, 1916
Received from Robert Montgomery	
One and $\frac{25}{100}$	~~~~~ Dollars
for balance due on catcher's mask.	
George Sinelair	

A receipt should contain the place and date, the name of the one paying the money (the *payer*), the amount written both in words and in figures, a statement of the reason for paying the money (the nature of the debt), and the signature of the one to whom the money is paid (the *payee*).

WRITTEN EXERCISE

Write receipts for the following, inserting dates and names:

1. \$1.15 for a pair of skates.
2. \$1.20 for three baseball bats.
3. \$0.75, balance due on a football.
4. \$1.35, balance due on a mandolin.
5. \$0.80, balance due on a tennis racket.
6. \$1.60, balance due on a tennis racket and a tennis net.
7. \$6.50 for 2 days' labor on sidewalk and fence.
8. \$0.72 for 26 papers at 2¢ each, and 4 papers at 5¢ each.

REVIEW AND DRILL

I. DRILL SECTION

WRITTEN EXERCISE

1. Write in our common numerals the number forty-seven million, eight hundred six and thirty-seven thousandths.

Find the curious results of the following :

2.		3.		4.		
7×7		4×4		$1 \times 8 + 1$		
67×67		34×34		$11 \times 8 + 11$		
667×667		334×334		$111 \times 8 + 111$		
6667×6667		3334×3334		$1111 \times 8 + 1111$		
66667×66667		33334×33334		$11111 \times 8 + 11111$		
666667×666667		333334×333334		$111111 \times 8 + 111111$		
I	II	III	IV	V	VI	VII
$a \frac{2}{3}$	$\frac{7}{8}$	$\frac{3}{5}$	$\frac{3}{4}$	$\frac{3}{8}$	$\frac{7}{8}$	$\frac{4}{5}$
$b 1\frac{1}{2}$	$2\frac{1}{3}$	$1\frac{1}{4}$	$4\frac{1}{5}$	$1\frac{1}{8}$	$6\frac{1}{10}$	$8\frac{1}{8}$
$c 34.8$	4.32	0.72	382	0.48	1237	$62\frac{1}{2}$
$d 42.1$	5.71	0.79	468	0.51	2608	$81\frac{1}{4}$

In each of the seven columns perform the following operations, carrying division of decimals to two decimal places :

- | | | | |
|--------------|---------------|--------------------|------------------|
| 5. $a + b$. | 8. $b - a$. | 11. $a \times b$. | 14. $d \div c$. |
| 6. $a + c$. | 9. $c - a$. | 12. $a \times c$. | 15. $d \div b$. |
| 7. $a + d$. | 10. $d - c$. | 13. $a \times d$. | 16. $d \div a$. |

Teachers are asked to read carefully the suggestions on page 83.

II. PROBLEMS WITHOUT NUMBERS

ORAL EXERCISE

1. If you are multiplying dollars and cents by a whole number, how do you know where to place the decimal point in the product?

2. If you are dividing dollars and cents by a whole number, how do you know where to place the decimal point in the quotient?

3. If you are multiplying one decimal fraction by another, how many decimal places must you have in the product? Why is this the case? Illustrate by an example.

4. If you are dividing a decimal fraction by a whole number, how can you tell where to put the decimal point in the quotient? Illustrate by an example.

5. If you are dividing a decimal fraction, and the divisor is not a whole number, how may you make it such without altering the quotient? Give your reason.

6. If you know the length of each side of a triangle, how do you find the perimeter, or distance around the triangle? Illustrate by an example.

7. If you know the length of only one side of a square, how do you find the perimeter? Illustrate by an example.

8. If you know the length of only one side of a square, how do you find the area of the square? Illustrate by an example.

9. If you know the cost of one catcher's glove, how do you find the cost of several?

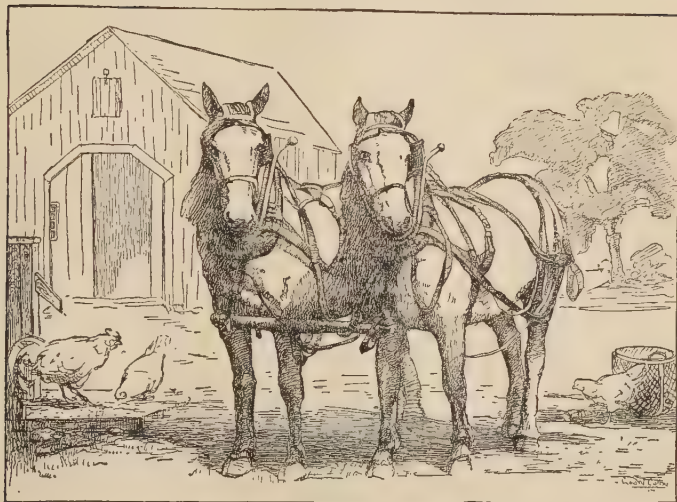
10. If you know the cost of several baseball bats, how do you find the cost of one?

11. If you know the cost of baseballs by the dozen, how do you find the cost of each ball? Could you probably buy one at this price? If not, why not?

12. If you know the cost of a certain number of tennis balls, how do you find the cost of a certain other number?

III. INDUSTRIAL PROBLEMS

GETTING HORSES READY FOR MARKET



1. To one lot of horses a dealer fed corn. In this case the average weight increased from 1436 lb. to 1674 lb. in 112 da. What was the average increase per day?

2. To another lot he fed three parts corn to one of oats, and the average weight increased from 1468 lb. to 1734 lb. in 112 da. What was the average increase per day?

3. To a third lot he fed half corn and half oats, the increase being from 1463 lb. to 1736 lb. in 112 da. What was the average increase per day in this case? From these experiments, what ration gives the largest increase per day?

Examples like this, representing scientific experiments, not only give the pupils exercise in computation but also give some idea of the way in which the government is trying to solve the problem of how to improve stock, so that the farmer may have the best advice as to what rations to feed.

THE PARCEL POST

1. Our school has a baseball team. The captain finds that the catalogue from which he orders supplies tells the weight of the articles. The parcel post rates (1915) for shipping in four zones are :

<i>Pounds</i>	<i>Local Rate</i>	<i>First Zone</i>	<i>Second Zone</i>	<i>Third Zone</i>	<i>Fourth Zone</i>
1	\$0.05	\$0.05	\$0.05	\$0.06	\$0.07
2	.06	.06	.06	.08	.11
3	.06	.07	.07	.10	.15
4	.07	.08	.08	.12	.19
5	.07	.09	.09	.14	.23
6	.08	.10	.10	.16	.27
7	.08	.11	.11	.18	.31
8	.09	.12	.12	.20	.35
9	.09	.13	.13	.22	.39
10	.10	.14	.14	.24	.43

The first zone is from 1 mi. to 50 mi., the second is from 50 mi. to 150 mi., the third is from 150 mi. to 300 mi., and the fourth is from 300 mi. to 600 mi. Our school is 75 mi. from the city where the captain buys the goods. He buys three bats at 45¢, the shipping weight of each being 50 oz. How much money does he have to send for the bats and the postage ?

Any weight between 1 lb. and 2 lb. is counted as 2 lb., and so on.

2. The captain wishes two catcher's mitts. They cost \$2 each, and the shipping weight of each is 25 oz. How much money does he have to send for the mitts and the postage ?

3. Our team is going to play a team from a school that is 170 mi. from the city where that team buys its supplies. The captain of that team sends for two baseball masks. They cost \$1.40 each, and the shipping weight of each is 24 oz. How much money must he send for the masks and the postage ?

4. If a team sends to a city 200 mi. away for five balls at 50¢ each, the shipping weight of each being 7 oz., how much money must be sent with the order to cover the price and the postage ?

GRADE VI. SECOND HALF

I. REVIEW

WRITTEN EXERCISE

Without copying these numbers, add them, and then write the results on paper, timing yourself:

1.	2.	3.	4.	5.
243	408	280	445	281
378	749	409	648	462
421	654	977	876	393
506	328	645	639	441
887	765	321	894	288
929	564	964	207	770
<u>642</u>	<u>239</u>	<u>573</u>	<u>776</u>	<u>906</u>
6.	7.	8.	9.	10.
\$4.73	\$5.42	\$4.04	\$6.60	\$5.44
5.82	6.81	5.00	7.87	8.28
6.04	9.78	6.78	4.34	9.96
7.70	3.44	8.92	5.45	6.76
3.95	2.72	3.83	.72	8.96
5.87	6.63	.42	8.00	7.54
7.62	4.41	6.82	8.93	6.55
<u>3.48</u>	<u>8.82</u>	<u>7.07</u>	<u>4.21</u>	<u>4.21</u>

Pupils should be drilled until they can add the numbers on this page, without copying the columns, in about three minutes. See pages 83, 93, 152, and 157.

WRITTEN EXERCISE

Add, subtract, multiply, and divide, in turn, dividing the larger by the smaller and finding each quotient to the nearest hundredth :

- | | |
|----------------------|----------------------|
| 1. 38.41 and 16.72. | 13. 54.47 and 29.99. |
| 2. 43.72 and 18.96. | 14. 83.84 and 43.48. |
| 3. 56.34 and 19.77. | 15. 92.31 and 55.75. |
| 4. 82.62 and 20.32. | 16. 8.346 and 2.009. |
| 5. 60.08 and 21.07. | 17. 7.028 and 1.987. |
| 6. 70.23 and 32.09. | 18. 6.124 and 2.423. |
| 7. 80.36 and 51.82. | 19. 9.287 and 5.342. |
| 8. 79.09 and 29.72. | 20. 428.3 and 1.426. |
| 9. 65.33 and 28.86. | 21. 507.4 and 2.836. |
| 10. 72.05 and 53.53. | 22. 9236 and 41.83. |
| 11. 88.76 and 59.63. | 23. 7802 and 63.48. |
| 12. 92.28 and 31.73. | 24. 5405 and 27.72. |

Multiply the following, finding each product only to the nearest tenth :

- | | | |
|--------------------------|--------------------------|----------------------------|
| 25. 2.97×3.42 . | 29. 3.43×24.9 . | 33. 0.423×0.764 . |
| 26. 4.26×8.99 . | 30. 4.28×63.4 . | 34. 2.837×9.692 . |
| 27. 7.08×3.42 . | 31. 5.39×48.7 . | 35. 2.432×8.887 . |
| 28. 6.02×4.31 . | 32. 6.08×30.9 . | 36. 0.098×9.607 . |

Add, subtract, multiply, and divide, in turn, dividing both the first fraction by the second and the second fraction by the first :

- | | | | |
|----------------------------------|----------------------------------|----------------------------------|----------------------------------|
| 37. $\frac{1}{2}, \frac{1}{4}$. | 40. $\frac{1}{2}, \frac{3}{8}$. | 43. $\frac{1}{4}, \frac{1}{8}$. | 46. $\frac{3}{4}, \frac{3}{8}$. |
| 38. $\frac{1}{2}, \frac{3}{4}$. | 41. $\frac{1}{2}, \frac{5}{8}$. | 44. $\frac{1}{4}, \frac{3}{8}$. | 47. $\frac{3}{4}, \frac{5}{8}$. |
| 39. $\frac{1}{2}, \frac{1}{8}$. | 42. $\frac{1}{2}, \frac{7}{8}$. | 45. $\frac{1}{4}, \frac{5}{8}$. | 48. $\frac{3}{4}, \frac{7}{8}$. |

WRITTEN EXERCISE

Find 10%, 20%, 25%, 50%, and 75% of each of the following :

- | | | |
|----------|--------------|-------------------|
| 1. 624. | 11. \$21.36. | 21. 4 ft. 8 in. |
| 2. 772. | 12. \$33.64. | 22. 7 ft. 4 in. |
| 3. 816. | 13. \$51.16. | 23. 3 yd. 4 in. |
| 4. 964. | 14. \$63.32. | 24. 5 yd. 8 in. |
| 5. 556. | 15. \$17.72. | 25. 3 lb. 8 oz. |
| 6. 348. | 16. \$37.12. | 26. 5 lb. 12 oz. |
| 7. 708. | 17. \$53.28. | 27. 3 mi. 24 rd. |
| 8. 924. | 18. \$70.04. | 28. 1 mi. 164 ft. |
| 9. 736. | 19. \$90.68. | 29. 108 sq. in. |
| 10. 996. | 20. \$79.96. | 30. 1164 cu. in. |

Using short methods (page 170), multiply the following :

- | | | |
|--|--|--|
| 31. $248 \times 50 \text{ ¢}$. | 36. $664 \times 37\frac{1}{2} \text{ ¢}$. | 41. $558 \times 16\frac{2}{3} \text{ ¢}$. |
| 32. $576 \times 50 \text{ ¢}$. | 37. $684 \times 75 \text{ ¢}$. | 42. $996 \times 16\frac{2}{3} \text{ ¢}$. |
| 33. $369 \times 33\frac{1}{3} \text{ ¢}$. | 38. $776 \times 25 \text{ ¢}$. | 43. $775 \times 20 \text{ ¢}$. |
| 34. $396 \times 66\frac{2}{3} \text{ ¢}$. | 39. $996 \times 25 \text{ ¢}$. | 44. $905 \times 20 \text{ ¢}$. |
| 35. $748 \times 12\frac{1}{2} \text{ ¢}$. | 40. $776 \times 12\frac{1}{2} \text{ ¢}$. | 45. $924 \times 25 \text{ ¢}$. |

Using cancellation, divide the following :

- | | |
|-------------------------------|--------------------------------|
| 46. 7.8×1.68 by 7.8. | 49. 1.1×0.21 by 0.77. |
| 47. 2.8×1.2 by 0.32. | 50. 13.5×12.6 by 0.9. |
| 48. 1.1×4.8 by 2.2. | 51. 3.6×5.5 by 3.3. |

Express the following as inches :

- | | | | |
|-----------|------------------------|------------------------|-----------------|
| 52. 7 ft. | 54. 4 yd. | 56. $2\frac{1}{4}$ yd. | 58. 3 ft. 6 in. |
| 53. 9 ft. | 55. $3\frac{1}{2}$ yd. | 57. $3\frac{3}{4}$ yd. | 59. 2 ft. 8 in. |

WRITTEN EXERCISE

1. Write the following in Roman numerals:

17 22 16 37 41 49 62 98

2. Write the following in common numerals:

XXI XLVII XXXIX LXXXVIII L

3. Express the following fractions as eighths:

 $\frac{1}{2}$ $\frac{1}{4}$ $\frac{3}{4}$ $\frac{4}{16}$ $\frac{8}{16}$ $\frac{10}{16}$ $\frac{14}{16}$ $\frac{4}{32}$ $\frac{16}{32}$ $\frac{20}{32}$

4. Express the following fractions as fourths:

 $\frac{1}{2}$ $\frac{4}{8}$ $\frac{8}{8}$ $\frac{6}{12}$ $\frac{12}{16}$ $\frac{24}{16}$ $\frac{8}{16}$ $\frac{8}{32}$ $\frac{24}{32}$ $\frac{16}{64}$

5. Express the following mixed numbers as improper fractions:

 $2\frac{1}{2}$ $3\frac{1}{4}$ $4\frac{2}{3}$ $3\frac{3}{4}$ $2\frac{3}{8}$ $5\frac{5}{8}$ $4\frac{7}{8}$ $3\frac{5}{16}$

6. Express the following common fractions as decimal fractions, carrying each result to the nearest hundredth:

 $\frac{1}{2}$ $\frac{1}{4}$ $\frac{3}{4}$ $\frac{3}{8}$ $\frac{5}{8}$ $\frac{1}{3}$ $\frac{2}{3}$ $\frac{5}{6}$ $\frac{7}{8}$ $\frac{8}{16}$

7. Express the following as common fractions:

0.8 0.24 0.36 0.75 0.125 0.375

The purpose of the review work in this grade is to establish a strong foundation for future mathematical study by making pupils sure and skillful in the use of the essential processes previously studied.

At the outset the teacher should make sure that every pupil can add and subtract accurately and use the multiplication combinations automatically. All cases of "arrested development" — such as counting in adding, adding instead of using the multiplication combinations, and writing down numbers to be added to the next higher order in multiplication — should be earnestly sought out and corrected. The teacher, particularly of this grade, should feel that unless this fundamental work is thoroughly accomplished his work in arithmetic is, to a considerable extent, a failure.

ORAL EXERCISE

1. Express as decimal fractions:

$$\frac{1}{2} \quad \frac{1}{4} \quad \frac{3}{4} \quad \frac{1}{3} \quad \frac{2}{3} \quad \frac{1}{5} \quad \frac{2}{5} \quad \frac{3}{5} \quad \frac{4}{5}$$

2. Express as common fractions:

$$.25 \quad .50 \quad .75 \quad .33\frac{1}{3} \quad .66\frac{2}{3} \quad .12\frac{1}{2} \quad .16\frac{2}{3}$$

3. Add the following fractions:

$$\begin{array}{ccccc} \frac{1}{2} + \frac{1}{2} & \frac{1}{4} + \frac{1}{4} & \frac{1}{4} + \frac{3}{4} & \frac{3}{4} + \frac{3}{4} & \frac{5}{4} + \frac{3}{4} \\ \frac{1}{2} + \frac{1}{4} & \frac{1}{2} + \frac{3}{4} & \frac{1}{4} + \frac{1}{8} & \frac{1}{4} + \frac{3}{8} & \frac{1}{4} + \frac{5}{8} \end{array}$$

4. Subtract the following fractions:

$$\begin{array}{ccccc} \frac{1}{2} - \frac{1}{2} & \frac{1}{2} - \frac{1}{4} & \frac{3}{4} - \frac{1}{4} & \frac{3}{4} - \frac{1}{2} & \frac{5}{4} - \frac{1}{4} \\ \frac{1}{4} - \frac{1}{8} & \frac{3}{8} - \frac{1}{4} & \frac{5}{8} - \frac{1}{4} & \frac{7}{8} - \frac{3}{8} & \frac{7}{8} - \frac{1}{2} \end{array}$$

5. Multiply the following fractions:

$$\begin{array}{ccccc} \frac{1}{2} \text{ of } \frac{1}{2} & \frac{1}{2} \text{ of } \frac{1}{4} & \frac{1}{2} \text{ of } \frac{3}{4} & \frac{1}{2} \text{ of } \frac{1}{8} & \frac{1}{2} \text{ of } \frac{3}{8} \\ \frac{1}{2} \text{ of } \frac{5}{8} & \frac{1}{2} \text{ of } \frac{7}{8} & \frac{1}{4} \text{ of } \frac{1}{8} & \frac{1}{4} \text{ of } \frac{4}{5} & \frac{1}{2} \text{ of } \frac{2}{8} \end{array}$$

6. Divide the following fractions:

$$\begin{array}{ccccc} \frac{1}{2} \div \frac{1}{2} & \frac{1}{2} \div \frac{1}{4} & \frac{1}{4} \div \frac{1}{2} & \frac{1}{4} \div \frac{1}{4} & \frac{3}{4} \div \frac{1}{2} \\ \frac{1}{2} \div \frac{3}{8} & \frac{3}{8} \div \frac{1}{2} & \frac{1}{2} \div \frac{5}{8} & \frac{5}{8} \div \frac{1}{2} & \frac{7}{8} \div \frac{1}{2} \end{array}$$

WRITTEN EXERCISE

Add each fraction in line I, in turn, to each fraction in line II; subtract each fraction in line I, in turn, from each fraction in line II; multiply and divide each fraction in line I, in turn, by each fraction in line II:

I.	$\frac{1}{5}$	$\frac{2}{5}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{6}$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{1}{16}$	$\frac{3}{16}$
II.	$\frac{1}{2}$	$\frac{3}{5}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{5}{6}$	$\frac{5}{8}$	$\frac{7}{8}$	$\frac{9}{16}$	$\frac{11}{16}$

II. PERCENTAGE

169. Per Cent. We have already learned, in our study of common fractions and decimal fractions, that another name for hundredths is *per cent*, and that the symbol for per cent is written thus: %.

Therefore

$$1\% = \frac{1}{100}, \text{ or } 0.01;$$

$$2\frac{1}{2}\% = \frac{2\frac{1}{2}}{100}, \text{ or } 0.02\frac{1}{2}, \text{ or } 0.025;$$

$$100\% = \frac{100}{100}, \text{ or } 1;$$

and

$$125\% = \frac{125}{100}, \text{ or } 1.25, \text{ or } 1\frac{1}{4}.$$

170. Relation to Fractions. Since 6% means $\frac{6}{100}$, which equals 0.06, or $\frac{3}{50}$, we see that we may express per cent as a decimal fraction or as a common fraction.

It is often convenient to use one form, and often another. Thus, if we are multiplying by 13.5%, it is more convenient to think of the multiplier as 0.135; but if we are multiplying by $33\frac{1}{3}\%$, it is better to think of it as $\frac{1}{3}$ instead of $0.33\frac{1}{3}$.

171. Per Cents as Common Fractions. Since $62\frac{1}{2}\% = \frac{62\frac{1}{2}}{100} = \frac{125}{200} = \frac{5}{8}$, we see the truth of the following rule:

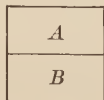
To express per cent as a common fraction, write the number indicating the per cent for the numerator and 100 for the denominator, and reduce this fraction to lowest terms.

For example, 50% means 0.50, or $\frac{50}{100}$, and this equals $\frac{1}{2}$.

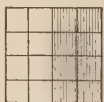
Pupils should realize that work in percentage is the same as work in common and decimal fractions. The terms "base," "rate," and "percentage" are seldom, if ever, used outside the schoolroom. Their use in the schoolroom should therefore rapidly disappear. The terms "per cent," "interest" (infrequently "rate of interest"), "commission," etc. should be used. The term "percentage" is often used to designate that part of arithmetic in which per cents are used.

ORAL EXERCISE

1. If this square is called 100, how much is A ? how much is B ? If the square is called 1, how many hundredths is A ? how many hundredths is B ?



2. Read 0.50, using the words "per cent"; using the word "hundredths." How many halves are there in 1? Then how many times is 50% contained in 1?



3. If this square is called 16, how much is the shaded part? How many sixteenths of the square are shaded? How many hundredths of it are shaded?

4. If this square is called 1, how much is the shaded part? If the square is called 100, how much is the shaded part? The shaded part is how many hundredths of the square?



5. Read 0.25, using the words "per cent"; using the word "hundredths." How many fourths are there in 1? Then how many times is 25% contained in 1?

6. If this square is called 1, how much is the shaded part? If the square is called 100, how much is the shaded part? What per cent of the square is shaded?



7. How many eighths are there in 1? Then how many times is $12\frac{1}{2}\%$ contained in 1?

8. Draw a picture on the blackboard and shade 10% of it. Draw another picture and shade 75% of it.

9. How many times is $16\frac{2}{3}\%$ contained in 1? in 2?

10. How many times is $6\frac{1}{4}\%$ contained in 1? in 3?

11. How many times is 20% contained in 1? in 5?

12. How many times is 50% contained in 7? in 12?

13. How many times is 25% contained in 9? in 20?

ORAL EXERCISE

1. Half of a square is what per cent of the square?
2. A fourth of a square is what per cent of the square?
3. An eighth of a square is what per cent of the square?
4. A sixteenth of a square is what per cent of the square?

172. Important Per Cents. We have found that

$$\begin{array}{ll} 50\% = \frac{1}{2} & 25\% = \frac{1}{4} \\ 12\frac{1}{2}\% = \frac{1}{8} & 6\frac{1}{4}\% = \frac{1}{16} \end{array}$$

All such per-cent equivalents should become so familiar that when the per-cent form is thought the common-fraction form will be thought simultaneously, and vice versa, and the two forms should be used interchangeably in all work.

5. How much is 50% of 20? of 40? of 50? of 7?
6. How much is 25% of 32? of 60? of 120? of 5?
7. How much is $12\frac{1}{2}\%$ of 40? of 72? of 88? of 100?
8. How much is $6\frac{1}{4}\%$ of 32? of 16? of 160? of 320?

WRITTEN EXERCISE

Find the values of the following:

- | | |
|-------------------|---------------------------------------|
| 1. 50% of 632. | 6. 50% of 978 bu. |
| 2. 25% of 592. | 7. 25% of 96 ft. 8 in. |
| 3. 50% of 333. | 8. $12\frac{1}{2}\%$ of \$91.28. |
| 4. 25% of \$3416. | 9. $12\frac{1}{2}\%$ of \$73.68. |
| 5. 50% of \$4917. | 10. $12\frac{1}{2}\%$ of 72 lb. 8 oz. |
11. A hawk caught $12\frac{1}{2}\%$ of 152 chickens. How many chickens were caught? How many were left?
 12. Of 1408 sheep owned by a farmer $6\frac{1}{4}\%$ were sold. How many sheep were sold? How many remained?

ORAL EXERCISE

1. If this circle is called 100, how much is the shaded part? how much is the unshaded part? If the circle is called 1, what per cent is the shaded part?

2. Read $0.33\frac{1}{3}$, using the words "per cent"; using the word "hundredths." How many thirds are there in 1? Then how many times is $33\frac{1}{3}\%$ contained in 1?



3. If this circle is called 100, the shaded part is what part of $33\frac{1}{3}$? How much is this? Then the shaded part is what per cent of the circle?



4. How much is $\frac{1}{2}$ of $16\frac{2}{3}\%$? The shaded part of this circle is what per cent of the whole circle? Then $\frac{1}{12}$ is what per cent?



173. Important Per Cents. We have found that

$$33\frac{1}{3}\% = \frac{1}{3}$$

$$66\frac{2}{3}\% = \frac{2}{3}$$

$$16\frac{2}{3}\% = \frac{1}{6}$$

$$8\frac{1}{3}\% = \frac{1}{12}$$

5. How much is $33\frac{1}{3}\%$ of 90? of 120? of 45? of 60?

6. How much is $66\frac{2}{3}\%$ of 30? of 66? of 90? of 300?

7. How much is $16\frac{2}{3}\%$ of 36? of 24? of 66? of 72?

WRITTEN EXERCISE

Find the values of the following:

1. $33\frac{1}{3}\%$ of 891.

6. $66\frac{2}{3}\%$ of 747 rd.

2. $66\frac{2}{3}\%$ of 711.

7. $16\frac{2}{3}\%$ of 9774 bu.

3. $16\frac{2}{3}\%$ of 504.

8. $33\frac{1}{3}\%$ of 222 ft. 9 in.

4. $16\frac{2}{3}\%$ of \$41.10.

9. $16\frac{2}{3}\%$ of 726 bu. 6 qt.

5. $8\frac{1}{3}\%$ of \$159.72.

10. $66\frac{2}{3}\%$ of 141 lb. 15 oz.

ORAL EXERCISE

1. How much is $\frac{200}{100}$? 200% ? $\frac{100}{100}$? 100% ?
2. Anything is what per cent of itself? of half itself?
3. If we call column *A* *one*, what is *B*? *C*? *D*? If we call *A* 100% , what per cent is *B*? *C*? *D*?
4. If we call *A* *two*, what is *B*? *C*? *D*? If we call *A* 200% , what per cent is *B*? *C*? *D*?
5. If we call *C* 100% , what per cent is *D*? *B*? *A*? *B* + *D*? *A* + *B* + *D*?
6. If we call *D* 100% , what per cent is *C*? *B*? *A*?
7. If we call *B* 100% , what per cent is *D* + *C*? *C* + *A*?
8. Because $2.75 = 2\frac{75}{100} = \frac{275}{100}$, then $2.75 =$ what per cent?



174. Reading Per Cents. We have found that 2.75 may be read "two and seventy-five hundredths" or "two hundred seventy-five per cent."

WRITTEN EXERCISE

Express as decimals:

- | | | | | |
|--------------|------------------------|----------------------|-----------------------|---------------|
| 1. 25% . | 3. $33\frac{1}{3}\%$. | 5. 1% . | 7. $2\frac{1}{2}\%$. | 9. 7.5% . |
| 2. 125% . | 4. $\frac{1}{2}\%$. | 6. $\frac{3}{4}\%$. | 8. 200% . | 10. 0.3% . |

Express as common fractions reduced to lowest terms:

- | | | | | |
|--------------|------------------------|---------------|---------------|---------------------------|
| 11. 32% . | 13. 24% . | 15. 36% . | 17. 17% . | 19. $82\frac{1}{2}\%$. |
| 12. 68% . | 14. $6\frac{1}{4}\%$. | 16. 125% . | 18. 375% . | 20. $33.3\frac{1}{3}\%$. |

Express as per cents:

- | | | | | |
|---------------------|---------------------|---------------------|----------------------|----------------------|
| 21. $\frac{1}{2}$. | 23. $\frac{7}{8}$. | 25. $\frac{2}{3}$. | 27. $1\frac{2}{5}$. | 29. $2\frac{1}{4}$. |
| 22. $\frac{4}{5}$. | 24. $\frac{1}{8}$. | 26. $\frac{3}{4}$. | 28. $1\frac{1}{2}$. | 30. $2\frac{3}{4}$. |

ORAL EXERCISE

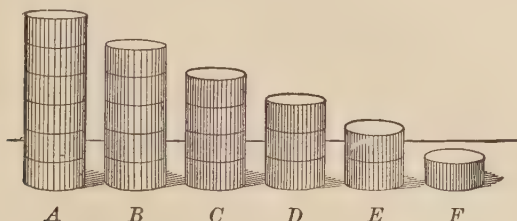
1. In the figures given below, F is what part as large as E ? what per cent?

2. F is what part as large as D ? what per cent?

3. F is what part as large as C ? what per cent?

4. F is what part as large as B ? what per cent?

5. F is what part as large as A ? what per cent?



In each pair tell what per cent of the second is the first:

6. $E, C.$ 10. $E, D.$ 14. $D, C.$ 18. $D, A.$ 22. $B, A.$

7. $E, A.$ 11. $E, B.$ 15. $C, B.$ 19. $D, B.$ 23. $B, C.$

8. $E, F.$ 12. $D, F.$ 16. $C, F.$ 20. $C, A.$ 24. $B, E.$

9. $A, D.$ 13. $C, E.$ 17. $D, E.$ 21. $C, D.$ 25. $B, F.$

26. Find 10% of 50; 175; 212; \$327; \$475.50.

27. Find 50% of 86.4; \$16.44; \$244.50; \$862.20.

28. Find $12\frac{1}{2}\%$ of 16; 80; 96; 728; 488; 648; \$840.

29. Find $16\frac{2}{3}\%$ of 36; 66; 90; 720; 366; 540; \$636.

30. Find 20% of 75; 90; 250; 305; 555; 750; \$3.50.

31. Find 25% of 64; 96; 324; 4.40; 2.40; 320; \$840.

32. Find $33\frac{1}{3}\%$ of 63; 96; \$3.33; \$1.50; 630; 900; 324;

1200; 3600; 3300; 3333; 9630.

Sam is 6 years old, Nora is 9, and Tom is 12. Tell the following per cents :

33. Sam's age is what per cent of Tom's ?
34. Sam's age is what per cent of Nora's ?
35. Nora is what per cent older than Sam ?
36. Sam is what per cent younger than Nora ?
37. Nora's age is what per cent of Tom's ? of Sam's ?
38. Tom is what per cent older than Sam ? than Nora ?
39. Nora's age is what per cent of the sum of Sam's and Tom's ?
40. Nora's age is what per cent of the product of Sam's and Tom's ?
41. Sam's age is what per cent of the difference between Nora's and Tom's ?

It is 3000 miles from London to New York, 1000 miles from New York to Chicago, and 2000 miles from Chicago to San Francisco. Tell the following per cents :

42. The distance from Chicago to San Francisco is what per cent of the distance from London to Chicago ?
43. The distance from New York to Chicago is what per cent of the distance from London to San Francisco ?
44. The distance from New York to Chicago is what per cent of the distance from Chicago to San Francisco ?
45. The distance from New York to San Francisco is what per cent of the distance from London to San Francisco ?
46. The distance from London to New York is what per cent greater than the distance from New York to Chicago ?
47. The distance from London to New York is what per cent less than the distance from London to San Francisco ?

ORAL EXERCISE

1. Recite the per-cent equivalents of the eighths.

2. $87\frac{1}{2}\%$ is how many times $12\frac{1}{2}\%$?

3. Find the value of $75\% \div 62\frac{1}{2}\%$.

Divide the equivalent common fractions.

4. Find the value of $62\frac{1}{2}\% \div 50\%$.

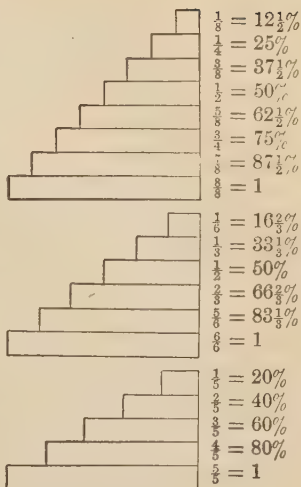
5. Recite the per-cent equivalents of the sixths.

6. Divide $83\frac{1}{3}\%$ by $33\frac{1}{3}\%$.

7. Divide 50% by $16\frac{2}{3}\%$.

8. Recite the per-cent equivalents of the fifths.

9. Divide 1 by 80% .



WRITTEN EXERCISE

Draw figures as above and learn the tables of equivalents of:

- | | | |
|---|--------------------|--------------------|
| 1. The sevenths. | 3. The twelfths. | 5. The tenths. |
| 2. The ninths. | 4. The sixteenths. | 6. The twentieths. |
| 7. Divide 20% by $16\frac{2}{3}\%$. | | |

Divide the equivalent common fractions and write the result both as a common fraction and as a decimal.

Perform the following operations:

- | | | |
|-----------------------------------|-----------------------------------|--|
| 8. $50\% \div 37\frac{1}{2}\%$. | 12. $83\frac{1}{3}\%$ of 60% . | 16. 80% of $16\frac{2}{3}\%$. |
| 9. $80\% \div 50\%$. | 13. 60% of $33\frac{1}{3}\%$. | 17. $62\frac{1}{2}\%$ of $83\frac{1}{3}\%$. |
| 10. $33\frac{1}{3}\%$ of 75% . | 14. $50\% \div 12\frac{1}{2}\%$. | 18. $66\frac{2}{3}\%$ of 50% . |
| 11. $12\frac{1}{2}\%$ of 80% . | 15. $25\% \div 60\%$. | 19. $33\frac{1}{3}\%$ of 40% . |

175. Finding Per Cents. Since 90%, 0.90, and $\frac{90}{100}$ mean the same, 90% of a number is the number multiplied by 0.90. Thus

$$85\% \text{ of } 70 = 0.85 \text{ of } 70 = 59.5,$$

$$\text{and } 12\frac{1}{2}\% \text{ of } 40 = 0.125 \text{ of } 40 = \frac{1}{8} \text{ of } 40 = 5.$$

ORAL EXERCISE

1. If you have 50 words in a spelling test and are marked 98%, how many words did you spell correctly?

2. If you have 10 questions in arithmetic and are marked 80%, how many questions did you answer correctly?

3. If you have 12 questions in geography and are marked 75%, how many questions did you answer correctly?

4. If 60% of a class of 40 have not missed a day this year, how many have been present every day?

5. If a merchant pays 2¢ apiece for pencils and sells them at a profit of 50%, how much does he receive for each?

6. If a milk dealer pays 5¢ a quart for milk and sells it at a profit of 20%, how much does he receive per quart?

7. If a butcher buys meat at 18¢ a pound and sells it at a profit of $16\frac{2}{3}\%$, at what rate does he sell it per pound?

WRITTEN EXERCISE

Find the values of the following:

1. 22% of 5475.

2. 6% of \$2496.

3. 95% of \$625.

4. 72% of \$345.

5. 35% of 5280 ft.

6. 62% of 3500 mi.

7. 65% of \$9200.

8. 87% of 4700 ft.

9. $66\frac{2}{3}\%$ of \$4131.

10. $17\frac{1}{2}\%$ of \$4000.

11. 31% of 4920 lb.

12. $87\frac{1}{2}\%$ of \$14.40.

ORAL EXERCISE

1. From these columns show that $20\% = \frac{1}{5}$, $40\% = \frac{2}{5}$, $60\% = \frac{3}{5}$, and $80\% = \frac{4}{5}$.

2. Show that 20% is $\frac{1}{2}$ of 40% , $\frac{1}{3}$ of 60% , $\frac{1}{4}$ of 80% , and $\frac{1}{5}$ of 100% .

3. Show that 40% is $\frac{2}{3}$ of 60% , $\frac{1}{2}$ of 80% , and $\frac{2}{5}$ of 100% .

4. Show that 60% is $1\frac{1}{2}$ times 40% , 3 times 20% , $\frac{3}{4}$ of 80% , and $\frac{3}{5}$ of 100% .

				20%
			20%	20%
	20%	20%	20%	20%
20%	20%	20%	20%	20%
<u>20%</u>	<u>40%</u>	<u>60%</u>	<u>80%</u>	<u>100%</u>

These columns should be written on the board. Columns of other per cents, such as $12\frac{1}{2}\%$, $16\frac{2}{3}\%$, 25% , and $33\frac{1}{3}\%$, are also useful for rapid drill work.

WRITTEN EXERCISE

1. Find $12\frac{1}{2}\%$ of 984; of 6116; of \$177.76.

2. Find $33\frac{1}{3}\%$ of 729; of \$923.22; of 5280 ft.

3. Find $16\frac{2}{3}\%$ of 5280 ft.; of \$1234.56; of 1728 cu. in.

4. A man bought a bicycle for \$40 and sold it at a loss of 35%. How much did he lose?

5. A man bought 83 sheep at \$6 apiece and sold them at a profit of 15%. How much did he gain?

6. A dealer buys 25 cows at \$32 each and sells them at a profit of 8%. How much does he gain on them all?

7. A man bought an automobile for \$700, and after using it two seasons sold it for 65% less than it cost. What was the selling price?

Teachers should from time to time require the pupils to estimate their answers in advance, comparing their results with their estimates.

8. A farmer bought 75 A. of land at \$65 an acre, and 12 cows at \$35 a head. He sold the land at a profit of 10% and the cows at a loss of 5%. Did he gain or lose on the whole transaction, and how much?

9. In a school of 450 pupils 8% were in the sixth grade. How many pupils were there in that grade?

10. In a school of 350 pupils 12% are in the third grade and 14% in the first. How many pupils are there in each of these grades?

11. Of 220 pupils 5% have been tardy or absent this month. How many pupils have not been tardy or absent?

12. If there should be an increase of $3\frac{1}{3}\%$ next year in a school of 330 pupils, how many pupils would there then be?

13. In a school of 250 pupils there are 4% fewer pupils than four years ago. How many pupils were there then?

14. In a school of 246 pupils $16\frac{2}{3}\%$ were not in the school last year. How many new pupils are there this year?

15. An ice wagon started out with a load of ice. One cake weighed 190 lb. at starting, but before it was delivered lost 4% by melting. How much did it then weigh?

16. The area of Illinois is 56,600 sq. mi. Of this 63% is underlaid with coal beds. How many square miles of coal lands are there in the state?

17. A class in industrial arts bought 36 yd. of cloth, and when the cloth was washed the shrinkage was $\frac{1}{2}\%$. How much did the cloth shrink? What was the length after shrinkage?

18. In the class in cooking it was found that some food costing \$8.50 had deteriorated in value 5% by being kept for a week. What was the amount of the deterioration expressed in cents?

176. The Product and One Factor. The most important thing in percentage is to find some per cent of a number. With this important operation you are now very familiar, and you know that it requires us to find the product of two given factors.

For example, 50% of 120 = 0.50×120 ,
 30% of 240 = 0.30×240 .

But you have learned from the relation

$$2 \times 5 = 10,$$

that we can obtain $10 \div 2 = 5$,

and $10 \div 5 = 2$.

That is, *given the product of two factors, and one of the factors, the other factor may be found by dividing the product by the given factor.*

We shall consider a few of these cases before we apply this principle to percentage.

ORAL EXERCISE

The first number is the product of two factors; the second is one of the factors; find the other factor.

- | | | | |
|--------------|--------------|-------------------------|-------------------------|
| 1. 48, 4. | 11. 640, 20. | 21. 4, 2. | 31. $3, \frac{1}{2}$. |
| 2. 48, 12. | 12. 640, 64. | 22. $\frac{4}{10}, 2$. | 32. $5, \frac{1}{4}$. |
| 3. 72, 8. | 13. 740, 74. | 23. 0.4, 2. | 33. $7, \frac{1}{8}$. |
| 4. 72, 9. | 14. 820, 82. | 24. 0.40, 2. | 34. $3, \frac{5}{10}$. |
| 5. 70, 10. | 15. 820, 41. | 25. 0.40, 4. | 35. $3, 0.5$. |
| 6. 90, 9. | 16. 440, 11. | 26. 40, 8. | 36. $3, 0.50$. |
| 7. 200, 20. | 17. 440, 10. | 27. 0.40, 8. | 37. $5, 0.50$. |
| 8. 300, 10. | 18. 440, 44. | 28. $3, \frac{1}{10}$. | 38. $6, 50\%$. |
| 9. 450, 45. | 19. 440, 22. | 29. $3, 0.1$. | 39. $6, 25\%$. |
| 10. 600, 20. | 20. 880, 22. | 30. $3, 10\%$. | 40. $8, 25\%$. |

177. Second Important Percentage Problem. The most practical problem in percentage is to find some per cent of a number. A second important problem is to find what per cent one number is of another.

In the first case we have two factors given, to find the product:

$$50\% \text{ of } 400 = 0.50 \times 400, \text{ or } \frac{1}{2} \text{ of } 400.$$

In the second case we have a product and one factor given, to find the other factor.

For example, if some per cent of 400 is 200, then 200 is a product, 400 is one of the factors, and we find the other factor by dividing.

$$\text{That is,} \quad 200 \div 400 = 0.50 = 50\%.$$

$$\text{Therefore} \quad 200 = 50\% \text{ of } 400.$$

ORAL EXERCISE

1. Some per cent of 50 is 25. What is it?
2. Some per cent of 40 is 10. What is it?
3. Some per cent of 80 is 60. What is it?

Find what per cent the second number is of the first:

- | | | | |
|----------|-------------|-------------|---------------------------|
| 4. 2, 1. | 9. 60, 30. | 14. 64, 32. | 19. 25, 5. |
| 5. 4, 1. | 10. 70, 35. | 15. 64, 16. | 20. 25, 2.5. |
| 6. 4, 2. | 11. 90, 45. | 16. 64, 48. | 21. 25, $12\frac{1}{2}$. |
| 7. 4, 3. | 12. 60, 15. | 17. 72, 54. | 22. 25, 25. |
| 8. 8, 6. | 13. 32, 8. | 18. 36, 27. | 23. 25, 50. |

24. 4 is what per cent of 8? of 16? of 6?

25. 5 is what per cent of 10? of 20? of 100?

26. If a baseball team wins 6 games out of 10, what per cent of games does it win? what per cent of games does it lose?

178. Application to Written Exercises. You now understand the second important kind of problem in percentage. We shall consider once more the first kind of problem, and then the application of the second kind to written exercises.

1. An agent bought an automobile for \$600 and sold it so as to gain 20% of the cost. How much did he gain?

Here we have to find 20% of \$600, or 0.20 of \$600. That is, we have two factors given, 0.20 and \$600, to find the product. Therefore we have

$$0.20 \times 600 = \$120.$$

This is the first kind of problem. We shall now consider the second kind.

2. An agent bought an automobile for \$600 and sold it at a profit of \$120. What per cent of the cost did he gain?

Here we have \$120 equal to some per cent of \$600.

That is, we have the product (120) of two factors, and one of the factors (600) given, to find the other factor. Therefore we have

$$120 \div 600 = 0.20 = 20\%.$$

That is, the agent gained 20% of the cost.

WRITTEN EXERCISE

1. \$130 is what per cent of \$2600? of \$3900?
2. \$11.25 is what per cent of \$375? of \$56.25?
3. \$10.12 is what per cent of \$253? of \$202.40?
4. \$29.10 is what per cent of \$465.60? of \$174.60?
5. A foot is what per cent of a yard? of a rod?
6. A quart is what per cent of a gallon? of 25 gal.?
7. $\frac{2}{3}$ is what per cent of $\frac{1}{4}$? $\frac{3}{8}$ is what per cent of $\frac{1}{4}$?
8. 25% is what per cent of 75%? of 125%? of 250%?
9. $33\frac{1}{3}\%$ is what per cent of $66\frac{2}{3}\%$? of 1? of $133\frac{1}{3}\%$?
10. $16\frac{2}{3}\%$ is what per cent of $33\frac{1}{3}\%$? of $66\frac{2}{3}\%$? of 1?

11. If Ralph Jenkins is at bat 40 times and makes 12 base hits, what is his per cent of base hits?

12. If the seventh grade devotes 45 min. a day to arithmetic, and the sixth grade 42 min., what is the per cent of increase?

13. If a class had 24 examples to solve on Monday and 29 on Tuesday, what was the per cent of increase?

14. If you have read 78 pages in a book of 300 pages, what per cent of the pages have you read?

15. If the last chapter of a book is numbered XXXII, and you have finished reading Chapter XXIV, what per cent of the chapters have you read?

16. A man's income is \$1650 a year, and he spends \$693. What per cent of his income does he save?

17. In a certain village 576 out of the 1200 pupils in school are boys. What per cent are boys? What per cent are girls?

18. A man invests \$6750 and gains \$810. His gain is what per cent of the amount invested?

19. The purity of gold is measured in carats, or twenty-fourths, 18 carats meaning $\frac{18}{24}$ pure gold. What is the per cent of pure gold in an 18-carat ring?

20. What is the per cent of pure gold in a watch case that is 16 carats fine? in a chain that is 12 carats fine?

21. If 2 members of a class of 32 have not been either tardy or absent during the year, and 4 members have not been absent, what per cent have not been either tardy or absent? what per cent have not been absent?

22. A man's income is \$2175 a year, and he spends \$652.50. What per cent of his income does he spend? What per cent does he save? What is the sum of these two per cents?

179. Third Important Percentage Problem. There is another kind of problem in percentage, depending on the principle stated on page 233. It will be understood from the following:

An agent bought an automobile and sold it at a profit of \$120, which was 20% of the cost. Find the cost.

Here we have 20% of the cost = \$120.

That is, we have the product (120) of two factors, and one of the factors (0.20) given, to find the other factor.

Therefore we have $120 \div 0.20 = 600$.

That is, the automobile cost the agent \$600.

Problems of this kind are not required in the New Jersey course of study, and may therefore be omitted if desired. The course is a minimum one, however, and since these problems involve exactly the same principle that is followed on page 234, they are inserted for such schools as may care to use them.

WRITTEN EXERCISE

1. If 20% of the inhabitants of a town are voters, and there are 236 voters, what is the population of the town?
2. Of what number is 487 equal to 5%?
3. \$1.05 is 15% of what amount? 10% of what amount?

Find the numbers of which the following are the given per cents:

- | | | |
|-------------|------------------------------|--------------------------------|
| 4. 64, 8%. | 11. 111, 3%. | 18. 147, $87\frac{1}{2}\%$. |
| 5. 72, 9%. | 12. 112, 2%. | 19. 42.06, $66\frac{2}{3}\%$. |
| 6. 88, 11%. | 13. 97, $12\frac{1}{2}\%$. | 20. 12.48, $16\frac{2}{3}\%$. |
| 7. 85, 5%. | 14. 89, $3\frac{1}{3}\%$. | 21. 57.8, 34%. |
| 8. 84, 6%. | 15. 62, $33\frac{1}{3}\%$. | 22. 90.2, 110%. |
| 9. 91, 7%. | 16. 156, $37\frac{1}{2}\%$. | 23. 65.72, 106%. |
| 10. 64, 4%. | 17. 125, $62\frac{1}{2}\%$. | 24. 622.6, 5.5%. |

25. If a farmer sold a horse for 80% of its cost and received \$152, how much did the horse cost?

26. A poultry raiser set some eggs, and 15% failed to hatch. The number failing to hatch being 33, how many eggs did the poultry raiser set?

27. $112\frac{1}{2}$ ft. is 45% of what length? $66\frac{2}{3}\%$ of what length?

28. A man saves \$675.20 a year, which is 32% of his income. How much is his income?

29. A school has 20% of its pupils in the sixth grade, which numbers 29. How many pupils are there in the school?

30. If a school is in session 50% of the days of a certain year, and is in session 183 days, is that year a leap year or not?

31. A man buys some lumber and sells it for \$60, thereby gaining 20% on the cost. The \$60 is what per cent of the cost? What is the cost?

32. A certain school has 119 boys, which is 85% of the number of girls. How many girls are there in the school?

33. A man sells 41 sheep, which is $33\frac{1}{3}\%$ of all he owned. He receives \$5.50 a head. At this rate, what is the value of the sheep remaining in his flock?

34. A player reaches first base 23 times, or $33\frac{1}{3}\%$ of the number of times he was at bat. How many times was he at bat?

35. A player reaches home base 16 times, or $8\frac{1}{3}\%$ of the number of times he was at bat. How many times was he at bat?

36. A player made a base hit 24 times, or 30% of the number of times he was at bat. How many times was he at bat?

37. If you go on a camping trip and contribute $37\frac{1}{2}\%$ of the expenses, your contribution being \$3.12, what are the expenses of the trip?

180. Miscellaneous Problems. We are now prepared to consider miscellaneous problems, and here you will find it necessary to decide upon the method of solution. You will find it best to ask yourself exactly what is given, and then to consider carefully whether you have to find some per cent of a number (two factors given), or whether you have a product and one factor given, to find the other factor.

It will be noticed that only two principles are involved, those suggested by the New Jersey course of study. The examples after Ex. 21 involve the variation of the principle mentioned on page 237, and may be omitted if desired. They are inserted for the use of those who may wish to carry the work into more practical fields.

WRITTEN EXERCISE

1. A cubic foot of water weighs $62\frac{1}{2}$ lb. From a tank containing 16 cu. ft. there is drawn off an amount of water that would weigh 125 lb. What per cent of the water is drawn off?

2. To a class of 32 pupils 8 more are admitted. What per cent of the original number are admitted? What per cent of the present number are admitted? If 10 pupils now leave the class, what per cent leave the class?

3. In an arithmetic class it is planned to solve 550 problems in a certain number of days. All but 44 problems have been solved. What per cent of the problems remain to be solved? What per cent of them have been solved?

4. If 45% of the 240 pupils that attend a certain school are boys, how many are girls?

5. A man purchased 530 head of cattle and sold 30% of them. How many head of cattle had he left?

6. If an automobile going at the rate of 14 mi. an hour increases its speed 37%, at what rate will it then be traveling?

7. A cubic foot of platinum weighs 1220 lb., and a cubic foot of pressed gold weighs 10 lb. less. Platinum is what per cent heavier than gold?

8. A bank averaged \$1200 a day of new deposits in November, and 26% more a day in December. What were the average daily deposits in December?

9. A clerk's salary is \$12 a week. If he spends 20% of it for rent, 10% for clothes, and 25% for other expenses, and saves the rest, how much does he save in a year of 50 weeks?

10. If a baker uses 639 lb. of flour in making bread and adds to the flour 213 lb. of liquid, the weight of the liquid is what per cent of the weight of the mixture?

11. If $3\frac{1}{2}$ teaspoons of sugar make a tablespoon, a teaspoon of sugar is what per cent of a tablespoon?

12. If there are 16 teaspoons to a cup, a teaspoon is what per cent of a cup?

13. A teaspoon of butter is $\frac{1}{4}$ of a tablespoon. Express a teaspoon of butter as a per cent of a tablespoon.

14. The record of the champion team in baseball in a certain year was 52 games won, 14 lost. What per cent of the games did the champion team win that year?

15. One of the best records in baseball of recent years was when Brooklyn won the pennant with a total of 82 games won and 54 lost. What was the per cent of games won?

16. Two of the best running averages in baseball were 597 times at bat, 139 runs, and 543 times at bat, 145 runs. What was the per cent of runs in each case?

17. If the expenses of a certain village were \$8250 last year and \$8002.50 this year, what is the per cent of decrease?

18. If the rainfall in a certain place was 7.2 in. in April and $6\frac{1}{4}$ in. in March, what was the per cent of increase?

19. In 1800, of the 161,800,000 people who spoke European languages, 31,450,000 spoke French. Of the 401,700,000 in 1900, those who spoke French numbered 51,200,000. What per cent spoke French in each of these years?

20. A dressmaker bought a silk dress pattern for \$45 less 20%. She sold it for \$45. What per cent did she gain on what she paid for it?

21. Allowing 8 oz. of Paris green to 50 gal. of water in preparing a spray to destroy leaf-eating insects, and considering a gallon of water to weigh 8.4 lb., the weight of the Paris green is what per cent of the weight of the water?

22. A dealer sold pencils at 5¢ each, which gave him a profit of 25%. How much did they cost him a dozen?

23. A farmer purchased some sheep and sold them for \$252, which gave him a profit of 20%. What did they cost him?

24. A dealer sells some goods for \$31.05, thus making a profit of 15% on the cost. How much did the goods cost him?

25. A manufacturer sells a suit of clothes to a merchant at a profit of $12\frac{1}{2}\%$. The merchant sells it to you for \$12 and makes a profit of $33\frac{1}{3}\%$. How much does the suit cost the merchant? How much does it cost the manufacturer?

Notice that $133\frac{1}{3}\%$ of the cost to the merchant is \$12.

26. An agent bought 100 doz. eggs, which he sold to a grocer at a profit of 20%. The grocer sold the eggs at retail for \$22.44 and made a profit of 10%. What did the eggs cost the grocer? What did they cost the agent?

Notice that 110% of the cost to the grocer is \$22.44.

The following numbers are the given per cents more than what numbers? That is, 72 is 20% more than what number?

- | | | |
|-----------------------------|-------------------------------|-------------------------------|
| 27. 72, 20%. | 31. 14.7, $16\frac{2}{3}\%$. | 35. 27.3, $62\frac{1}{2}\%$. |
| 28. 75, 25%. | 32. 12.1, $37\frac{1}{2}\%$. | 36. 16.5, $87\frac{1}{2}\%$. |
| 29. 78, 30%. | 33. 6.48, $33\frac{1}{3}\%$. | 37. 129.6, 80%. |
| 30. 99, $12\frac{1}{2}\%$. | 34. 1.56, 50%. | 38. 375, 100%. |

39. A man's salary has been increased 22% this year. It is now \$1525. What was it last year?

40. If I have added 60% to the size of my farm and now have 240 acres, how many acres did I have before the increase?

41. A man adds $9\frac{1}{2}\%$ to the size of his farm and now has 284.7 acres. How many acres did he have before?

42. A railroad adds $8\frac{1}{2}\%$ to its track mileage and now has 299.46 mi. How many miles did it have before?

43. If an army has been increased 21% and now numbers 73,810, how many did it number before the increase?

44. A man sold some goods for \$192.50, making a profit of 10%. What did the goods cost him?

45. A dealer sold a horse for \$181.50, making a profit of 10%. What did the horse cost him?

46. A village has a population of 1853, an increase of 9% over ten years ago. What was the population ten years ago?

47. A village has a population of 2664, an increase of 11% over ten years ago. What was the population ten years ago?

48. If you weigh 78.1 lb. and have increased 42% in weight in the last five years, how much did you weigh five years ago?

49. If a library has increased in size $16\frac{2}{3}\%$ in the past five years and now has 8323 volumes, how many volumes had it before the increase?

Find the numbers of which :

50. 35% is 7.

53. 90% is 198.

56. 13% is 22.36.

51. 4% is 8.

54. 10% is 100.

57. 17% is 42.5.

52. 75% is 75.

55. 20% is 440.

58. 33% is 10.89.

59. A man sold an automobile for \$490, which was 30% less than it cost. How much did it cost?

Just as

$$\$1 - \$0.30 = \$0.70,$$

so

$$\text{cost} - 0.30 \text{ of cost} = 0.70 \text{ of cost.}$$

If

$$0.70 \text{ of cost} = \$490,$$

then

$$\begin{aligned}\text{cost} &= \$490 \div 0.70 \\ &= \$700.\end{aligned}$$

That is, we have again the product (490) and one factor (70%) given, to find the other factor (700).

60. After losing 15% of its population a town had 5950 inhabitants. How many had it before losing the 15%?

61. A dealer sold a bicycle for \$24.50, which was 30% less than the marked price. What was the marked price?

62. A man bought a set of furniture for \$52, which was 20% less than the marked price. What was the marked price?

The following numbers are the given per cents less than what numbers? That is, 30.4 is 5% less than what number?

63. 30.4, 5%.

67. 38.22, 9%.

71. 55, $16\frac{2}{3}\%$.

64. 31.96, 6%.

68. 43.2, 10%.

72. 75, $37\frac{1}{2}\%$.

65. 33.48, 7%.

69. 51, 15%.

73. 9.6, $62\frac{1}{2}\%$.

66. 38.64, 8%.

70. 77, $12\frac{1}{2}\%$.

74. 14.7, $87\frac{1}{2}\%$.

75. A man sold a house for \$5040, at a profit of 20%. How much did the house cost him?

76. If the boys in a certain school are 45% of the whole number of pupils, and there are 66 girls, how many boys are there?

The following numbers are the given per cents more than what numbers and the same per cents less than what other numbers? That is, 390 is $62\frac{1}{2}\%$ more than what number, and $62\frac{1}{2}\%$ less than what other number?

77. 390, $62\frac{1}{2}\%$.81. 350, $16\frac{2}{3}\%$.

85. 210, 40%.

78. 150, $87\frac{1}{2}\%$.82. 143, $8\frac{1}{3}\%$.

86. 9.1, 30%.

79. 640, $33\frac{1}{3}\%$.

83. 2.4, 20%.

87. 17.6, 60%.

80. 175, $66\frac{2}{3}\%$.

84. 42.3, 50%.

88. 56.1, 70%.

89. What number decreased by $2\frac{1}{2}\%$ of itself equals 11.7?

90. If a man sells 12% of his hens and has 66 left, how many had he at first?

91. If a man spends 5% of his salary for clothes and then has \$855 left, what is his salary?

92. If you have read all but 40% of a book and have read 234 pages, how many pages has the book?

93. If the width of a schoolhouse is 10% less than the length, and the width is 46.8 ft., what is the length?

94. If you lose 10% of a kite string and have 585 ft. left, how long was the string? How many feet did you lose?

95. A man sold some damaged goods for \$1547, at a loss of 9%. What did the goods cost him?

96. A village has a population of 1692, having lost 6% in ten years. What was its population ten years ago?

97. A man sold a horse for \$168, at a loss of 4%. What did the horse cost him?

98. A man gained \$180, which was 12% on the money he invested. How much did he invest?

99. A man sold some property for \$1680, at a profit of 12%. How much did the property cost him?

181. Per Cents off. If you have a catcher's glove or a tennis racket that you have used for a year, you may be glad to sell it at half price so as to get part of the money that it cost and have this much toward buying a new one. In that case you would sell it at a *discount* of 50%.

Sometimes merchants have bargain sales to dispose of odd pieces or of goods that are going out of style. At such times they usually sell goods at a certain "per cent off."

For example, goods marked \$2.50, but sold 10% off, sell at \$2.50 - 10% of \$2.50, or \$2.25.

\$2.50	\$2.50
.10	0.25
<u>\$0.25</u>	<u>\$2.25</u>

Merchants who sell at wholesale, that is, in large quantities to dealers, sell at a certain per cent off the price as stated in their catalogues, or the *list price*, as it is called.

182. Discount. A deduction made on a price or amount is called a *discount*. Discounts are usually reckoned at some rate per cent of the amount from which the discount is made. If you should offer to sell for \$2 a tennis racket that cost \$3, you would be offering to sell it at a discount of $\frac{1}{3}$, or of $33\frac{1}{3}\%$.

\$245.50	list price
0.12	rate of discount
<u>\$29.46</u>	discount

\$245.50	
29.46	
<u>\$216.04</u>	selling price

183. Rate of Discount. The per cent of discount is called the *rate of discount*.

What is the discount and the selling price (or *net price*) when the list price of a shipment of furniture is \$245.50 and the rate of discount is 12%?

$$12\% \text{ of } \$245.50 = \$29.46, \text{ discount.}$$

$$\$245.50 - \$29.46 = \$216.04, \text{ selling price.}$$

ORAL EXERCISE

1. How much is \$200 less \$50? \$100 less 25% of itself?
2. If from 1 we take $\frac{1}{4}$, what is left? If from 1 we take 25%, what is left?
3. How much is 1 less $\frac{1}{5}$? $\frac{100}{100} - \frac{20}{100}$? $1 - 0.20$? $1 - 20\%$?
4. If you buy a \$1 book for 10% less than the list price, how much does it cost you?

Find the price that must be paid for goods marked as stated in these examples, and discounted at the rates given:

- | | | |
|--------------|-------------------------------|------------------------------|
| 5. \$2, 10%. | 9. \$50, 30%. | 13. \$6, $33\frac{1}{3}\%$. |
| 6. \$6, 20%. | 10. \$60, $33\frac{1}{3}\%$. | 14. \$9, $33\frac{1}{3}\%$. |
| 7. \$5, 20%. | 11. \$1.60, 10%. | 15. \$10, 75%. |
| 8. \$4, 25%. | 12. \$100, 30%. | 16. \$1.50, 10%. |

WRITTEN EXERCISE

Find the price that must be paid for goods marked as stated in these examples, and discounted at the rates given:

- | | | |
|----------------|------------------|-------------------|
| 1. \$635, 6%. | 4. \$125, 35%. | 7. \$375.25, 4%. |
| 2. \$350, 15%. | 5. \$21.25, 8%. | 8. \$175.50, 20%. |
| 3. \$275, 15%. | 6. \$45.50, 20%. | 9. \$225.75, 20%. |
10. How much will a set of Scott's works, marked \$14, cost at 20% discount? Check by taking 80% of \$14.
 11. If you can buy a \$7 suit of clothes at 15% discount, how much will you save on the marked price?
 12. If the list price of your books, pencils, and paper is \$2.50 a year, how much will they cost at 12% discount?
 13. At a bargain sale an \$8 chair was sold at 30% discount. For how much did it sell?

Find the discounts on goods of which the list price and rate of discount are as follows :

- | | | |
|-----------------|----------------------------------|----------------------------------|
| 14. \$430, 15%. | 17. \$1235, 17%. | 20. \$42.64, 25%. |
| 15. \$281, 20%. | 18. \$2350, 14%. | 21. \$6336, $6\frac{1}{4}\%$. |
| 16. \$425, 10%. | 19. \$43.26, $33\frac{1}{3}\%$. | 22. \$34.54, $16\frac{2}{3}\%$. |

Find the selling price, given the list price and rate of discount as follows :

- | | | |
|--------------------------------|-------------------|----------------------------------|
| 23. \$275, 20%. | 27. \$425, 12%. | 31. \$52.84, $16\frac{2}{3}\%$. |
| 24. \$575, 5%. | 28. \$320, 14%. | 32. \$26.32, 50%. |
| 25. \$462, 3%. | 29. \$12.75, 20%. | 33. \$4235, 22%. |
| 26. \$650, $12\frac{1}{2}\%$. | 30. \$6202, 14%. | 34. \$14.43, $33\frac{1}{3}\%$. |

35. A \$17.50 set of china is marked down 15%. What is the selling price?

36. If you buy \$1.80 worth of fireworks at 15% discount, how much do they cost?

37. A book dealer buys a 65¢ arithmetic at 22% discount. How much does it cost him?

38. A \$31 overcoat is marked "25% off" at a bargain sale. What is the selling price?

39. If you buy a \$1.20 sled at 10% discount, how much does it cost? Suppose the discount is 20%; 25%.

40. How much will 2 dozen blackboard erasers cost at 45¢ a dozen, discount 10%? discount 20%?

41. A man paid \$150 for a carriage, and after using it for a year sold it at a discount of $16\frac{2}{3}\%$. What did he receive for it?

42. A man bought a \$900 automobile at 15% discount. After using it for a season he sold it for 15% less than he paid for it. How much did he receive for it?

184. Debtor and Creditor. When one man owes another he is called a *debtor* (Dr.). The man to whom he owes money is called his *creditor* (Cr.). So if Rob has bought a baseball bat from Fred and owes him 50¢, Rob is a debtor and Fred is a creditor.

185. Debit and Credit. The amount owed on a bill is called the *debit*, and any payments that have been made form the *credit*.

186. Bills. Bills usually begin as follows:

Mr. A. B. Jefferson, 17 Monroe St.

Bought of ROE & DOE, Grocers

No. 4 Washington Street

They may also read: "To Roe & Doe, Dr." This means that Mr. Jefferson is debtor to Roe & Doe for the goods mentioned in the bill.

Bills of goods sold by a wholesale dealer to a retail dealer are usually discounted.

The following is an example of a bill filled out, footed, and receipted:

Princeton, N.J., [Date]

Mrs. John Doe, 47 Tenth Ave.

To RICHARD ROE & CO., Dr.

Dry Goods Dealers, No. 10 Grand St.

.....[Year]....	$\frac{1}{2}$ yd. silk	.75		38
.....[Date]....	$\frac{3}{4}$ " "	1.25		94
"	$7\frac{1}{8}$ " velour	3.50	26	25
			27	57

Received payment,

Richard Roe & Co.

WRITTEN EXERCISE

Date, fill, foot, and receipt the following bills, inserting the name and address of some purchaser and of some dealer whom you know :

1. Bill for furniture: 6 chairs @ \$3.00; 2 armchairs @ \$6.25; 1 rocking-chair @ \$5.50; 1 table @ \$22.75.

An item like "6 chairs @ \$3.00" means \$3.00 each.

2. Bill for dry goods: $3\frac{1}{2}$ yd. satin @ \$1.50; 2 yd. flannel @ 50¢; $1\frac{1}{2}$ yd. canvas @ 20¢; $1\frac{1}{2}$ yd. ribbon @ 23¢.

3. Bill for meat: $2\frac{1}{2}$ lb. steak @ 32¢; $3\frac{1}{2}$ lb. fish @ 20¢; $4\frac{3}{4}$ lb. roast beef @ 36¢; 8 lb. turkey @ 31¢.

4. Bill for dry goods: $17\frac{1}{2}$ yd. embroidery @ 10¢; 3 yd. lawn @ 20¢; $7\frac{1}{2}$ yd. percale @ 25¢; $\frac{1}{8}$ yd. satin @ \$1.50.

5. Bill for crockery: $\frac{1}{2}$ doz. cups and saucers @ \$6.00; 3 doz. plates @ \$4.25; $1\frac{1}{2}$ doz. tumblers @ 90¢; 2 doz. individual butter plates @ 70¢; $\frac{1}{2}$ doz. bowls @ \$2.30.

An item like " $\frac{1}{2}$ doz. cups @ \$6.00" means \$6.00 a dozen.

6. Bill for groceries: 10 lb. sugar @ 8¢; 2 qt. berries @ 11¢; 4 doz. eggs @ 26¢; $5\frac{1}{2}$ lb. butter @ 32¢; $1\frac{1}{2}$ lb. cheese @ 22¢; $\frac{1}{2}$ doz. bars soap @ 44¢.

7. Bill for dry goods: 7 yd. muslin @ 18¢; 6 handkerchiefs @ 30¢; 3 pairs kid gloves @ \$1.50; 12 yd. sheeting @ $12\frac{1}{2}$ ¢; 16 yd. pongee @ 70¢.

8. Bill for stationery: 500 envelopes @ \$3.50 per M (that is, per 1000); 1 box writing paper @ \$2.25; $\frac{1}{2}$ doz. pencils @ 35¢; 3 penholders @ 5¢; 1 bottle ink @ 30¢.

9. Bill for groceries: 5 lb. coffee @ 32¢; 2 lb. tea @ 45¢; 8 lb. ham @ 15¢; 7 lb. lard @ 8¢; $3\frac{1}{2}$ lb. butter @ 33¢; 3 qt. currants @ 7¢; 2 doz. eggs @ 31¢; $1\frac{3}{4}$ lb. cheese @ 16¢; $\frac{1}{2}$ gal. sirup @ 65¢.

187. Order for Goods. The following is a model order for goods:

WOOD & ROBERTS

Booksellers

Hoboken, N.J., Feb. 15, 1918

Messrs. Ginn and Company, Publishers

70 Fifth Ave., New York City

Dear Sirs: Please send at once, by express,
75 Wentworth-Smith Complete Arithmetics, Pt. II,
60 " " " Arithmetics, Pt. I.

Yours truly,

WOOD & ROBERTS

188. Model Bill. The following is the form of the bill that Ginn and Company would send in reply:

Feb. 16, 1918

Messrs. Wood & Roberts

Hoboken, N.J.

Bought of GINN AND COMPANY

Educational Publishers

70 FIFTH AVE., NEW YORK

Terms of this Invoice: 1/6 Cash

75	Wentworth-Smith Complete Arithmetics, Pt. II	\$.40	\$30.00
60	" " " Arithmetics, Pt. I	.40	24.00
			<u>\$54.00</u>
		1/6	<u>9.00</u>

\$45.00

It is customary to state per cents like $16\frac{2}{3}\%$, $12\frac{1}{2}\%$, 20% , and 25% in the common-fraction form, as above. In this case $16\frac{2}{3}\%$ is stated as $\frac{1}{6}$. If the discount had been $12\frac{1}{2}\%$, it would have been stated as $\frac{1}{8}$.

189. Sample Price List. The following is a price list of certain school supplies, with the discounts allowed to schools and dealers where the prices are not net:

Rulers,	\$0.35 per doz., net
Composition books,	4.50 per gross, less 5%
Thumb tacks,	0.40 per 100, less 40%
Drawing pencils,	4.71 per gross, less 20%
Drawing paper, 9 × 12,	1.10 per package, less $\frac{1}{10}$
Pens,	0.61 per gross, less $\frac{1}{4}$
Tubes of paste,	4.05 per gross, less 10%
Penholders,	3.20 per gross, less 12%
Drawing compasses,	1.65 per doz., less 10%

WRITTEN EXERCISE

1. How much must your school pay for $\frac{1}{2}$ gross of penholders?
2. The school wishes to buy 5 packages of drawing paper and 300 thumb tacks. How much will they cost?
3. How much will 8 gross of pens and $\frac{1}{2}$ gross of composition books cost? 2 gross of pens and 1 doz. rulers?
4. There are 40 pupils in a class, and each needs drawing compasses and a ruler. How much will they cost the school? How much would they cost for 60 pupils?
5. If a dealer sells pens at a cent apiece, how much does he gain per gross? If he sells penholders at 2¢ each, how much does he gain per gross?
6. A dealer buys a gross of rulers and a gross of drawing pencils. He sells both at 5¢ each. What does he gain on the lot?
7. If a dealer buys a gross of tubes of paste for mounting pictures and sells the tubes at 5¢ each, how much does he gain on the purchase? How much would he gain on 3 gross?

WRITTEN EXERCISE

Write both orders and bills for the following goods bought of yourself by the person named or by some one you know:

1. Bought 2 doz. tennis rackets @ \$21; 8 doz. tennis balls @ \$3; $\frac{3}{4}$ doz. tennis nets @ \$19.20. Discount 12%.

2. Jones & Co., 3497 Wabash Ave., Chicago: 2 bbl. Porto Rico molasses @ \$12; 200 lb. Rio tapioca @ 6¢; 150 lb. Mocha coffee @ 20¢. Discount 6%.

3. B. Franklin & Co., Philadelphia: 5 doz. bottles ink @ \$3; 300 lb. paper @ 30¢; 2 doz. bottles mucilage @ \$4; 6 gross pencils @ \$2.75. Discount 3%.

4. George Lloyd, Lincoln, Nebr.: 18 brass bedsteads, No. 142, @ \$17.25; 15 woven-wire springs, No. 16, @ \$3.45; 3 sideboards, No. 196, @ \$17.25. Discount $\frac{1}{6}$.

5. R. B. Homer, Buffalo: 75 bags potatoes @ \$1.48 per bag; 50 bags potatoes @ \$1.28 per bag; 125 bags potatoes @ \$1.42 per bag; 130 bags potatoes @ 1.32 per bag; 1620 lb. Jersey potatoes in bulk @ \$1.47 per 180 lb. Discount $\frac{1}{6}$.

Potatoes are often sold in bags of about 150 lb. to 180 lb.

6. Newton & Co., 2831 Spring St., New York: 21 Ohio steers, 1226 lb. (average), @ \$5.10 (per 100 lb.); 20 Ohio steers, 1247 lb., @ \$5.10; 20 Ohio steers, 1112 lb., @ \$4.95. Discount $\frac{1}{10}$.

7. Sherman & Culver, New York: 20 Illinois steers, 1157 lb. (see Ex. 6), @ \$4.60; 28 Illinois steers, 996 lb., @ \$4.12 $\frac{1}{2}$; 20 Kentucky steers, 1230 lb., @ \$4.80; 7 Indiana steers, 1078 lb., @ \$4.70. Discount $\frac{1}{10}$.

In such examples as the above, teachers are advised to have the pupils bring to class problems based on local prices. See also the note on page 130.

ORAL EXERCISE

1. If a man borrows money of another, how does he pay him for the loan, besides returning the money?

2. If you put money in a savings bank and leave it 6 mo. or 1 yr., will they pay you back only what you put in?

3. If any one where you live wished to borrow \$100 for a year, what interest would he have to pay?

190. Interest. The price paid for the use of money is called *interest*. For example, if I borrow \$50 for a year and pay \$3 for the use of it, the \$3 is interest.

191. Principal. The sum of money for which interest is to be charged is called the *principal*.

192. Reckoning Interest. Interest is ordinarily reckoned as a certain per cent of the principal. If the interest is 6% of the principal, and the principal is \$200, the interest for 1 yr. is 6% of \$200, or \$12.

193. Rate of Interest. The per cent of interest is called the *rate*. That is, if the interest is 6% of the principal the rate is 6%.

WRITTEN EXERCISE

1. What is the interest on \$375 for 1 yr. at 6%? at 4%?
2. What is the interest on \$950 for 1 yr. at 5%? at 3%?
3. What is the interest on \$235 for 1 yr. at 4%? at 5%?
4. What is the interest on \$750 for 1 yr. at 6%? at 5%?
5. If you had \$27 in a bank that paid 3% a year, how much interest would you receive for a year?
6. How much are the interest and principal together of \$475 at the end of 1 yr., the rate of interest being 6%?

ORAL EXERCISE

Find the interest on the following sums for 1 yr.:

- | | | |
|-----------------|------------------|-------------------|
| 1. \$100 at 4%. | 5. \$900 at 3%. | 9. \$70 at 5%. |
| 2. \$300 at 6%. | 6. \$50 at 4%. | 10. \$1000 at 4%. |
| 3. \$60 at 3%. | 7. \$400 at 6%. | 11. \$3000 at 4%. |
| 4. \$500 at 5%. | 8. \$1200 at 3%. | 12. \$90 at 6%. |

Find the interest on the following sum for 6 mo., first finding the interest for 1 yr., and then dividing by 2:

- | | | |
|------------------|------------------|-------------------|
| 13. \$100 at 6%. | 17. \$250 at 4%. | 21. \$70 at 3%. |
| 14. \$200 at 5%. | 18. \$60 at 4%. | 22. \$800 at 4%. |
| 15. \$40 at 6%. | 19. \$600 at 5%. | 23. \$1000 at 3%. |
| 16. \$400 at 3%. | 20. \$400 at 5%. | 24. \$80 at 5%. |

WRITTEN EXERCISE

Find the interest on the following sums for 1 yr.:

- | | | |
|-------------------------------|-------------------------------|--------------------------------|
| 1. \$652 at 4%. | 6. \$42 at $4\frac{1}{2}\%$. | 11. \$275.20 at 5%. |
| 2. \$375 at 5%. | 7. \$275 at 4%. | 12. \$84 at $4\frac{1}{2}\%$. |
| 3. \$27 at $3\frac{1}{2}\%$. | 8. \$450.25 at 4%. | 13. \$535 at 5%. |
| 4. \$325 at 3%. | 9. \$75 at $2\frac{1}{2}\%$. | 14. \$625.20 at 5%. |
| 5. \$433 $\frac{1}{3}$ at 3%. | 10. \$725 at 6%. | 15. \$72 at $3\frac{1}{8}\%$. |

Find the interest on the following sums for 6 mo.:

- | | | |
|---------------------------------|---------------------------------|------------------|
| 16. \$82 at 5%. | 21. \$675 at 3%. | 26. \$395 at 3%. |
| 17. \$175 at 4%. | 22. \$45 at 4%. | 27. \$864 at 3%. |
| 18. \$350 at $2\frac{1}{2}\%$. | 23. \$450 at 6%. | 28. \$475 at 6%. |
| 19. \$75 at 6%. | 24. \$450 at $4\frac{1}{2}\%$. | 29. \$750 at 5%. |
| 20. \$325 at 5%. | 25. \$35 at 2%. | 30. \$890 at 6%. |

194. Finding Interest. We have found the interest on money, at various rates, for one year, and also for half a year. We shall now find it for other lengths of time.

What is the interest on \$250 for 10 mo. at 6%?

The interest for 1 yr. is 6% of \$250, and for 10 mo. it is $\frac{10}{12}$, or $\frac{5}{6}$, of this sum. We therefore have

$$\frac{5}{6} \text{ of } 6\% \text{ of } \$250 = \$12.50.$$

In this particular case we see that $\frac{5}{6}$ of 6% is $5 \times 1\%$, and so we simply have $5 \times 1\%$ of \$250 = $5 \times \$2.50 = \12.50 . Advantage of such short methods should always be taken. In finding interest, consider a month as 30 da.

ORAL EXERCISE

1. If the interest for 1 yr. is \$16, how much is it for $\frac{1}{2}$ yr.?
2. If the interest for 1 yr. is \$24, how much is it for 1 mo.?
3. If the interest for 1 yr. is \$8, how much is it for 3 yr.?
4. At 6%, how much is the interest on \$500 for 6 mo.?

WRITTEN EXERCISE

1. What is the interest on \$436, at 6%, for 1 yr.? for 6 mo.? for 4 mo.? for 8 mo.? for 10 mo.?
2. What is the interest on \$360, at 5%, for 1 yr.? for 6 mo.? for 10 mo.? for 1 mo.? for 9 mo.?
3. If the interest on a certain sum is \$93.84 for 1 yr., how much is the interest for 3 mo.? for 6 mo.? for 9 mo.?
4. If the interest on a certain sum is \$36.48 for 1 yr., how much is the interest for 1 mo.? for 5 mo.? for 10 mo.?
5. What is the interest on \$540, at 4%, for 1 yr.? for 1 mo.? for $\frac{1}{2}$ mo.? for 2 yr.? for 2 yr. 6 mo.?
6. What is the interest on \$175.50, at 4%, for 1 yr.? for 6 mo.? for 4 mo.? for 3 mo.? for 1 mo.? for 2 yr. 4 mo.?

ORAL EXERCISE

Find the interest on :

- | | |
|----------------------------|-----------------------------------|
| 1. \$100, at 5%, for 1 yr. | 9. \$100, at 6%, for 6 mo. |
| 2. \$300, at 6%, for 1 yr. | 10. \$600, at 3%, for 6 mo. |
| 3. \$500, at 4%, for 1 yr. | 11. \$700, at 4%, for 6 mo. |
| 4. \$800, at 5%, for 1 yr. | 12. \$500, at 3%, for 4 mo. |
| 5. \$200, at 6%, for 2 yr. | 13. \$100, at 4%, for 6 mo. |
| 6. \$500, at 5%, for 2 yr. | 14. \$240, at 5%, for 11 mo. |
| 7. \$120, at 5%, for 1 mo. | 15. \$200, at 4%, for 2 yr. 6 mo. |
| 8. \$300, at 5%, for 6 mo. | 16. \$300, at 4%, for 2 yr. 1 mo. |

195. Interest for Years and Months. We have seen that to find the interest for 2 yr. 6 mo., we find the interest for 1 yr., and multiply by $2\frac{1}{2}$. That is,

Find the interest for 1 year, and multiply by the number of years.

Thus, to find the interest on \$125 for 2 yr. 3 mo. at 5% we proceed as follows:

5% of \$125 = \$6.25, interest for 1 yr.

$2\frac{1}{4} \times \$6.25 = \14.06 , interest for 2 yr. 3 mo.

\$125
.05
\$6.25
$2\frac{1}{4}$
1 56 $\frac{1}{4}$
12 50
\$14.06

WRITTEN EXERCISE

Find the interest on :

- | | |
|--|------------------------------------|
| 1. \$250, at 6%, for 6 mo. | 6. \$45.50, at 4%, for 2 yr. 3 mo. |
| 2. \$300, at 5%, for 8 mo. | 7. \$750, at 5%, for 2 yr. 4 mo. |
| 3. \$8200, at 5%, for $2\frac{1}{2}$ yr. | 8. \$75.50, at 6%, for 2 yr. 6 mo. |
| 4. \$625, at 4%, for $3\frac{1}{2}$ yr. | 9. \$225, at 6%, for 3 yr. 6 mo. |
| 5. \$35, at 6%, for $1\frac{1}{2}$ yr. | 10. \$4250, at 4%, for 1 yr. 6 mo. |

Find the interest on \$250, at 6%, for:

- | | | |
|-----------------|-----------------|-----------------|
| 11. 3 yr. 6 mo. | 13. 1 yr. 9 mo. | 15. 4 yr. 1 mo. |
| 12. 2 yr. 4 mo. | 14. 3 yr. 3 mo. | 16. 3 yr. 7 mo. |

Find the interest on \$375, for 3 yr. 4 mo., at:

- | | | | | |
|---------|---------|---------|---------|---------|
| 17. 6%. | 18. 5%. | 19. 4%. | 20. 3%. | 21. 2%. |
|---------|---------|---------|---------|---------|

Find the interest for 2 yr. 7 mo., at 6%, on:

- | | | | | |
|------------|------------|------------|------------|------------|
| 22. \$175. | 23. \$325. | 24. \$475. | 25. \$685. | 26. \$750. |
|------------|------------|------------|------------|------------|

27. A man borrows \$575, agreeing to pay 6% interest. He pays the principal and interest in 1 yr. 8 mo. How much money does he pay?

28. A man buys 7 head of cattle at \$85 a head, and agrees to pay for them in 8 mo., with interest at 6%. How much does he pay?

29. Mr. St. John hires Mr. Sinclair to paint his house, agreeing to pay him \$65 for the work. Mr. Sinclair agrees to wait 3 mo. for his money if Mr. St. John will pay interest at the rate of 5%. How much is due at the end of the 3 mo.?

30. Higgins & Son are dealers in dry goods. They need \$1250 with which to buy holiday goods. They borrow it Nov. 1 and agree to pay it back on Feb. 1 with interest at 5%. How much do they owe on Feb. 1?

31. McDowell and Perkins buy \$725 worth of goods from a manufacturer, agreeing to pay for the goods in 5 mo., with interest at 6%. How much do they owe at the end of the 5 mo.?

32. On March 1, Mr. Roberts buys a secondhand automobile from Mr. Grant, agreeing to pay \$475 for it on July 1, with interest at 6%. How much interest is due on July 1? What is the total amount due?

ORAL EXERCISE

1. How much is \$20 plus 10% of itself? plus 1% of itself?
2. How much is \$3 plus 30% of itself? plus 6% of itself?
3. How much is \$15 plus 1% of itself? plus $33\frac{1}{3}\%$ of itself?
4. If you bought \$10 worth of eggs for a dealer, receiving 5% for your work, how much would you make?
5. If you bought \$2 worth of eggs for a dealer and he paid you 10% more than cost, how much would you make?

196. Commission. The pay received by one person for buying or selling for another is often reckoned at a certain per cent of the price, and is called a *commission*.

For example, if a man buys \$1000 worth of oats for another man, at 5% commission, he would receive 5% of \$1000, or \$50.

WRITTEN EXERCISE

1. Required the commission on an \$850 sale, at $2\frac{1}{2}\%$.
2. Required the commission on a \$1200 sale, at $5\frac{1}{2}\%$.
3. Required the commission on a \$2000 purchase, at $4\frac{1}{2}\%$.
4. A commission merchant sells for a factory 7850 lb. of cheese at $13\frac{1}{2}\phi$ a pound, charging 5% commission. What sum should he remit to the factory?
5. A man sells at an auction \$292 worth of property for another man and receives 10% commission. How much does he get? How much is left for the other man?
6. A dealer in fruit in a city sends a man to the fruit region to buy for him. He pays him 2% commission. The buyer purchases \$2500 worth in one place, \$3200 worth in another, and \$4100 worth in another. What is his total commission?

ORAL EXERCISE

1. If you sold \$2 worth of papers on a commission of 20%, how much would you make?
2. At 30%, how much is the commission on a \$120 sale?
3. At 10% commission, how much does a collection agent receive for collecting a debt of \$95.70?
4. An agent sells 100 school desks at \$3.75 each and receives a commission of 10%. How much does he receive?

WRITTEN EXERCISE

1. How much is an architect's commission on a house costing \$9472, at 5%?
2. How much is an auctioneer's commission for selling \$358 worth of furniture, at 6%?
3. A real-estate dealer sells 168 acres of land at \$40 an acre. How much is his commission at $2\frac{1}{2}\%$?
4. An agent sells our school \$65 worth of maps, and his firm pays him $7\frac{1}{2}\%$ commission. How much is this?
5. A farmer sends some butter and eggs to the city, and they are sold for \$72.50. He is charged 8% by his agent. What is the net amount received by the farmer?
6. A real-estate dealer rents 21 city apartments for the owner, at \$1500 each, and charges a commission of $2\frac{1}{2}\%$. How much is the commission?
7. A salesman's sales in a department store in a city amounted during the six days of one week to \$257.70, \$103.10, \$175.02, \$132.30, \$302.25, \$50.63. He receives $2\frac{1}{2}\%$ on his sales. How much did he receive that week?

III. OPERATIONS WITH COMPOUND NUMBERS

197. Addition of Compound Numbers. The general principles of adding compound numbers are similar to those of adding ordinary whole numbers. The only difference arises from the fact that in whole numbers ten units of each order always make one unit of the next higher order, while in compound numbers the number of units of a lower denomination required to make one unit of the next higher varies with the denomination we are using.

Required to add 2 hr. 38 min. 42 sec., 3 hr. 25 min. 36 sec., and 5 hr. 17 min. 58 sec.

The sum of the seconds is 136 sec., or 2 min. 16 sec. We write the 16 sec. under the seconds, and add the 2 min. to the minutes. The sum of the minutes is 82 min., or 1 hr. 22 min. Writing the 22 min. in the column of minutes, we add the 1 hr. to the hours. The sum of the hours is 11 hr. Hence the result is 11 hr. 22 min. 16 sec.

hr.	min.	sec.
2	38	42
3	25	36
5	17	58
11	22	16

In practical life to-day we rarely have to use numbers of more than two denominations, and hence the examples given in this book are generally of not more than two denominations.

WRITTEN EXERCISE

Add the following :

1.		2.		3.		4.	
ft.	in.	sq. ft.	sq. in.	cu. ft.	cu. in.	lb.	oz.
5	8	94	30	47	823	148	8
9	10	82	85	96	538	862	13
5.		6.		7.		8.	
ft.	in.	sq. ft.	sq. in.	cu. ft.	cu. in.	lb.	oz.
28	11	38	48	38	795	385	13
17	9	56	84	47	874	421	12

Add the following:

9.		10.		11.		12.		
gal.	qt.	gal.	pt.	ft.	in.	hr.	min.	sec.
35	2	48	6	27	6	3	27	46
34	2	26	1	36	8	2	38	45
26	3	45	4	35	5	4	16	20
18	1	73	1	44	3	3	56	49
35	2	83	5	28	9	6	48	35

13.		14.		15.		16.		
gal.	qt.	bu.	pk.	ft.	in.	°	'	"
35	3	26	2	45	9	4	32	40
28	2	26	2	29	6	6	26	13
24	1	45	1	43	9	3	57	41
53	2	33	3	85	6	1	40	32
52	3	48	2	52	1	2	15	9

17.		18.		19.		20.		
qt.	pt.	pk.	qt.	wk.	da.	hr.	min.	sec.
16	1	24	7	10	2	12	20	22
26	0	15	6	12	6	13	25	45
45	1	13	5	14	5	14	17	25
28	1	14	4	12	5	13	29	39
15	1	12	5	23	3	45	6	23

21. Add $26^{\circ} 37' 42''$, $47^{\circ} 58' 26''$, $38^{\circ} 39' 26''$, $24^{\circ} 14' 27''$.

22. Add 3 A. 40 sq. rd., 2 A. 80 sq. rd., 5 A. 30 sq. rd.

23. Add 4 sq. rd. 6 sq. yd., 3 sq. rd. $20\frac{1}{4}$ sq. yd., 32 sq. rd. 4 sq. yd., 9 sq. rd. 2 sq. yd.

24. Add 4 cu. yd. 18 cu. ft., 24 cu. ft. 920 cu. in., 8 cu. yd. 12 cu. ft., 808 cu. in.

198. Subtraction of Compound Numbers. The general principles of subtracting compound numbers are quite similar to those of subtracting ordinary whole numbers.

For example, from 22 hr. 16 min. 27 sec. required to subtract 14 hr. 25 min. 40 sec.

Since we cannot subtract 40 sec. from 27 sec., we take 1 min. (60 sec.) of the 16 min. Then 60 sec. + 27 sec. = 87 sec., and 87 sec. - 40 sec. = 47 sec.

Since we cannot subtract 25 min. from 15 min., we take 1 hr. (60 min.) of the 22 hr. Then 60 min. + 15 min. = 75 min., and 75 min. - 25 min. = 50 min. Then 21 hr. - 14 hr. = 7 hr. The difference is 7 hr. 50 min. 47 sec.

hr.	min.	sec.
22	16	27
14	25	40
7	50	47

WRITTEN EXERCISE

Subtract the following :

1.		2.		3.		4.	
ft.	in.	yd.	ft.	cu. ft.	cu. in.	gal.	qt.
62	8	32	2	225	455	32	1
15	10	18	1	128	1326	18	3
5.		6.		7.		8.	
ft.	in.	bu.	qt.	cu. ft.	cu. in.	qt.	pt.
91	$41\frac{1}{2}$	31	23	325	46	71	0
38	$8\frac{1}{4}$	17	17	278	699	39	1
9.		10.		11.		12.	
sq. ft.	sq. in.	A.	sq. rd.	lb.	oz.	bu.	pk.
96	72	64	75	350	8	61	1
38	107	43	119	274	13	37	2
13.		14.		15.		16.	
sq. ft.	sq. in.	hr.	min.	lb.	oz.	min.	sec.
81	$35\frac{1}{4}$	20	21	274	0	201	37
38	$16\frac{1}{2}$	13	25	39	12	59	48

Add the following :

17.		18.		19.		20.	
ft.	in.	lb.	oz.	rd.	ft.	ft.	in.
37	9	23	7	82	5	17	4
42	7	48	9	75	7	16	9 ^b
67	6	61	4	62	12	32	11
31	4	37	5	9	3 $\frac{1}{2}$	87	10
21.		22.		23.		24.	
qt.	pt.	yd.	in.	bu.	pk.	yd.	in.
24	1	91	27	42	3	37	29
3	0	61	14	27	1	81	32
45	1	33	31	63	2	43	8
12	1	42	9	48	3	9	0
25.		26.		27.		28.	
gal.	pt.	mi.	rd.	mi.	rd.	mi.	ft.
23	2	23	120	17	192	32	260
41	1	46	57	26	36	27	627
6	0	29	230	13	240	33	45
83	3	80	175	4	48	41	605

Add, and also subtract :

29.		30.		31.		32.	
ft.	in.	lb.	oz.	rd.	ft.	rd.	ft.
93	7	47	8	32	4	67	8 $\frac{1}{4}$
61	8	26	9	26	5	39 $\frac{1}{2}$	
33.		34.		35.		36.	
mi.	rd.	yd.	in.	ft.	in.	mi.	rd.
81	7	87	4	65 $\frac{1}{2}$		89	80
65	98	69	8	48	9	69 $\frac{1}{2}$	

199. Multiplication of Compound Numbers. The general principles of multiplying compound numbers are similar to those of multiplying ordinary whole numbers.

1. Multiply 4 lb. 9 oz. by 6.

We might multiply as with other numbers and obtain 24 lb. 54 oz., and reduce this to 27 lb. 6 oz. But it is easier to say, "6 times 9 oz. = 54 oz. = 3 lb. 6 oz.; 6 times 4 lb. = 24 lb., which added to 3 lb. = 27 lb."

lb.	oz.
4	9
	6
27	6

2. Multiply 243 ft. 7 in. by 27.

Multiplying as with other numbers, the result is 6561 ft. 189 in., which is reduced to 6576 ft. 9 in.

In this case it is not easy to reduce while we are multiplying, as we did in Ex. 1. We therefore multiply as with whole numbers and reduce the product to a simpler form.

The tendency at present is to avoid the multiplication of compound numbers except in very simple cases, chiefly those involving feet and inches. Instead of multiplying 5 lb. 4 oz. by 8, we multiply $5\frac{1}{4}$ lb. by 8, and similarly in other cases.

ft.	in.
243	7
	27
1701	49
486	14
6561	189
6576	9

ORAL EXERCISE

Multiply the following :

1. 3 ft. 4 in. by 2.

2. 7 ft. 2 in. by 4.

3. 8 ft. 1 in. by 9.

4. 9 yd. 6 in. by 3.

5. 8 yd. 3 in. by 3.

6. 6 yd. 2 in. by 7.

7. 8 lb. 2 oz. by 6.

8. 7 rd. 1 ft. by 3.

9. 11 lb. 3 oz. by 4.

10. 9 gal. 1 qt. by 3.

11. 12 gal. 1 qt. by 2.

12. 10 mi. 200 ft. by 5.

13. 10 yd. 18 in. by 2.

14. 12 yd. 1 ft. by 3.

15. 15 yd. 9 in. by 4.

16. 11 ft. 2 in. by 5.

17. 10 ft. 2 in. by 6.

18. 10 lb. 2 oz. by 8.

WRITTEN EXERCISE

Multiply the following:

1. 4 ft. 6 in. by 5.
2. 4 yd. 2 in. by 6.
3. 9 ft. 3 in. by 4.
4. 4 lb. 4 oz. by 9.
5. 6 yd. 3 in. by 8.
6. 7 lb. 15 oz. by 4.
7. 8 lb. 13 oz. by 6.
8. 7 lb. 11 oz. by 9.
9. 6 ft. 8 in. by 7.
10. 7 ft. 11 in. by 4.
11. 5 ft. $10\frac{1}{2}$ in. by 2.
12. 4 yd. 27 in. by 3.
13. 6 yd. 33 in. by 9.
14. 27 lb. 3 oz. by 4.
15. 4 mi. 3 ft. by 8.
16. 73 ft. 2 in. by 5.
17. 39 lb. 5 oz. by 7.
18. 30 yd. 1 in. by 8.
19. 3 gal. 2 qt. by 2.
20. 8 gal. 2 qt. by 9.
21. 6 gal. 3 qt. by 7.
22. 4 mi. 113 rd. by 3.
23. 6 mi. 109 rd. by 7.
24. 5 mi. 1250 ft. by 3.
25. If each side of a five-sided figure (pentagon) is 1 ft. $6\frac{1}{2}$ in., what is the perimeter?
26. If each side of a six-sided figure (hexagon) is 1 ft. $7\frac{2}{3}$ in., what is the perimeter?
27. If a cubic foot of water weighs 62 lb. 8 oz., what is the weight of the water in a tank that contains 48 cu. ft.?
28. If the height of each story above the first, or ground, story in a city office building is 11 ft. 9 in., and the height of the first story is 15 ft. 8 in., and there are nine stories in all, what is the height of the building?
29. A man travels in an automobile 6 da. a week for 3 wk. The first week he averages 8 gal. 2 qt. of gasoline a day, the second week 7 gal. 3 qt., and the third week 8 gal. 1 qt. How much gasoline does he use in the three weeks?

ORAL EXERCISE

1. How much is $\frac{1}{2}$ of 10 ft. ? $\frac{1}{2}$ of 10 ft. 6 in. ?

2. How much is $\frac{1}{4}$ of 8 lb. ? $\frac{1}{4}$ of 8 lb. 12 oz. ?

Divide the following :

3. 24 ft. 6 in. $\div$ 2.

7. 48 ft. 8 in. $\div$ 4.

4. 36 ft. 6 in. $\div$ 6.

8. 21 ft. 7 in. $\div$ 7.

5. 81 lb. 9 oz. $\div$ 9.

9. 72 ft. 8 in. $\div$ 8.

6. 50 ft. 10 in. $\div$ 5.

10. 27 lb. 12 oz. $\div$ 3.

11. How many times is 2 ft. contained in 4 ft. ?

12. How many times is 9 lb. contained in 27 lb. ?

Find how many times the following divisors are contained in the dividends :

13. 56 lb. $\div$ 7 lb.

19. 156 mi. $\div$ 12 mi.

14. 64 ft. $\div$ 8 ft.

20. 777 ft. $\div$ 7 ft.

15. 108 yd. $\div$ 12 yd.

21. 609 in. $\div$ 29 in.

16. 72 in. $\div$ 9 in.

22. 78 yd. $\div$ 13 yd.

17. 54 rd. $\div$ 6 rd.

23. \$792 $\div$ \$4.

18. 225 ft. $\div$ 25 ft.

24. 648 ft. $\div$ 8 ft.

200. Two Kinds of Division. We see from the above examples that there are two kinds of division.

Because	$9 \times 8 \text{ ft.} = 72 \text{ ft.},$
therefore	$72 \text{ ft.} \div 9 = 8 \text{ ft.},$
and	$72 \text{ ft.} \div 8 \text{ ft.} = 9.$

Practically, as stated on page 15, we do not label the numbers, all numbers being essentially abstract. But in order to understand clearly the operations with compound numbers, it is necessary to label them and to consider these two kinds of division.

201. Division of a Compound Number by an Abstract Number.

Required to divide 7 hr. 5 min. 24 sec. by 2.

Arranging as in ordinary division, we have $7 \text{ hr.} \div 2 = 3 \text{ hr.}$ with remainder 1 hr., or 60 min. Then $60 \text{ min.} + 5 \text{ min.} = 65 \text{ min.}$, and $65 \text{ min.} \div 2 = 32 \text{ min.}$ with remainder 1 min., or 60 sec. Then $60 \text{ sec.} + 24 \text{ sec.} = 84 \text{ sec.}$, and $84 \text{ sec.} \div 2 = 42 \text{ sec.}$ The quotient is 3 hr. 32 min. 42 sec.

In most cases in practical business the dividend has only two denominations. Indeed, it is so usual to write $2\frac{1}{2}$ bu., $7\frac{1}{2}$ lb., 28.25 A., instead of 2 bu. 2 pk., 7 lb. 8 oz., 28 A. 40 sq. rd.,

that compound numbers are not nearly so important as they were formerly.

	hr.	min.	sec.
2)	7	5	24
	3	32	42

202. Division of One Compound Number by Another. Required to divide 41 ft. 3 in. by 2 ft. 9 in.

In this case it is advisable to reduce both numbers to the same denomination, and it makes little difference whether we use feet or inches.

Reducing to inches, we have $41 \text{ ft. } 3 \text{ in.} = 495 \text{ in.}$, and $2 \text{ ft. } 9 \text{ in.} = 33 \text{ in.}$ Then $495 \text{ in.} \div 33 \text{ in.} = 15$. Or reducing to feet, we have

$$41\frac{1}{4} \text{ ft.} \div 2\frac{3}{4} \text{ ft.} = \frac{A}{11} \text{ of } \frac{165}{A} = 15.$$

We aim to choose the denomination that makes the work the shorter.

WRITTEN EXERCISE

Divide the following :

1. $72 \text{ ft. } 6 \text{ in.} \div 3$.

2. $81 \text{ ft. } 9 \text{ in.} \div 3$.

3. $73 \text{ ft. } 8 \text{ in.} \div 4$.

4. $87 \text{ ft. } 6 \text{ in.} \div 6$.

5. $96 \text{ ft. } 3 \text{ in.} \div 5$.

6. $127 \text{ ft. } 2 \text{ in.} \div 7$.

7. $135 \text{ ft. } 4 \text{ in.} \div 8$.

8. $141 \text{ ft. } 9 \text{ in.} \div 9$.

9. $7 \text{ lb. } 8 \text{ oz.} \div 10$.

10. $6 \text{ lb. } 3 \text{ oz.} \div 11$.

11. $27 \text{ lb. } 12 \text{ oz.} \div 12$.

12. $36 \text{ lb. } 12 \text{ oz.} \div 21$.

13. $2 \text{ mi. } 120 \text{ rd.} \div 38$.

14. $3 \text{ mi. } 40 \text{ rd.} \div 125$.

Divide the following :

15. 26 hr. 26 min. $\div$ 13.

18. 36 hr. 52 min. $\div$ 14.

16. 17 wk. 6 da. $\div$ $12\frac{1}{2}$.

19. 15 wk. 5 da. $\div$ $5\frac{1}{2}$.

17. 2 yr. 5 mo. $\div$ $14\frac{1}{2}$.

20. 3 yr. 7 mo. $\div$ $21\frac{1}{2}$.

21. How many times is 3 ft. 7 in. contained in 14 ft. 4 in.?

22. By what must 7 lb. 6 oz. be multiplied to make $14\frac{3}{4}$ lb.?23. If the perimeter of an equilateral (equal-sided) triangle is 11 ft. $4\frac{1}{2}$ in., what is the length of each side?

24. The perimeter of a square is 6 ft. 10 in. Find the length of each side.

25. The perimeter of a certain equal-sided figure is 7 ft. 6 in. The length of each side is 1 ft. 3 in. How many sides has it?

Divide the following :

26. 53 ft. 8 in. $\div$ 7 ft. 8 in.

30. 11 ft. 9 in. $\div$ 3 ft. 11 in.

27. 55 ft. 6 in. $\div$ 6 ft. 2 in.

31. 100 ft. 10 in. $\div$ 9 ft. 2 in.

28. 10 yd. 16 in. $\div$ 2 yd. 22 in.

32. 20 yd. 12 in. $\div$ 3 yd. 14 in.

29. $7^{\circ} 51' 9'' \div 2^{\circ} 37' 3''$.

33. $42^{\circ} 10' 20'' \div 8^{\circ} 26' 4''$.

34. If a man walks at the rate of a mile in 19 min., how many miles can he walk in 1 hr. 54 min.?

35. If a man can build 30 ft. 8 in. of fence a day, how many days will it take him to build 214 ft. 8 in. of fence?

36. A steamship changed its longitude $52^{\circ} 30'$ in 7 days. How many degrees of longitude did it sail per day?

37. If each tile has a surface area of 1 sq. ft. 12 sq. in., how many tiles will be required to cover a space containing 156 sq. ft.?

38. The perimeter of a certain equal-sided figure is 2 yd. 16 in. The length of each side lacks an inch of being a foot. How many sides has the figure?

WRITTEN EXERCISE

1. At the rate of a mile in 4 min., how far will an automobile go in 3 hr.? in $3\frac{1}{2}$ hr.?
2. How far will a horse go in 3 hr., traveling at the rate of a mile in 9 min.?
3. If a wheel is $10\frac{1}{2}\frac{3}{5}$ ft. in circumference, how many times will it turn in going 25 mi.?
4. At 21¢ a foot, how much would it cost to inclose by a wire fence a rectangular field 150 rd. by 160 rd.?
5. If a cubic foot of water weighs 62 lb. 8 oz., what is the weight of the water in a cistern that holds 44 cu. ft.?
6. If a man wishes to enrich the soil of a 75-acre meadow, how many tons of fertilizer would he use at the rate of 75 lb. to the acre?
7. A woman buys $3\frac{1}{4}$ lb. of coffee for \$1.04. How much does she pay per pound?
8. If a man walks at the rate of a mile in $19\frac{1}{2}$ min., how many miles can he walk in $37\frac{1}{2}$ min.?
9. If a cubic foot of gold weighs 1187 lb. 8 oz., and if gold is 19 times as heavy as water, how much does a cubic foot of water weigh?
10. If it takes a man 8 hr. 31 min. 24 sec. to walk a certain distance, how long will it take an automobile to travel the same distance if it goes 12 times as fast?
11. A man can row across a lake $1\frac{3}{4}$ mi. in 28 min. At this rate how long will it take him to row 1 mi.?
12. On a trip of 3 mi. an automobile took 2 min. 34 sec. for the first mile, 2 min. 17 sec. for the second, and 1 min. 56 sec. for the third. How long did it take for the whole trip?

WRITTEN REVIEW

1. How many feet are there in $\frac{1}{8}$ mi. ? in $\frac{1}{4}$ mi. ?
2. How many yards are there in $\frac{3}{8}$ mi. ? in $\frac{3}{4}$ mi. ?
3. How many yards are there in 5 rd. ? in $7\frac{1}{2}$ rd. ?
4. How many yards are there in 79 ft. ? in 113 in. ?
5. If a river is $7\frac{1}{2}$ fathoms deep, what is its depth in feet ?
6. Taking the knot as $1\frac{3}{20}$ mi., how many miles has a ship sailed after it has gone 160 knots ?
7. How many square inches in $7\frac{1}{2}$ sq. ft. ?

Find the number of square inches in :

- | | | |
|---------------------------|-----------------------------|--------------------------|
| 8. $9\frac{1}{4}$ sq. ft. | 10. $16\frac{1}{2}$ sq. ft. | 12. 3 sq. ft. 75 sq. in. |
| 9. $3\frac{1}{3}$ sq. yd. | 11. $24\frac{2}{3}$ sq. yd. | 13. 5 sq. yd. 13 sq. in. |

Find the number of square feet in :

- | | | |
|-----------------------------|-----------------|-------------------------|
| 14. $15\frac{1}{3}$ sq. yd. | 15. 576 sq. in. | 16. 2 sq. yd. 7 sq. ft. |
|-----------------------------|-----------------|-------------------------|

Find the number of cubic inches in :

- | | | |
|---------------|----------------------------|--------------------------|
| 17. 3 cu. ft. | 18. $4\frac{1}{2}$ cu. ft. | 19. 2 cu. ft. 50 cu. in. |
|---------------|----------------------------|--------------------------|

Find the number of cubic feet in :

- | | | |
|------------------|----------------------------|-------------------------|
| 20. 5184 cu. in. | 21. $7\frac{1}{3}$ cu. yd. | 22. 9 cu. yd. 7 cu. ft. |
|------------------|----------------------------|-------------------------|

Find the number of pounds in :

- | | | |
|-------------|--------------|-------------------|
| 23. 176 oz. | 24. 1584 oz. | 25. 9 T. 1200 lb. |
|-------------|--------------|-------------------|

Find the number of quarts in :

- | | | |
|------------|------------|-------------------|
| 26. 96 gi. | 27. 98 pt. | 28. 31 gal. 2 qt. |
|------------|------------|-------------------|

Find the number of pecks in :

- | | | |
|-------------|------------|-------------------|
| 29. 176 qt. | 30. 96 bu. | 31. 108 bu. 1 pk. |
|-------------|------------|-------------------|

Find the number of minutes in :

- | | | | |
|--------------|------------------------|------------|------------------------|
| 32. 900 sec. | 34. 1320 sec. | 36. 17 hr. | 38. $9\frac{3}{4}$ hr. |
| 33. 1 da. | 35. $1\frac{1}{2}$ da. | 37. 1 wk. | 39. $1\frac{1}{2}$ wk. |

Find the number of cents in :

- | | | | |
|----------------|---------------|-----------|-------------|
| 40. 120 mills. | 41. 17 dimes. | 42. \$25. | 43. \$2.75. |
|----------------|---------------|-----------|-------------|

Find the number of minutes in :

- | | | | |
|-----------|------------------------------|--------------------|--------------------|
| 44. 360". | 45. $2\frac{1}{2}^{\circ}$. | 46. 45° . | 47. 90° . |
|-----------|------------------------------|--------------------|--------------------|

Find the number of dozens in :

- | | | | |
|----------|-----------|-----------|------------|
| 48. 288. | 49. 1728. | 50. 7 gr. | 51. 36 gr. |
|----------|-----------|-----------|------------|

52. A hand is 4 in. If a horse is 15 hands high, what is its height in inches? What is its height in feet?

53. Taking $24\frac{3}{4}$ cu. ft. as a perch of stone, how many perches in 99 cu. ft.?

54. Taking, as is often done, 25 cu. ft. as a perch of stone, how many cubic feet in $17\frac{1}{2}$ perches?

55. A gallon contains 231 cu. in. How many gallons will it take to fill an aquarium containing $1\frac{1}{2}$ cu. ft.?

56. Allowing $31\frac{1}{2}$ gal. to the barrel, how many gallons will it take to fill a tank containing $16\frac{1}{2}$ bbl.?

57. Allowing $2150\frac{1}{2}$ cu. in. to the bushel, how many cubic inches in a bin that holds 16 bu.?

58. Allowing $67\frac{1}{5}$ cu. in. to the dry quart, how many cubic inches in a peck? how many cubic inches in a pint?

59. Allowing $2150\frac{1}{2}$ cu. in. to the bushel, how many cubic feet in a bin that holds 3456 bu. when full?

60. Allowing $67\frac{1}{5}$ cu. in. to the dry quart, how many quarts of oats will it take to fill a box that contains $604\frac{4}{5}$ cu. in.?

IV. MEASUREMENTS

203. Common Figures. We are already familiar with most of the common figures that are used in measuring. We shall now review some of these and shall study and learn how to measure others.



Quadrilateral



Rectangle



Square

204. Quadrilateral. A plane figure bounded by four straight lines is called a *quadrilateral*.

205. Rectangle. A quadrilateral all of whose angles are right angles is called a *rectangle*.

206. Square. A rectangle whose sides are all equal is called a *square*.

207. Polygon. A plane figure bounded by straight lines is called a *polygon*.

The quadrilateral, rectangle, and square, as shown above, are all special kinds of polygons, but a polygon may also have three, five, six, or any other number of sides.

As a preliminary to this chapter the tables on pages 194-204 should be reviewed; and in order that this review should be thorough, much oral work is provided on pages 273-274.

In this work, provision is made for a careful review of the rectangle, the square, the triangle, and the cube. The pupils are given exercise in the reading of exact definitions, in distinguishing the various forms from one another, and in associating the geometrical names with the figures.

The problems in measurement should be practical, and much care has been taken to make such problems practical in this book. Pupils should, however, frequently be required to make their own measurements in order to obtain the conditions for the problems they are to solve. For example, they should be required to find the dimensions of the schoolroom, the area of the floor, walls, and ceiling, the window surface, the blackboard surface, and the cost of sodding the school yard and inclosing it with a fence.

ORAL EXERCISE

1. Express 640 rd. as miles.
2. Express 64 oz. as pounds.
3. Express 2 A. as square rods.
4. Express 33 ft. as rods ; as yards.
5. Express 54 cu. ft. as cubic yards.
6. Express 9 ft. as inches ; as yards.
7. Express 11 da. as hours ; as weeks.
8. Express 17 qt. as pints ; as gallons.
9. Express 9 pk. as quarts ; as bushels.
10. How many cubic inches in 10 cu. ft. ?
11. How many rods in 3 mi. ? in 10 mi. ?
12. How many yards in 8 rd. ? in 10 rd. ?
13. How many cubic feet in 2 cd. of wood ?
14. Express 300 min. as hours ; as seconds.
15. How many quarts in a barrel of $31\frac{1}{2}$ gal. ?
16. How many pints in a 60-gallon hogshead ?
17. Express 80 sq. rd. as a fraction of an acre.
18. How many right angles in 270° ? in 180° ?
19. How many days from May 23 to June 25 ?
20. How many days from June 11 to July 17 ?
21. What part of a 16-carat ring is pure gold ?
22. Express 11 yd. as rods ; as feet ; as inches.
23. Express 30 pt. as quarts ; as gills ; as gallons.
24. How many feet in 2 rd. 8 ft. ? in 2 rd. 10 ft. ?
25. How many feet in 10 rd. ? in 5 rd. ? in 2 rd. ?
26. How many degrees in $540'$? in $420'$? in $660'$? in $720'$?

27. How many mills in \$2.75?
28. How many days from August 15 to October 15?
29. How many acres in a tract of land 10 mi. square?
30. How many days from January 17 to February 17?
31. How many square feet in 10 sq. yd. ? in 5 sq. yd. ?
32. Using 6 qt. a day, how long will 3 bu. of oats last a horse?
33. What is the perimeter of a square that is $4\frac{1}{2}$ ft. on a side?
34. What is the side of a square whose perimeter is 14 ft. 4 in. ?
of a square whose perimeter is $4\frac{1}{2}$ ft. ?
35. Since a gallon contains 231 cu. in., how many gallons in 693 cu. in. ?
36. How many square feet in a surface having an area of 720 sq. in. ?
37. Express a long ton (2240 lb.) as a short ton and hundredths of a short ton.
38. What is the perimeter of an equilateral triangle that is 3 ft. 8 in. on a side?
39. If school closes June 21, how many days from that day to the Fourth of July?
40. How many quarts will a 10-bushel bin hold ? a 5-bushel bin ? How many pecks in each ?
41. A man buys a building lot that has a frontage of 2 rd. How many square feet of sidewalk 5 ft. wide must he have ?
42. A cellar 15 ft. by 21 ft. is to be excavated to a depth of 6 ft. How many loads (cubic yards) of earth must be removed ?
43. If a man buys a load of coal weighing 1 T. 500 lb., how many pounds of coal does he buy ? Express this weight in tons and tenths of a ton.

DRAWING TO SCALE

1. Draw a plan of a floor 16 ft. long and 12 ft. wide, representing 1 ft. by $\frac{1}{8}$ in.

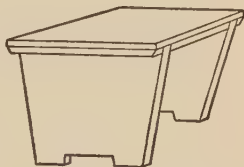
2. What part of 1 ft. is $\frac{1}{8}$ in.? Then the drawing in Ex. 1 is what part as long as the floor that it represents?

3. Draw a plan of a box cover 6 in. by 10 in., representing 1 in. of length by $\frac{1}{4}$ in. That is, draw the plan to the scale $\frac{1}{4}$.

4. Draw a plan of a school garden 56 ft. long by 48 ft. wide, representing 1 ft. by $\frac{1}{8}$ in. That is, draw the plan to the scale of $\frac{1}{8}$ in. to 1 ft.

5. Draw a plan of a rug that the class is weaving. The rug is to be 10 in. by 16 in. Use the scale $\frac{1}{4}$.

6. This is a drawing of a doll's table made by the class. The top is to be 10 in. by 12 in. The end supports are to be 10 in. at the top, 7 in. at the bottom, and 8 in. high. Draw plans for the top and ends, using the scale $\frac{1}{4}$. The end supports are indented 1 in. at the distance of 2 in. from each bottom corner.



7. Draw a plan of a book cover that the boys are making. The cover is to be 5 in. by 8 in. Use the scale $\frac{1}{2}$.

8. A corridor in a school building is 128 ft. long and 12 ft. wide. Draw a floor plan of this corridor, using the scale of $\frac{1}{16}$ in. to 1 ft.

9. A playground near our school is 125 ft. long and 65 ft. wide. Draw a plan of the playground, using the scale of $\frac{1}{16}$ in. to 1 ft.

10. A schoolroom is 40 ft. long and 20 ft. wide. Draw a plan of the floor, using the scale $\frac{1}{80}$.

208. Changing Denominations. We often wish to change from feet to inches, ounces to pounds, and so on, as in the following examples, and as already explained in this book.

1. Express $35\frac{1}{2}$ yd. as feet.

Since $1 \text{ yd.} = 3 \text{ ft.},$
therefore $35\frac{1}{2} \text{ yd.} = 35\frac{1}{2} \times 3 \text{ ft.}, \text{ or } 106\frac{1}{2} \text{ ft.}$

2. Express 81 oz. as pounds and ounces.

Since $1 \text{ oz.} = \frac{1}{16} \text{ lb.},$
therefore $81 \text{ oz.} = 81 \times \frac{1}{16} \text{ lb.} = 5\frac{1}{16} \text{ lb.}, \text{ or } 5 \text{ lb. } 1 \text{ oz.}$

3. Express 2 ft. 8 in. as inches.

Since $1 \text{ ft.} = 12 \text{ in.},$
therefore $2 \text{ ft.} = 2 \times 12 \text{ in.} = 24 \text{ in.},$
and $2 \text{ ft. } 8 \text{ in.} = 24 \text{ in.} + 8 \text{ in.} = 32 \text{ in.}$

WRITTEN EXERCISE

1. Express 92 rd. 3 ft. as feet; as inches.
2. Express 32 yd. 2 ft. as feet; as inches.
3. Express $3\frac{1}{2}$ A. as acres and square rods.
4. Express 21 sq. ft. 82 sq. in. as square inches.
5. Express 95 cu. ft. as cubic yards; as cubic inches.
6. Express 27 sq. rd. 4 sq. yd. as square yards; as square feet.
7. Express 8 sq. yd. 7 sq. ft. as square feet; as square inches.
8. Express 35 cu. yd. 15 cu. ft. as cubic yards; as cubic feet.
9. Express 4 T. 1735 lb. as tons and the decimal part of a ton; as pounds; as ounces.
10. Express 39,426 lb. as tons and the decimal part of a ton; as tons and pounds; as ounces.
11. Express 27 qt. 1 pt. as quarts and the decimal part of a quart; as gallons and the decimal part of a gallon; as pints.

209. Equilateral Triangle. A triangle whose three sides are equal is called an *equilateral* triangle.



Equilateral



Isosceles



Scalene

210. Isosceles Triangle. A triangle two of whose sides are equal is called an *isosceles* triangle.

211. Scalene Triangle. A triangle of which no two sides are equal is called a *scalene* triangle.



Right



Acute



Obtuse

212. Right Triangle. A triangle having one of its angles a right angle is called a *right* triangle. The side opposite the right angle is called the *hypotenuse*.

213. Acute Triangle. A triangle having three acute angles is called an *acute* triangle.

214. Obtuse Triangle. A triangle having one of its angles an obtuse angle is called an *obtuse* triangle.

ORAL EXERCISE

1. How long is the side of an equilateral triangle whose perimeter is 45 in.? 9 ft. 6 in.? $9\frac{1}{2}$ yd.? 63 ft. 9 in.?

2. How long is the side of a square whose perimeter is 84 in.? 8 ft. 4 in.? 16 yd. 8 in.? $60\frac{1}{2}$ rd.? 120 in.?

3. How long is the fourth side of a quadrilateral whose perimeter is 85 in. and three of whose sides are 10 in., 20 in., 30 in.?

WRITTEN EXERCISE

1. What is the perimeter of an equilateral triangle $275\frac{2}{3}$ ft. on a side?

Draw figures of the correct shape in all examples on this page, not to scale but only approximately.

2. A triangle has a perimeter of 342 ft. 8 in. One side is 90 ft. and another is 50% more; how long is the third?

3. One side of an isosceles triangle is 35 ft., and each of the equal sides is 35% longer. What is the perimeter?

4. Cut off the three angles of a triangle and place them together. The sum is equal to how many right angles?

Find the sides of the equilateral triangles whose perimeters are as follows:

5. 113 ft. 3 in.

6. 127 yd.

7. 88 ft. 3 in.

Find the sides of the squares whose perimeters are as follows:

8. 110 ft. 4 in.

10. 61 yd. 2 ft.

12. 118 ft. 8 in.

9. 42 ft. $3\frac{1}{2}$ in.

11. 3 yd. 2 ft. 4 in.

13. 227 mi. 40 rd.

14. The perimeter of an isosceles triangle is 100 ft., and the unequal side is 33 ft. Find the length of one of the equal sides.

15. The perimeter of an isosceles triangle is 88 ft., and the unequal side is 20% of the perimeter. Required the sides.

16. The perimeter of a triangle is 24 ft. One side is $\frac{1}{3}$ of the perimeter and another is $\frac{2}{3}$ of the perimeter. How many feet are there in each of the three sides?

17. The perimeter of a triangle is 120 ft. One side is $\frac{1}{4}$ of the perimeter and another is $\frac{1}{3}$ of the perimeter. How many feet are there in each of the three sides?

ORAL EXERCISE

1. How many square inches in a rectangle 1 in. high and 10 in. long? in one that is 4 in. high and 10 in. long?



State the areas of the following rectangles :

- | | | |
|--------------------------------|--------------------------------|--------------------------------|
| 2. 5 ft. by 9 ft. | 5. $4\frac{1}{2}$ ft. by 6 ft. | 8. 8 in. by 1 ft. |
| 3. 7 ft. by 8 ft. | 6. $8\frac{1}{3}$ in. by 9 in. | 9. $7\frac{1}{3}$ in. by 9 in. |
| 4. $1\frac{1}{2}$ yd. by 4 yd. | 7. 10 ft. by 1 rd. | 10. 1 rd. by 10 yd. |

215. Area of a Rectangle. If a rectangle is 4 in. by 10 in., we find the area by taking 4 times 10 sq. in., or $4 \times 10 \times 1$ sq. in. We do not multiply 10 in. by 4 in., but we multiply 1 sq. in. by 4×10 , the product being 40 sq. in.

The area of a rectangle equals the product of the base and height.

WRITTEN EXERCISE

Find the areas of the following rectangles, using short methods :

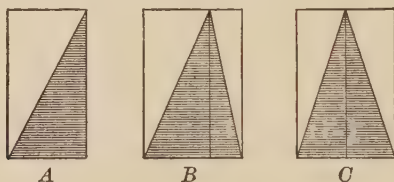
- | | |
|----------------------------------|-----------------------------------|
| 1. $37\frac{1}{2}$ ft. by 48 ft. | 9. 125 ft. by 160 ft. |
| 2. $33\frac{1}{3}$ yd. by 42 yd. | 10. 250 ft. by 324 ft. |
| 3. 75 ft. by 96 ft. | 11. 625 ft. by 720 ft. |
| 4. $66\frac{2}{3}$ yd. by 48 yd. | 12. $87\frac{1}{2}$ in. by 96 in. |
| 5. 12.5 in. by 24.8 in. | 13. 748 ft. by 825 ft. |
| 6. 50 ft. by 96.6 ft. | 14. $37\frac{1}{2}$ in. by 48 in. |
| 7. $16\frac{2}{3}$ yd. by 36 yd. | 15. 69 ft. by 127 ft. |
| 8. 3.75 mi. by 4.8 mi. | 16. 36 rd. by 125 rd. |

216. Base. The line on which a figure appears to stand is called the *base*.

217. Height or Altitude. The shortest distance from the highest point above the base to the base is called the *height* or *altitude* of a triangle or parallelogram.

ORAL EXERCISE

1. How is the area of a rectangle found?



2. How do the areas of these triangles compare with the areas of the rectangles? Then a triangle is what part of a rectangle of the same base and the same height?

3. If the rectangle *A* is 2 in. wide and 4 in. high, what is its area? What is the area of the shaded triangle in *A*?

4. If the rectangle *B* is 3 ft. wide and 5 ft. high, what is its area? What is the area of the shaded triangle in *B*?

5. If the rectangle *C* is 4 yd. wide and 6 yd. high, what is its area? What is the area of the shaded triangle in *C*?

6. What is the area of a rectangle with base 6 ft. and height 9 ft.? of a triangle with base 6 ft. and height 9 ft.?

7. How do you find the area of a triangle?

Find the areas of triangles whose bases and heights are as follows:

8. 4 in., 3 in.

10. 5 in., 4 in.

12. 8 in., 10 in.

9. 8 in., 6 in.

11. 7 in., 6 in.

13. 9 ft., 4 ft.

218. Area of a Triangle. As shown on the preceding page,

The area of a triangle equals half the product of the base and height.

For example, what is the area of a triangle of base 18 in. and altitude 7 in.?

$$\frac{1}{2} \text{ of } 18 \times 7 \text{ sq. in.} = 63 \text{ sq. in.}$$

ORAL EXERCISE

Using aliquot parts, state the area of the triangles whose bases and altitudes are as follows:

- | | | |
|--------------------|--------------------------------|---------------------------------|
| 1. 80 in., 75 in. | 3. 60 in., $33\frac{1}{3}$ in. | 5. 44 in., 50 in. |
| 2. 16 in., 125 in. | 4. 80 in., 25 in. | 6. 120 in., $16\frac{2}{3}$ in. |

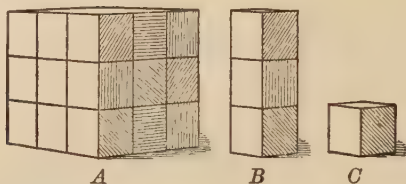
WRITTEN EXERCISE

Find the areas of the triangles whose bases and altitudes are as follows:

- | | |
|---------------------|---|
| 1. 43 ft., 32 ft. | 4. $37\frac{1}{4}$ ft., $42\frac{1}{2}$ ft. |
| 2. 7.9 in., 8.7 in. | 5. 2 yd. 32 in., 27 in. |
| 3. 8.7 in., 6.3 in. | 6. 3 ft. 4 in., 2 ft. 8 in. |
7. What is the area of an equilateral triangle whose perimeter is 37 ft. 6 in. and whose altitude is 10.8 ft.?
8. What is the area of a triangle whose altitude is 6.4 rd. and base 17 rd.? altitude 7.2 in. and base 17 in.?
9. A right triangle has the two sides forming the right angle 30 rd. and 40 rd. What is its area in square rods? in acres?
10. What is the number of square feet of slating required for a steeple, the surface being made up of 6 triangles, each with base 20 ft. and altitude 65 ft.?

ORAL EXERCISE

1. If $C = 1$ cu. in., what is the volume of B ? of A ?



2. Find the volume of a box 4 in. by 3 in. by 2 in.

State the volumes of solids of the following dimensions:

- | | |
|------------------------|-------------------------|
| 3. 4 in., 5 in., 6 in. | 5. 2 in., 3 in., 10 in. |
| 4. 3 in., 3 in., 7 in. | 6. 6 in., 8 in., 10 in. |

219. Rectangular Solid. A solid having six sides, each side being a rectangle, is called a *rectangular solid*. If its sides are all squares it is called a *cube*.

220. Volume of a Rectangular Solid. *The volume of a rectangular solid equals the product of its three dimensions.*

In finding volumes we do not write 4 in. $\times$ 3 in. $\times$ 2 in., but $4 \times 3 \times 2$ cu. in. It is customary, however, in giving dimensions, to write 4 in. $\times$ 3 in. $\times$ 2 in., or $4'' \times 3'' \times 2''$, reading it "4 inches by 3 inches by 2 inches."

ORAL EXERCISE

Find the volumes of solids of the following dimensions:

- | | |
|----------------------------------|---|
| 1. $3'' \times 4'' \times 7''$. | 5. $3' \times 8' \times 11'$. |
| 2. $6' \times 8' \times 10'$. | 6. $11'' \times 11'' \times 10''$. |
| 3. $20' \times 30' \times 5'$. | 7. 6 yd. $\times$ 8 yd. $\times$ 2 yd. |
| 4. $2'' \times 8'' \times 9''$. | 8. 6 ft. $\times$ 8 ft. $\times$ 11 ft. |

WRITTEN EXERCISE

1. How many cubic inches in an aquarium 16 in. long, $8\frac{1}{2}$ in. wide, and $9\frac{1}{2}$ in. deep?
2. How many cubic feet of masonry in a wall 40 ft. long, $30\frac{1}{2}$ ft. high, and $1\frac{1}{2}$ ft. thick?
3. A cellar 24 ft. by 39 ft. by 6 ft. is to be excavated. How much will it cost at 40¢ a load (cubic yard)?
4. How many cords in a pile of 4-foot wood 5 ft. high and 56 ft. long? 6 ft. high and 128 ft. long?
5. How many loads of earth must be taken out in excavating a cellar 24 ft. by 18 ft., and 7 ft. deep?
6. How much will it cost to excavate a ditch 150 ft. by 2 ft. by 6 ft., at 50¢ per cubic yard? at $62\frac{1}{2}$ ¢ per cubic yard?
7. A schoolroom is 32 ft. 8 in. long, $18\frac{1}{3}$ ft. wide, and $12\frac{1}{2}$ ft. high. What is the number of cubic feet in the room?
8. A tunnel 720 ft. long has a cross-section area of 180 sq. ft. How much earth and rock were excavated? (Imagine the cross section 10 ft. by 18 ft.)
9. The dimensions of a room are 14 ft., 12 ft., and 9 ft. How many square feet of floor? of walls? of ceiling?
10. In preparing a flower bed in a park it was necessary to fill in a space 60 ft. by 100 ft. to an average depth of $1\frac{1}{2}$ ft. The earth cost $33\frac{1}{3}$ ¢ a load. What was the total cost of the earth used?
11. The interior of a freight car is 8 ft. wide, 34 ft. long, and 7 ft. high. How many cubic feet does it contain? If it is filled with grain to a height of 5 ft., what is the weight of the grain at 60 lb. to the bushel, allowing $1\frac{1}{4}$ cu. ft. to the bushel?

Teachers should frequently require the pupils to estimate results in advance.

12. A rectangular fishpond has been excavated to a depth of 5 ft. The dimensions of the bottom are 40 ft. and 30 ft. How much earth was excavated?

13. The size of an ordinary farm-wagon box is 10 ft. by 3 ft., and 24 in. to 26 in. deep. Find the contents in cubic feet for each of these depths; also in cubic inches.

14. Such a wagon box 24 in. deep contains 50 bu. of shelled corn. How many bushels of shelled corn will such a box 26 in. deep contain?

15. If a wagon box contains 50 bu. of wheat worth 89¢ a bushel, how many loads must be drawn to carry \$311.50 worth? to carry \$133.50 worth?

16. Prairie hay is heavier than Eastern meadow hay, a cube of the former 7 ft. on an edge weighing a ton (the measurement taken 30 days after the hay is stacked). If it takes $33\frac{1}{3}\%$ more in bulk of a certain Eastern meadow hay to make a ton, how many cubic feet to a ton of the latter?

17. What is the weight of prairie hay that will fill a space 30 ft. long, 17 ft. 6 in. wide, and 11 ft. deep, taking the number of cubic feet to the ton suggested in Ex. 16?

18. What is the weight of the Eastern meadow hay mentioned in Ex. 16 required to fill the space mentioned in Ex. 17?

19. A haystack is estimated to contain 2600 cu. ft. If it is prairie hay, what does it weigh, to the nearest half ton? If it is Eastern meadow hay (Ex. 16), what does it weigh?

20. A schoolroom is 30 ft. long, 20 ft. wide, and 15 ft. high. If it is occupied by 29 children and the teacher, how many cubic feet of air are allowed to each? If 2400 cu. ft. of fresh air per hour should be allowed to each person, how many times an hour should the air be changed?

WRITTEN EXERCISE ON EXCAVATIONS

1. How many cubic yards of earth must be removed in digging a tunnel 492 ft. long, 39 ft. wide, and 19 ft. 6 in. high?
2. If a dirt car is 27 ft. long, 6 ft. wide, and $3\frac{1}{4}$ ft. deep, how many cubic yards will it carry? How many will 12 cars carry?
3. If a laborer can shovel a cubic yard of earth into a car in 30 min., how long will it take him to load a car? how long will it take 15 men? (See Ex. 2.)
4. How many cars of the dimensions in Ex. 2 will it take to remove the earth from the tunnel mentioned in Ex. 1? How many trains of 9 cars each? Allowing 30 ft. for the total length of a car, how long a train of cars would be needed?
5. If one laborer can remove, on an average, the earth from a space 20 ft. long, 6 ft. wide, and 6 ft. deep in a day, how long will it take 100 men to remove the earth from the tunnel mentioned in Ex. 1?
6. If a laborer is paid at the rate of 10¢ per load (cubic yard) of earth removed, how much will he earn in a day, removing the amount specified in Ex. 5?
7. In the first great subway in New York City, 9733 cu. yd. of brick were used. Taking the size of the brick as $8\frac{1}{4}'' \times 4'' \times 2''$, how many bricks were used?
8. What will be the earnings of a laborer who can excavate a space a yard wide, a yard deep, and 12 ft. long in solid rock, in one day, at 75¢ per cubic yard? How much at $87\frac{1}{2}$ ¢ per cubic yard?
9. How much will it cost to excavate a cellar 38 ft. long, 26 ft. wide, and 6 ft. deep, at 96¢ per cubic yard? How much more would it cost if the excavation were 1 ft. deeper?

221. Practical Equivalents. In the problems given below, refer to these measurements without memorizing them.

31.5 gal. = 1 barrel	$7\frac{1}{2}$ gal. = 1 cu. ft.
231 cu. in. = 1 gal.	$4\frac{1}{5}$ cu. ft. = 1 barrel
$24\frac{3}{4}$ cu. ft. = 1 perch	$1\frac{1}{4}$ cu. ft. = 1 bu.

The three values on the right are the ones usually taken, but they are not exact. For example, a bushel is 2150.42 cu. in.

In estimating brickwork, take 22 bricks with the mortar as 1 cu. ft.

A ton of hay varies from about 450 cu. ft. to 550 cu. ft., depending on the quality and on how much the hay has settled, but 500 cu. ft. may be taken in solving the problems. The volume of a ton of coal depends upon the size and kind of the coal, but 35 cu. ft. may be taken for our purposes.

In examples like the following, all measures refer to inside dimensions.

WRITTEN EXERCISE

1. How many bushels in a bin 7 ft. by 6 ft. by 4 ft.?
2. How many bushels in a bin 9 ft. by 5 ft. by 3 ft.?
3. How many bushels in a box 18 in. wide, 3 ft. long, and 2 ft. deep?
4. How many gallons in a tank 6 ft. by 5 ft. by 4 ft.?
5. How many gallons in an aquarium 14 in. long, 9 in. wide, and 11 in. deep?
6. How many gallons in a tank $6\frac{1}{4}$ ft. long, $3\frac{1}{2}$ ft. wide, and $2\frac{1}{3}$ ft. deep?
7. A cistern is 6 ft. square at the bottom and 5 ft. deep. How many barrels will it hold?
8. An iron tank is 8 ft. long, $6\frac{1}{2}$ ft. wide, and $4\frac{3}{4}$ ft. deep. How many barrels does it contain?
9. A rectangular watering trough is 6 ft. long, 18 in. wide, and 14 in. deep. How many barrels does it contain?

10. A coal bin is 21 ft. long, 13 ft. wide, and 8 ft. deep. About how many tons of coal will it hold?

11. A coal bin is 14 ft. long, $8\frac{1}{2}$ ft. wide, and $6\frac{1}{4}$ ft. deep. About how many tons of coal will it hold?

12. A haymow is 22 ft. long, 14 ft. wide, and 9 ft. high. About how many tons of hay will it hold?

13. A barn has two haymows, each 24 ft. long, 15 ft. 6 in. wide, and 10 ft. high. About how many tons of hay will it hold?

14. How many cords of wood in a pile 4 ft. wide, 7 ft. high, and $32\frac{1}{2}$ ft. long?

15. How many cords of wood in a pile 4 ft. wide, $6\frac{1}{2}$ ft. high, and 44 ft. long?

16. If a schoolroom is 30 ft. long, 15 ft. wide, and 12 ft. high, and 180 cu. ft. should be allowed to each person, how many will the room accommodate?

17. A tank holds 82,500 gal. About how many cubic feet does it contain?

18. An aquarium contains $3\frac{4}{5}$ cu. ft. About how many gallons does it hold?

19. An aquarium is 18 in. long, $9\frac{1}{4}$ in. wide, and $10\frac{1}{8}$ in. deep. How many gallons does it hold?

20. A can containing 1732.5 cu. in. is full of milk. What is the milk worth at 6¢ a quart?

21. A wall is $32\frac{1}{2}$ ft. long, 2 ft. thick, and $32\frac{1}{2}$ ft. high. How many perches of masonry does it contain?

22. A brick wall is to be built 42 ft. long, $1\frac{1}{2}$ ft. thick, and $6\frac{1}{4}$ ft. high. About how many bricks will be needed?

23. How much will a mason charge for laying a wall 20 ft. long, $1\frac{1}{2}$ ft. thick, and $5\frac{1}{2}$ ft. high, at \$1.87 $\frac{1}{2}$ a perch?

24. The standard Florida orange box for shipping is 12 in. by 12 in. by $26\frac{5}{8}$ in. How many cubic inches does it contain?

Use cancellation in all such cases.

25. In shipping Georgia peaches a crate 8 in. by $12\frac{1}{2}$ in. by 22 in. is used. This crate contains how many cubic inches more than 1 bu.?

26. In New Jersey a bushel cranberry box is 12 in. by $8\frac{3}{4}$ in. by 22 in. How does this differ from the standard bushel?

27. California apples and pears for local use are shipped in a 50-pound box 22 in. by 10 in. by 12 in. How does this differ in capacity from the standard bushel?

28. A California cantaloupe crate is 38 in. long, 16 in. wide, and 15 in. deep. How many cubic inches does it contain?

29. The California orange box is sometimes $26\frac{1}{2}$ in. by $11\frac{1}{4}$ in. by $11\frac{1}{4}$ in., and sometimes 22 in. by $7\frac{3}{4}$ in. by $17\frac{1}{2}$ in. Which holds the more, and how many cubic inches more?

30. Dried fruits are shipped in 50-pound or 25-pound boxes. The 50-pound box is $15\frac{1}{4}$ in. by 9 in. by 9 in. How many cubic inches in the box?

31. The 25-pound box mentioned in Ex. 30 is sometimes $13\frac{3}{4}$ in. by $9\frac{3}{4}$ in. by $5\frac{3}{4}$ in., and sometimes 16 in. by 9 in. by 5 in. Which box holds the more, and how much more?

32. For shipping fruit out of the state, the sizes of some of the boxes used in California are as follows:

Plums, $18\frac{1}{2}$ in. by $11\frac{3}{4}$ in. by 4 in.

Cherries, $16\frac{1}{2}$ in. by $10\frac{3}{8}$ in. by $2\frac{1}{2}$ in.

Pears, $18\frac{1}{2}$ in. by $11\frac{3}{4}$ in. by $8\frac{1}{2}$ in.

Peaches, $18\frac{1}{2}$ in. by $11\frac{3}{4}$ in. by $4\frac{7}{8}$ in.

Find the number of cubic inches in each box.

222. Other Problems Relating to Volumes. Instead of having the three dimensions of a box given to find the volume, we may have the volume and certain dimensions given to find other dimensions.

1. How thick is a block 5 ft. long and 4 ft. wide, containing 60 cu. ft.?

$$\frac{60}{5 \times 4} = 3$$

Since it is 5 ft. long and 4 ft. wide, the area of the base is 20 sq. ft. If it were 1 ft. high the volume would be 20 cu. ft.

Therefore it is as many times 1 ft. high as 60 cu. ft. $\div$ 20 cu. ft., or 3. Therefore it is 3 ft. high.

The number of cubic units of volume, divided by the product of the numbers of units of two dimensions, equals the number of units of the third dimension.

This is sometimes expressed thus: *The volume divided by two dimensions equals the third dimension.*

2. What is the area of the base of a block $2\frac{1}{2}$ in. high, containing 70 cu. in.?

If it were 1 in. high the volume would be 70 cu. in. $\div$ $2\frac{1}{2} = 28$ cu. in. But the base of such a block is 28 sq. in.

$$70 \div \frac{5}{2} = \frac{2 \times 70}{5} = 28$$

This might be briefly stated as follows: *The volume divided by the height equals the area of the base.*

3. What is the thickness of a block containing 100 cu. in., the base containing 30 sq. in.?

If it were 1 in. thick it would contain 30 cu. in. Therefore it is as many times 1 in. thick as 100 cu. in. $\div$ 30 cu. in. $= 3\frac{1}{3}$. Therefore it is $3\frac{1}{3}$ in. thick.

$$\frac{100}{30} = 3\frac{1}{3}$$

This might be briefly stated as follows: *The volume divided by the area of the base equals the height.*

WRITTEN EXERCISE

1. How many cubic feet in a hall 60 ft. by 80 ft. by 23 ft.?
2. A tank 9 ft. by $16\frac{2}{3}$ ft. contains 900 cu. ft. How deep is it?
3. A box 13 in. by 20 in. contains 3900 cu. in. How deep is it?
4. A tank 4 ft. deep contains 450 cu. ft. What is the area of the base?
5. A hall 22 ft. high and 30 ft. wide contains 28,600 cu. ft. Find the length.
6. A block containing 700 cu. ft. has a base area of 56 sq. ft. How high is it?
7. A hall contains 41,341.3 cu. ft. It is 41.3 ft. long and 36.4 ft. wide. How high is it?
8. A box contains 42.159 cu. in. The area of the bottom is 10.81 sq. in. How deep is the box?
9. A block of granite is 14 in. long and 8.3 in. wide. It contains 836.64 cu. in. How thick is it?
10. The floor of a hall contains 1386 sq. ft. The hall contains 15,246 cu. ft. Required the height.
11. A storeroom is 14.3 ft. long, 13 ft. 6 in. wide, and has a capacity of 2316.6 cu. ft. How high is the room?
12. The floor of a bedroom is square and contains 144 sq. ft. The room contains 1368 cu. ft. Required the three dimensions of the room.
13. A room contains 2110.68 cu. ft. The room is 16.4 ft. long, and the floor area is 234.52 sq. ft. What are the three dimensions of the room?

14. How many cubic feet in a bin that is 6 ft. 4 in. long, 3 ft. wide, and 3 ft. 6 in. deep?

15. Express in bushels the volume of a bin 6 ft. by 4 ft. by 3 ft. $6\frac{1}{2}$ in., allowing 2150 cu. in. to the bushel.

16. How many bars of soap, each 4 in. long, $2\frac{3}{4}$ in. wide, and $1\frac{1}{2}$ in. thick, can be packed in a box 16 in. long, $13\frac{3}{4}$ in. wide, and 1 ft. deep? Will the bars fit in the box?

17. What is the value of the bricks in a kiln 60 ft. long, 25 ft. wide, and 9 ft. high, each brick being 8 in. by 4 in. by 2 in., at \$9 a thousand?

18. The length of a tank is 100% greater than its width, and its width is 200% of its depth. If the width is 2 yd., how many gallons does it hold?

19. A cord of stone having the same volume as a cord of wood, what is a pile of stones 27 ft. long, 5 ft. wide, and 6 ft. high worth at \$4.20 a cord?

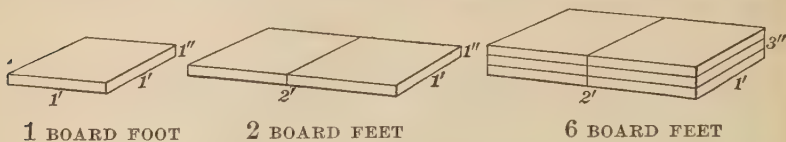
20. If it takes 550 cu. ft. of clover hay to make a ton, how many tons will fill a mow 32 ft. long, the width being double the depth and half the length?

21. Allowing 231 cu. in. to the gallon, how many gallons in a watering trough that is 6 ft. long and 16 in. wide, the depth of the water being $\frac{3}{4}$ the width?

22. A bin 21 ft. 6 in. long and 10 ft. wide is filled with wheat to the depth of 5 ft. Allowing 2150 cu. in. to the bushel, what is the wheat worth at 80¢ a bushel?

23. A cellar is 32 ft. long, 17 ft. wide, and 8 ft. 6 in. deep. How much will it cost to build the wall, 1 ft. thick, at 35¢ per cubic foot, no allowance being made for openings, and the length of the wall being determined by the outside measure, thus doubling the corners?

223. Board Measure. One board foot of lumber means a piece that is 1 sq. ft. on one surface and is 1 in. or less in thickness. In speaking of lumber a board foot is often called simply one foot.



A board 2' long, 1' wide, and 1" (or less) thick contains 2 board feet (usually written as 2 bd. ft.).

A plank 2' long, 1' wide, and 3" thick contains 6 bd. ft.

A board 18' long, 8" wide, and 1" or less thick contains $18 \times \frac{8}{12}$ bd. ft., or 12 bd. ft. A joist $12' \times 6" \times 2\frac{1}{2}"$ (the sign $\times$ here meaning "by") contains $2\frac{1}{2}' \times 12' \times \frac{1}{2}$ bd. ft., or 15 bd. ft.

WRITTEN EXERCISE

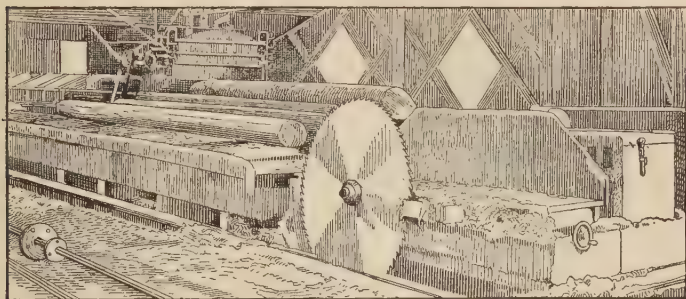
Find the number of feet, board measure, in :

1. 32 planks 3 in. thick, 16 ft. long, and 8 in. wide.
2. 24 planks $2\frac{1}{2}$ in. thick, 14 ft. long, and 10 in. wide.
3. 10 planks $1\frac{3}{4}$ in. thick, 12 ft. long, and 9 in. wide.
4. 4 sticks of timber 16 ft. long, 1 ft. wide, and 8 in. thick.
5. 36 joists 3 in. by 4 in. and 16 ft. long.
6. 12 beams 10 in. by 12 in. and 14 ft. long.
7. 60 boards $\frac{7}{8}$ in. thick, 8 in. wide, and 16 ft. long. Boards $\frac{7}{8}$ in. thick are considered for this purpose as 1 in. thick.
8. What is the cost of 30 spruce joists 2 in. by 6 in. and 14 ft. long, at \$34 per M (that is, per 1000 bd. ft.)?
9. What is the cost of 48 spruce joists 2 in. by 8 in. and 18 ft. long, at \$32.50 per M?

224. Measuring Lumber in the Log. We could find by careful measurement almost the exact number of cubic inches in a log, but for practical purposes a rule known as Doyle's Rule gives the number of board feet with sufficient accuracy.

From the number of inches in the smallest diameter of the log, take 4, multiply the remainder by half itself, multiply this result by the number of feet in the length of the log, and divide by 8.

The rule need not be learned, but it may be used in solving the problems.



For example, find the number of feet of lumber in a log 16 ft. long, with its smallest diameter 18 in. Since $18 - 4 = 14$, and $\frac{1}{2}$ of 14 is 7, the work is as here shown. The result is 196 ft.

$$\frac{14 \times 7 \times 16}{8} = 196$$

Here, as in all similar cases, cancellation should be employed.

WRITTEN EXERCISE

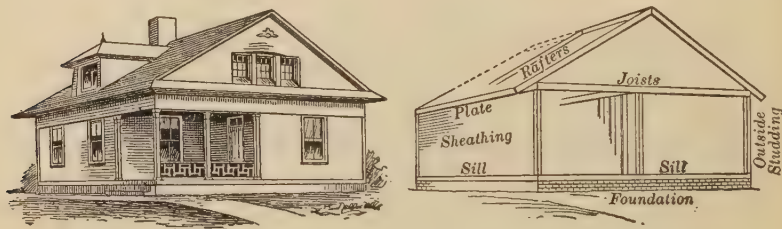
Determine the number of feet, board measure, of lumber in logs of the following lengths and smallest diameters:

- | | | |
|--------------|--------------|--------------|
| 1. 24', 12". | 4. 18', 18". | 7. 28', 30". |
| 2. 20', 14". | 5. 16', 24". | 8. 26', 18". |
| 3. 26', 16". | 6. 20', 22". | 9. 22', 20". |

225. House Building. The house shown in the picture below is to be $34' \times 27\frac{1}{2}'$. A builder estimates the cost of materials and labor. Part of the computation of cost and amount of materials is given in the following examples; the rest is left for the class. The specifications are taken from a recent government report.

Call a fraction of a foot a whole foot in any result.

The pupils should know, from the figure at the right, the meaning of the terms used.



WRITTEN EXERCISE

1. For sills the builder needs the following:

3 pieces, $6'' \times 8''$, 16' long, 192 bd. ft.

4 pieces, $6'' \times 8''$, 14' long, (?) bd. ft.

6 pieces, $6'' \times 8''$, 12' long, (?) bd. ft.

1 piece, $2'' \times 8''$, 14' long, 19 bd. ft.

1 piece, $2'' \times 9''$, 8' long, (?) bd. ft.

2. The builder needs the following joists:

30 pieces, $2'' \times 9''$, 16' long, (?) bd. ft.

18 pieces, $2'' \times 8''$, 12' long, (?) bd. ft.

13 pieces, $2'' \times 8''$, 6' long, (?) bd. ft.

27 pieces, $2'' \times 8''$, 16' long, (?) bd. ft.

27 pieces, $2'' \times 8''$, 12' long, (?) bd. ft.

27 pieces, $2'' \times 6''$, 16' long, (?) bd. ft.

3. The builder also needs the following studding:

- 5 pieces, $4'' \times 6''$, 14' long, (?) bd. ft.
- 100 pieces, $2'' \times 4''$, 14' long, 933 bd. ft.
- 35 pieces, $2'' \times 4''$, 10' long, 233 bd. ft.
- 25 pieces, $2'' \times 4''$, 18' long, (?) bd. ft.
- 45 pieces, $2'' \times 5''$, 12' long, (?) bd. ft.
- 50 pieces, $2'' \times 4''$, 8' long, 267 bd. ft.
- 50 pieces, $2'' \times 4''$, 14' long, 467 bd. ft.
- 40 pieces, $2'' \times 6''$, 18' long, (?) bd. ft.

4. For the roof the builder needs 1050 bd. ft., and this, with the amounts in Exs. 1-3, should make a total of 7847 bd. ft.

5. How much will the lumber mentioned in Exs. 1-4 cost at \$60 per M?

6. The builder also needs 2000 bd. ft. of sheathing at \$65 per M; 1950 bd. ft. of flooring at \$45; 125 bd. ft. of flooring for the porch at \$65; 1500 shingles for the gables at \$5.25 per 1000; 6500 shingles for the roof at \$7. What will this material cost?

7. He also needs 28 foundation posts at 40¢ each; water tables, cornice, corner boards, etc., \$35; porch columns, \$7; 16 doors at an average cost of \$5.75 each; 16 windows at an average cost of \$5.50 each; hardware, \$50; chimney and plastering, \$200; painting, \$148; tin work, \$25.50; the work of 4 carpenters for 30 days at \$4 a day each; excavating, \$130; grading, \$25. What is the total cost of these items?

8. How much will the flooring for a house cost, taking the measurement of each floor as $28' \times 32'$, and allowing for two floors, the lumber costing \$40 per M?

One good way of treating the general topic of practical measurements is to work out problems in connection with building and equipping a house or the school building. If a building is being erected in the neighborhood it should be used for this work in order that the problems may be real.

ORAL EXERCISE ON PLASTERING

1. What is the perimeter of a room 14 ft. by 16 ft.?
2. If the room is 9 ft. high, what is the area of the four walls, not allowing for openings?
3. What is the area of the ceiling of a room 18 ft. by 20 ft.? How many square yards in the ceiling?
4. What will it cost to plaster 60 sq. yd. at 30¢ a square yard? at 33¢ a square yard? at 35¢ a square yard?
5. A corridor is 30 ft. long, 4 ft. wide, and 9 ft. high above the baseboard. How many square feet of plaster are needed for the two sides and the top of the corridor, not allowing for any openings?

WRITTEN EXERCISE ON PLASTERING

1. How many square yards of plastering are there in the walls and ceiling of a room 16 ft. by 20 ft., and 9 ft. high, deducting 14 sq. yd. for openings and baseboard?
2. How many square yards of plastering are there in the walls and ceiling of a hall 60 ft. long, 40 ft. wide, and 28 ft. high, deducting 36 sq. yd. for openings and baseboard?
3. What will it cost to plaster a schoolroom 32 ft. long, 18 ft. wide, and 12 ft. high, at 32¢ a square yard, deducting 50% of the area of the walls on account of blackboards, baseboard, and openings?
4. On account of the danger from falling plaster, some school trustees decided to use pressed steel ceilings in a schoolhouse. There were 742 sq. yd. of ceiling in the building, and a contract was made for putting up the steel ceilings at 10¢ a square foot. How much did the ceilings cost?

ORAL EXERCISE ON CARPETING

1. How many strips of $\frac{3}{4}$ -yard carpet are needed for a room 18 ft. wide, not allowing for matching the pattern?
2. A carpet has a 12-inch pattern. Will there be any loss in matching if the strips are 16 ft. long? if they are $16\frac{1}{2}$ ft. long?
3. A carpet has an 18-inch pattern. Will there be any loss in matching if the strips are 15 ft. long? if they are 14 ft. long?

WRITTEN EXERCISE ON CARPETING

1. How many yards of carpet, 9-inch pattern, are needed for a room 11 ft. 3 in. wide and 12 ft. long, the carpet being 27 in. wide? Draw a plan showing how the strips lie.
2. How many yards of carpet, $\frac{1}{2}$ -yard pattern, are needed for a room 13 ft. 6 in. wide and 16 ft. long, the carpet being 27 in. wide? How much is wasted on each strip (except the first, with which the others must match)? Draw a plan. No allowance is necessary for the first strip unless one wishes to make it.

Find how many yards of carpet are needed for the following, allowing for matching each strip (except the first), as indicated:

3. 18 ft. by 20 ft., 27-inch carpet, allowing 9 in.
4. 16 ft. by 18 ft., 27-inch carpet, allowing $\frac{1}{2}$ yd.
5. 14 ft. by 15 ft., 36-inch carpet, allowing $\frac{1}{4}$ yd.

Find the cost of carpeting these rooms with 27-inch carpet, allowing 4 inches on each strip (except the first), @ \$1.20 a yard:

- | | |
|---------------------------|---|
| 6. 15 ft. by 18 ft. | 10. $13\frac{1}{2}$ ft. by 16 ft. 3 in. |
| 7. 14 ft. 3 in. by 18 ft. | 11. 4 ft. 6 in. by 23 ft. 8 in. |
| 8. 12 ft. by 13 ft. 6 in. | 12. 15 ft. 9 in. by 25 ft. 6 in. |
| 9. 17 ft. 6 in. by 18 ft. | 13. 22 ft. 3 in. by 28 ft. 6 in. |

226. Papering. The standard width for estimating wall paper is 18 in., although it varies greatly. A single roll is considered to contain 8 yd. of paper. In estimating, paper hangers usually count 3 rolls to 100 sq. ft. Parts of rolls cannot be bought.

ORAL EXERCISE

1. What will the border for a room 16 ft. by 20 ft. cost, at 25¢ a yard?
2. Estimate the cost of the paper for a ceiling 15 ft. by 20 ft., at 40¢ a roll.
3. A room has nearly 500 sq. ft. of wall to be papered. Estimate the cost of the paper at 50¢ a roll.
4. A plain wall 14 ft. long and 9 ft. high above the baseboard is to be papered. How many rolls should you allow?

WRITTEN EXERCISE

1. The walls of a room are 8 ft. 6 in. above the baseboard, and the total length of the four walls, allowing for openings, is 72 ft. Allowing 6 in. on each strip for matching, what will the paper cost at 40¢ a roll?
2. How many rolls of plain paper will it take to cover the four sides of a room 12 ft. high, 25 ft. long, and 20 ft. wide, the paper being 24 in. wide, allowing $1\frac{1}{2}$ rolls for windows and doors? At 35¢ a roll, what will it cost?
3. The walls of a room are to be papered with plain paper worth 30¢ a roll. They are 9 ft. high above the baseboard, and the total length of the four walls, allowing for windows and doors, is 60 ft. At the usual estimate, what will the paper cost?

The cost of papering rooms at home should be considered.

WRITTEN EXERCISE

1. The house described on page 294 was built on a lot 100 ft. wide and 200 ft. deep. The owner took a piece 40 ft. by 80 ft. for a vegetable garden, and one 60 ft. by 80 ft. for a fruit orchard. How many square feet were there in each of these two pieces?

2. Express the area of the lot and of all three parts as fractions of an acre.

3. What was the area occupied by the house?

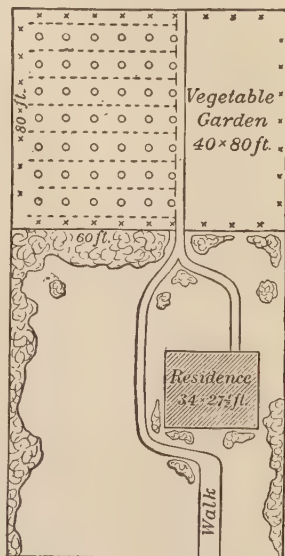
4. The excavation for the cellar was 5 ft. deep. How much earth was removed?

5. If the excavation for the cellar had been 6 ft. 4 in. deep, how much earth would have been removed?

6. If the owner fences the garden, the orchard, and the part on which his residence stands, how many feet of fencing will he need, not allowing for gates? how many rods of fencing? how many yards of fencing?

7. The owner planted a bed of asparagus 4 ft. wide and 36 ft. long; two beds of parsnips and two of beets, each $1\frac{1}{2}$ ft. wide and 36 ft. long; two beds of tomatoes and two of squash, each 3 ft. wide and 36 ft. long. How many square feet of area did he give to each of the beds?

8. He planted shrubs about the lawn. These, with the walks, occupied 2200 sq. ft. How many square feet were left for the lawn? See if you can improve this design.



WRITTEN REVIEW EXERCISE

1. Allowing $1\frac{1}{4}$ cu. ft. to a bushel, how many bushels of wheat can be put into a bin containing 900 cu. ft.?
2. A dry quart contains $67\frac{1}{5}$ cu. in. How many quarts in a measure containing 336 cu. in.?
3. A railway train goes $63\frac{9}{10}$ mi. in an hour and a half. What is its rate per hour?
4. If a farmer has cut 35 cords of 4-foot wood from his property and piled it 4 ft. high, how long is the pile?
5. If a rectangular lot 8 rd. by 40 rd. produces $27\frac{1}{2}$ bu. of corn to the acre, how many bushels does it produce in all?
6. If a cubic foot of water weighs 62 lb. 8 oz., what is the weight of the water in a cistern that contains $1\frac{7}{9}$ cu. yd.?
7. How many pounds do 5 cu. yd. 18 cu. ft. of earth weigh, if a cubic yard weighs 1 T. 825 lb.?
8. A hand is 4 in., and a certain horse is 14 hands high. Express this in inches; in feet and inches; in feet and a fraction.
9. A cubic foot of water weighs nearly 1000 oz. Express this weight in pounds and ounces, and in pounds and a fraction.
10. A farmer has a barn 60 ft. long, 30 ft. wide, and 15 ft. high to the eaves. The triangular gables on the ends, above the line of the eaves, have each an area of 150 sq. ft. How much will it cost to paint the sides of the barn at 20¢ a square yard? Illustrate the four sides by drawings.
11. If a cubic foot of ice weighs 57 lb. 8 oz., what will be the weight in tons of the mass of ice 1 ft. thick that covers a pond whose surface is 100,000 sq. yd.?
12. If you are in school 5 hr. a day, how many minutes are you in school during the 5 da. of the week? how many seconds?

V. SIMPLE ACCOUNTS

227. Simple Accounts. We have already learned (pages 126-129) how to keep simple accounts. We shall now review the subject. The following is a sample page from an account book :

1916		RECEIPTS			1916		PAYMENTS		
March	1	On hand . .	1	18	March	2	Papers . .		25
"	6	From father		25	"	3	Car fare .		10
"	14	Errand . . .		10	"	4	Pencil . .		05
"	20	Paper route .		75	"	31	On hand .	2	03
"	29	Errand . . .		15					
			2	43				2	43

In order to train pupils to keep simple accounts, an accomplishment that every one should possess, teachers are requested to have at least one exercise a week in which the children make a record of their income and expenses. This exercise need not occupy an entire period. Except when class instruction is needed, the records may be made before school or at odd times.

The accounts should be of actual moneys belonging to the pupils, if that is possible. It is better to have a briefer account and have it a real one than to fill it with imaginary transactions. The accounts should indicate briefly but adequately the sources and dispositions of income: not "Rec. from Aunt Mary," but "Gift from Aunt Mary"; not "Paid out," but "Paid for Candy"; not "Earned," but "Shoveling Snow." The headings at the top of the pages should be uniform through each book, either "Received," "Paid"; or "Income," "Outgo"; or "Receipts," "Payments." Balances should be computed at the close of each month. These balances should be properly brought down by the use of red lines, if possible, and carried forward to the next month's account.

Ask the pupils to bring to school bills that have been used in business transactions. This is review work, and pupils should become proficient in making and receipting bills of various sorts that are used in the common business of life. Make the work as practical as possible.

The examples in any textbook must necessarily be imaginative, and those given on page 302 may be used in case it is not possible to carry out the work in the spirit above set forth.

The dollar sign and the decimal point are commonly omitted when the ruling of the account book shows clearly, as in the above example, which column is dollars and which is cents.

WRITTEN EXERCISE

Rule a sheet of paper, or use an account book properly ruled, and write the following accounts, balancing each:

1. April 2, on hand, \$1.25. Receipts: April 9, gift from Uncle John, 30¢; April 20, errand, 15¢; April 24, errand, 10¢. Payments: April 13, ball, 15¢; April 14, bat, 60¢.

2. June 1, on hand, \$1.35. Receipts: June 8, from Rob for money borrowed, 10¢; June 18, paper route, 60¢; June 23, care of horse, 50¢. Payments: June 13, catcher's mask, \$1.15; June 24, 2 bats @ 60¢.

Write the following accounts, inserting dates and items, and balancing each:

3. On hand, \$1.60. Receipts: \$2.20, 20¢, 10¢, 15¢. Payments: \$1.50, 25¢, 75¢, 10¢.

4. On hand, \$1.25. Receipts: 20¢, 40¢, 75¢, 30¢. Payments: 20¢, 30¢, 5¢, 60¢.

5. On hand, \$3.60. Receipts: \$1, 45¢, 15¢, 25¢, 5¢. Payments: \$1.35, 25¢, 60¢, 10¢, 10¢.

6. On hand, \$4.30. Receipts: 60¢, \$1.30, 40¢, 20¢, 10¢, 5¢. Payments: \$1.75, 30¢, 40¢, 60¢, 10¢, 2¢, 1¢.

7. On hand, \$2.30. Receipts: \$5, 20¢, 30¢, 2¢, 10¢. Payments: \$1.25, \$1, 40¢, 65¢, 75¢, 2¢, 10¢.

8. On hand: \$3.35. Receipts: 5¢, 25¢, 60¢, \$1, \$2.25. Payments: \$2, \$1.20, 60¢, 2¢, 35¢, 4¢, 5¢.

9. On hand: \$2.75. Receipts: \$1, 75¢, 60¢, \$2, 40¢, 5¢. Payments: 40¢, 10¢, 3¢, 20¢, 25¢, 10¢, 15¢.

10. On hand: \$4.40. Receipts: 30¢, 70¢, \$1, 10¢, 15¢, 20¢. Payments: 25¢, \$2.50, 30¢, 45¢, 5¢, 15¢, 2¢.

REVIEW AND DRILL

228. Importance of this Review and Drill. You should now do what the merchant does at the end of the year—you should “take stock,” and see what you have on hand. This review and drill gives you the opportunity for such a stock-taking. It covers the essential features of the work in Grades V and VI.

Teachers should read carefully the suggestions on page 83.

I. DRILL SECTION

WRITTEN EXERCISE

1. Write CCXIV in ordinary numerals.
2. Write 89 and 1920 in Roman numerals.

	I	II	III	IV	V	VI	VII
<i>a</i>	$\frac{1}{16}$	$1\frac{1}{2}$	2.7	3754	47.8	1.276	0.237
<i>b</i>	$\frac{1}{8}$	$1\frac{3}{4}$	3.1	4017	52.9	24.31	0.706
<i>c</i>	$\frac{3}{8}$	$1\frac{3}{4}$	3.9	5291	57.1	30.60	0.819
<i>d</i>	$\frac{1}{2}$	$1\frac{7}{8}$	5.2	6807	61.6	49.19	1.273

In each of the seven columns above, find the results of the following operations to as many decimal places as the numbers have :

- | | | | |
|-------------------|-------------------|--------------------|--------------------|
| 3. $a + b$. | 7. $b + c$. | 11. $c + d$. | 15. $a + d$. |
| 4. $b - a$. | 8. $c - b$. | 12. $d - c$. | 16. $d - a$. |
| 5. $a \times b$. | 9. $b \times c$. | 13. $c \times d$. | 17. $a \times d$. |
| 6. $b \div a$. | 10. $c \div b$. | 14. $d \div c$. | 18. $d \div a$. |

Find, in turn, the sum, difference, product, and quotient of the following pairs of numbers, in every case of division dividing the greater by the less :

- | | | |
|-------------------|-------------------|-------------------------------------|
| 19. 67,956, 9708. | 22. 14.49, 20.7. | 25. $\frac{7}{8}$, $\frac{1}{2}$. |
| 20. 24,438, 4073. | 23. 273.6, 34.2. | 26. $\frac{5}{8}$, $\frac{1}{4}$. |
| 21. 45,081, 5009. | 24. 2.115, 0.423. | 27. $\frac{3}{4}$, $\frac{3}{8}$. |

28. To the difference between 4287.6 and 1939.8 add the sum of 428.6 and 9007.9.

29. Add 2 ft. 8 in., 3 ft. 6 in., 9 ft. 7 in., 12 ft. 4 in., 16 ft. 2 in., and 3 ft. 11 in.

Find the sums and also the differences :

30.		31.		32.		33.	
ft.	in.	ft.	in.	lb.	oz.	yd.	in.
29	2	30	1	16	4	38	14
9	7	18	9	8	7	19	29

Multiply, and also divide, the first number by the second :

- | | | |
|----------------------|-----------------------------------|-----------------------|
| 34. 22 ft. 8 in., 4. | 36. 36 yd. 4 in., 5. | 38. 12 lb. 8 oz., 5. |
| 35. 43 ft. 6 in., 6. | 37. 37 rd. $1\frac{1}{2}$ ft., 4. | 39. 30 yd. 26 in., 7. |

Find 50%, 25%, and also 75% of :

- | | | | |
|---------|---------|----------|-------------------|
| 40. 44. | 42. 72. | 44. 144. | 46. 4 ft. 8 in. |
| 41. 56. | 43. 96. | 45. 240. | 47. 16 yd. 24 in. |

Multiply by 9, and also by 11 :

- | | | | |
|-----------|-----------|-------------|-------------|
| 48. 1728. | 50. 9280. | 52. 12,365. | 54. 26,342. |
| 49. 5280. | 51. 3174. | 53. 14,807. | 55. 31,287. |

Multiply by 25, by 75, and also by $33\frac{1}{3}$:

- | | | | |
|----------|----------|-----------|-----------|
| 56. 120. | 58. 240. | 60. 1200. | 62. 4836. |
| 57. 360. | 59. 480. | 61. 1224. | 63. 4848. |

Find, in turn, the sum, difference, product, and quotient of the following pairs of numbers, dividing the greater by the less :

- | | | |
|-------------------|-------------------|-------------------|
| 64. 6531, 2177. | 67. 30,195, 6039. | 70. 31,104, 3456. |
| 65. 18,846, 3141. | 68. 56,080, 7010. | 71. 24,258, 4043. |
| 66. 29,547, 4221. | 69. 28,404, 7101. | 72. 69,993, 7777. |

Find, in turn, the sum, difference, product, and both quotients of the following pairs of fractions :

By "both quotients" is meant, in Ex. 73, $\frac{2}{3} \div \frac{3}{4}$ and $\frac{3}{4} \div \frac{2}{3}$.

- | | | | |
|----------------------------------|----------------------------------|----------------------------------|-----------------------------------|
| 73. $\frac{2}{3}, \frac{3}{4}$. | 75. $\frac{2}{3}, \frac{5}{8}$. | 77. $\frac{3}{4}, \frac{1}{8}$. | 79. $\frac{1}{2}, \frac{3}{16}$. |
| 74. $\frac{1}{3}, \frac{2}{8}$. | 76. $\frac{2}{3}, \frac{1}{8}$. | 78. $\frac{3}{4}, \frac{5}{8}$. | 80. $\frac{1}{8}, \frac{5}{16}$. |

Find the following :

- | | | |
|---------------|-----------------------------|---------------------------------------|
| 81. 7% of 48. | 83. $2\frac{1}{2}\%$ of 38. | 85. 50% of 9 ft. 8 in. |
| 82. 9% of 75. | 84. $3\frac{1}{8}\%$ of 69. | 86. $33\frac{1}{3}\%$ of 10 ft. 9 in. |

Find the number of which :

- | | | |
|-----------------|-----------------|------------------|
| 87. 3.78 is 9%. | 93. 2.48 is 8%. | 99. 4.8 is 15%. |
| 88. 4.14 is 9%. | 94. 3.72 is 6%. | 100. 73.2 is 4%. |
| 89. 72.8 is 8%. | 95. 73.2 is 6%. | 101. 6.66 is 3%. |
| 90. 65.6 is 8%. | 96. 97.5 is 5%. | 102. 9.75 is 3%. |
| 91. 15.4 is 7%. | 97. 3.45 is 5%. | 103. 73.2 is 2%. |
| 92. 37.1 is 7%. | 98. 64.8 is 4%. | 104. 61.8 is 2% |
105. Find 20% of 50% of \$1000.
106. Find 20% of 25% of 50% of \$1600.
107. Find 10% of 30% of 40% of \$1200.
108. Find 4% of 6% of 8% of \$1275.
109. Find 6% of 4% of 5% of \$1500.
110. Find 2% of 8% of 7% of \$2400.
111. Find 3% of 5% of 15% of \$3600.

I	II	III	IV	V	VI
<i>a</i> $\frac{3}{4}$	$\frac{4}{5}$	2454	3801	2 ft. 4 in.	6 ft. 1 in.
<i>b</i> $\frac{3}{8}$	$\frac{3}{5}$	3217	3504	3 ft. 7 in.	9 ft. 2 in.
<i>c</i> $\frac{2}{5}$	$\frac{5}{8}$	4286	5001	6 yd. 9 in.	7 yd. 1 in.
<i>d</i> $\frac{3}{5}$	$\frac{2}{3}$	9170	9309	4 bu. 2 pk.	9 bu. 1 pk.
<i>e</i> $\frac{1}{8}$	$\frac{2}{3}$	5007	5670	3 gal. 3 qt.	7 gal. 2 qt.
<i>f</i> $\frac{1}{4}$	$\frac{2}{5}$	2076	3803	6 lb. 7 oz.	9 lb. 6 oz.
<i>g</i> $\frac{3}{4}$	$\frac{4}{5}$	39.28	42.16	4 lb. 8 oz.	5 lb. 7 oz.
<i>h</i> $\frac{3}{8}$	$\frac{4}{5}$	41.77	50.23	3 ft. 3 in.	9 ft. 2 in.
<i>i</i> 4%	6%	31.72	50.01	5 yd. 2 in.	7 yd. 8 in.

In each of the nine lines above perform the following operations, carrying the quotients in columns III-VI to only two decimal places :

112. I + II.	117. III + IV.	122. V + VI.	127. IV ÷ I.
113. II - I.	118. IV - III.	123. VI - V.	128. I × V.
114. I × II.	119. III × IV.	124. III ÷ II.	129. I × VI.
115. I ÷ II.	120. III ÷ IV.	125. VI ÷ V.	130. II × V.
116. II ÷ I.	121. IV ÷ III.	126. III ÷ I.	131. II × VI.

In each of columns I-IV add the numbers in the following lines :

132. <i>a, b.</i>	137. <i>a, g.</i>	142. <i>c, d.</i>	147. <i>a, c, e.</i>
133. <i>a, c.</i>	138. <i>a, h.</i>	143. <i>b, e.</i>	148. <i>b, d, f.</i>
134. <i>a, d.</i>	139. <i>a, i.</i>	144. <i>b, f.</i>	149. <i>c, d, h.</i>
135. <i>a, e.</i>	140. <i>b, c.</i>	145. <i>b, g.</i>	150. <i>d, g, i.</i>
136. <i>a, f.</i>	141. <i>b, d.</i>	146. <i>b, h.</i>	151. <i>e, b, i.</i>

152. Find 4% of each number in column III.
 153. Find 12% of each number in column IV.
 154. Find 50% of each number in column V.
 155. Find $33\frac{1}{3}\%$ of each number in column VI.
 156. Find 12.5% of each number in column VI.

I	II	III	IV
<i>a</i> 3 ft. 6 in.	4 yd. 10 in.	3 lb. 4 oz.	9 bu. 2 pk.
<i>b</i> 5 ft. 9 in.	6 yd. 15 in.	9 lb. 6 oz.	7 bu. 3 pk.
<i>c</i> 8 ft. 7 in.	9 yd. 18 in.	7 lb. 5 oz.	5 bu. 1 pk.
<i>d</i> 7 ft. 8 in.	7 yd. 23 in.	3 lb. 9 oz.	4 bu. 2 pk.
<i>e</i> 9 ft. 4 in.	3 yd. 31 in.	6 lb. 8 oz.	6 bu. 1 pk.
<i>f</i> 4 ft. 5 in.	5 yd. 17 in.	8 lb. 6 oz.	4 bu. 3 pk.

In each of the four columns given above perform the following operations :

157. $a + b + c$.	172. $7 \times a$.	187. $a \div 2$.
158. $b + c + d$.	173. $6 \times b$.	188. $a \div 3$.
159. $c + d + e$.	174. $9 \times c$.	189. $b \div 4$.
160. $d + e + f$.	175. $3 \times d$.	190. $b \div 5$.
161. $a + c - b$.	176. $4 \times e$.	191. $c \div 6$.
162. $b + d - c$.	177. $5 \times f$.	192. $c \div 8$.
163. $d + f - e$.	178. $3 \times a$.	193. $d \div 12$.
164. $a + f - b$.	179. $7 \times b$.	194. $d \div 15$.
165. $b + f - c$.	180. $2 \times e$.	195. $e \div 7$.
166. $c + e - d$.	181. $5 \times d$.	196. $e \div 9$.
167. $e + f - d$.	182. $6 \times e$.	197. $f \div 12$.
168. $c + d - e$.	183. $9 \times b$.	198. $f \div 8$.
169. $a + d - e$.	184. $6 \times a$.	199. $c \div d$.
170. $a + d + f$.	185. $5 \times b$.	200. $9 \times a \div b$.
171. $a + e + f$.	186. $8 \times c$.	201. $10 \times b \div c$.

This Review and Drill is longer than some of the others. This is because it comes at the close of the sixth grade, when a book is being finished. It is intended to be available as a final drill in the work of Grades V and VI. At the close of each year teachers will find the time well spent that is given to a thorough review of the work of that year.

II. PROBLEMS WITHOUT NUMBERS

ORAL EXERCISE

1. If you know the cost of a certain number of things and wish to know the cost of one, how do you proceed?
2. If you know the cost of one thing and wish to find the cost of a certain number of things, how do you proceed?
3. If you know the cost of a fraction of anything and wish to know the cost of the whole, how do you proceed?
4. If you know the cost of a certain number of things and wish to know the cost of a certain other number of the same things, how do you proceed?
5. If you know the cost of a gallon of anything and wish to know the cost of a pint, how do you proceed?
6. If you know the price of cloth per yard, how do you find the cost of a certain number of inches?
7. If you know the price of a certain kind of cloth per yard, how do you find the cost of a certain number of yards plus a certain number of inches?
8. If you know the circumference of a wheel, how do you find how many times the wheel turns in going a mile?
9. If you know how many times a wheel turns in going a mile, how do you find the circumference?
10. If you know the freight rate per hundred pounds, how do you find the freight charge on a ton?
11. If you know the freight rate per hundred pounds, how do you find the freight charge on a certain number of tons plus a certain number of pounds?
12. If you have the product of several numbers to be divided by the product of several others, how may you perform the operations with least labor?

ORAL EXERCISE

1. If you have a given per cent of a number, how do you express it as a common fraction ?

2. If you are to find a certain fractional part of a number, how do you express this as a per cent of the number ?

3. If you have a certain rate per cent expressed with the per cent sign, how do you express it as a decimal fraction ?

4. If you have a decimal fraction, how do you express the same number with a per cent sign ?

5. If you know what a given per cent of a number is, how do you find the number ?

6. How do you find what per cent one number is of another ? Illustrate.

7. A man's salary two years ago was increased a certain per cent. Last year it was decreased the same per cent. Is his salary at present greater than or less than it was two years ago ?

8. A merchant marks his goods a certain per cent above cost. Owing to hard times he sells them at the same per cent below the marked price. Does he sell them for more than or less than cost ?

9. If you know the amount paid by an agent for certain property, and his rate of commission, how do you find the amount of his commission ?

10. If you know the net amount left after the commission has been deducted, and the price at which the property was sold, how do you find the rate of commission ?

11. If you know the length of time that a sum of money is on interest, the rate, and the sum of money itself, how do you find the interest ?

12. If you know what a dealer paid for some goods, and his selling price, how do you find the per cent of profit or loss ?

The boys in class should be asked how to find a baseball player's fielding average, batting average, and so on.

ORAL EXERCISE

1. The members of the class have measured the length, width, and height of a room. How do they find the number of rolls of paper needed to cover the walls and ceiling?

2. The members of the class have measured the length and width of a room. How do they find the number of yards of carpet needed to cover a floor?

3. If you know the dimensions of this room, how do you find the number of square yards of plastering needed for the walls and ceiling, making no allowance for doors and windows?

4. How do you find the number of feet of lumber, board measure, needed to cover the floor of your schoolroom with boards of the width of those now used?

5. How do you find the number of yards of carpeting required to cover the floor of this schoolroom?

6. If you know the length of one side of a square, how do you find the area?

7. If you know the lengths of two adjacent sides of a rectangle, how do you find the area?

8. If you know the base and altitude of a triangle, how do you find the area?

9. How would the girls in the class proceed to compute the cost of material and labor for making a dress for school wear?

10. Given the length, breadth, and thickness of a rectangular solid, how do you find the volume?

11. Given the length, breadth, and thickness of a rectangular solid, how do you find the total area of the faces?

12. Given the volume of a rectangular solid, and two dimensions, how do you find the third dimension?

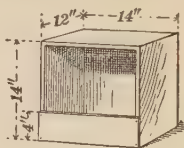
13. How do you proceed to estimate the number of thousand bricks needed to build a wall of given dimensions?

III. INDUSTRIAL PROBLEMS

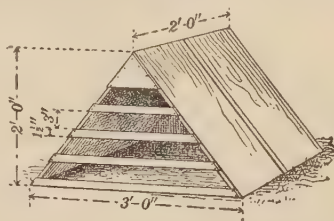
HOUSING THE POULTRY

1. Edward helped his father make some nests and coops. This is a picture of one of the nests for a sitting hen. Edward figured out the number of feet of lumber for each nest. How many feet were required?

2. Edward and his father made eight of these nests, using lumber that cost \$40 per M. How much did the lumber cost?

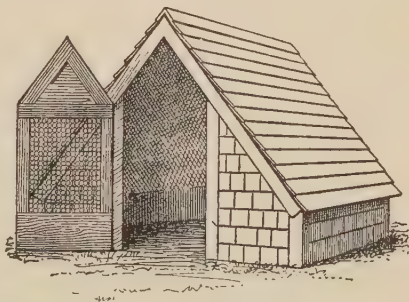


3. They made eight coops like the one pictured below for the hens and chickens. Edward's father suggested that he draw the figure to scale, 1 in. to 1 ft., and then measure the size of each part of the coop. Do this, and find the approximate number of feet of lumber required, and its value at \$40 per M.



Some approximation is allowed here, but this is exactly what happens in practical work, and is very desirable. The pupils may compare their answers and thus check one another's errors.

4. Edward's father builds enough poultry houses like this for his hens. The length is 9 ft., the width 7 ft., the height in the center 6 ft., and the height at the eaves 18 in. Allowing about 20 cu. ft. to each hen, how many hens will this house accommodate?



This is, of course, approximate. If Edward gets an answer like 11.8 hens, he will say that the house will accommodate a dozen. In other words, Edward will use his common sense with such an answer.

THE BOYS' CORN CLUB



1. A lot of boys on the farms of a certain county started corn clubs. Each boy had to charge to expenses \$5 an acre as rent. One of the boys in this picture cultivated $2\frac{3}{4}$ acres. How much did he charge to expenses as rent? His field was 40 rd. long. How wide was it?

2. He estimated his labor at 10¢ an hour, and the work of each horse at 25¢ an hour. In one month he put in 42 hr., during 4 hr. of which he had a horse. How much did he charge up that month?

3. He needed for seed 16 ears of corn to the acre, after rejecting all the doubtful seeds. How many ears did he need in all? He paid for the seed at the rate of $2\frac{1}{2}$ ¢ an ear. How much did he pay?

4. He planted the corn in hills 3 ft. 4 in. apart, the hills being in rows 3 ft. 6 in. apart. How many hills could he put in, making an allowance of 1 ft. 6 in. for vacant space at each end of the field and an allowance of 2 ft. 6 in. at each side?

5. It cost him \$12.48 per acre to raise the corn, including rental, seed, labor, and fertilizer. How much did it cost him in all? The corn being worth 40¢ a bushel, what was his profit on the $274\frac{3}{4}$ bu. that he raised? He received a prize of \$5. How much was his profit?

MAKING A LIVING

1. Mrs. Wiggs receives \$1.25 a day for doing washing. How much does she make in a week of six working days if she spends 5¢ a day for car fare? How much does she make in 51 wk.?

2. Mr. Wiggs left her some life insurance which she invested so as to receive \$45 a year. How much is this a month? If her month's rent is \$9, what part of the rent will this pay?

3. John helps his mother by working before and after school. He receives \$1.50 a week for delivering morning papers, and on Sunday mornings he earns on an average 85¢ more by selling extra papers. He averages 35¢ every evening by selling papers. If he also makes \$1.10 a week by selling the *Saturday Evening Post*, what is his total weekly contribution to the family income?

4. John's sister Jennie wishes to help too. A neighbor offers her 10¢ an hour for wheeling the baby. If she works on the average $1\frac{1}{2}$ hr. a day, including Sundays, missing only ten days, how much will she earn in a year?

5. Their brother Frank borrows \$1.50 from John to pay for plowing and seed, and \$1.50 from Jennie to pay for a spade and hoe. If he raises \$4.25 worth of cabbages, \$3.25 worth of tomatoes, \$6.15 worth of potatoes, and \$7.75 worth of other vegetables, and then pays back the money he borrowed, how much is he able to give his mother toward the family expenses during the year?

6. They all think they can do better by keeping chickens. They buy six hens and a rooster for \$5. Frank makes a coop out of a box that the grocer sells him for 15¢, and he spends 35¢ for nails, hinges, screws, and some nest eggs. What is the total investment?

7. Frank feeds the poultry half a peck of oats a week, costing 40¢ a bushel; $7\frac{1}{2}$ lb. of bran, costing \$1.30 per 100 lb.; and the table scraps. If the hens lay on an average 15 eggs a week, and Frank can sell the eggs at 36¢ a dozen, how much does he make in a week, after deducting expenses?

PAVING THE CITY STREETS



1. In preparing the street in front of his house for paving, John saw that the workmen took out dirt to the average depth of 14 in. The street is 40 ft. wide from curb to curb and is to be paved a distance of $\frac{1}{2}$ mi. How many cubic yards of dirt are to be removed, and how much will it cost at 50¢ per cubic yard?

2. John finds that they are going to put in a foundation of 8 in. of concrete costing \$1.05 per square yard. How much will this cost?

3. John finds that the concrete is to be covered with either brick or asphalt. Brick costs \$2.35 per square yard, and asphalt costs \$3.10 per square yard. What is the difference in cost for the street?

4. John also finds that the curbing for the street costs 78¢ per running foot, and that there is a curbing all along both sides, except where there are cross streets. He and Will measure and find that there is 480 ft. to be deducted on each side for intersecting streets. How much will the curbing cost?

5. John's father's lot has a frontage of 66 ft. on the street. If the city requires him to pay half the cost of the paving to the middle of the street, and half the cost of the curbing in front of his house, how much must he pay if brick is used for the paving?

PROBLEMS RELATING TO HEALTH

1. Typhoid fever and tuberculosis cause thousands of deaths every year, which could be prevented by care and cleanliness. There are probably about 392,000 cases of typhoid fever in the United States every year. If the deaths are $9\frac{1}{2}\%$ of this number, how many persons die from typhoid fever annually?

2. The United States Army requires all its men to be immunized against typhoid fever. Before this was done, out of 10,759 men in one army corps, there were 4422 cases of fever in one year, with 248 deaths. What was the per cent of cases and of deaths?

Carry the answer to Ex. 2 to thousandths, to Ex. 3 to hundred-thousandths.

3. Since the time when immunization was required in the army, out of 12,801 men in one maneuver, only one case of typhoid fever occurred, and no death. What was the per cent of cases?

4. In a recent year there were only two cases of typhoid fever among 80,000 officers and men, while the average number before immunization was required was three cases per thousand. What was the per cent of cases before immunization and during the year mentioned?

5. Typhoid-fever germs are carried from piles of filth to the food that we eat, and this is done by the common house fly. One fly lays on an average 120 eggs at one time, and will lay four such batches in a summer. If 90% of the eggs produce flies, how many flies will come from the eggs laid by 1000 flies?

6. Flies lay their eggs in the filth of stables. It has been found that 1200 flies will hatch in a pound of this filth. If even $\frac{1}{10}$ of this rate were kept up, how many flies would hatch in a summer from 6 tons of this filth?

7. If a man earns \$3.50 a day when he is well, but loses his wages and has to pay \$1.30 a day for care and medicines when he is ill, how much more is his net income when he is well and is working 6 da. in the week than when he is ill?

THRIFT IN THE HOME

1. Mrs. Brown finds that she can save at least 50¢ a week by going to the market and buying in person. By doing this, how much a year, at least, will she save?

2. Frank gets interested in the idea of helping to cut down the high cost of living. He says he will raise vegetables. Allowing \$1.25 for plowing, \$3.25 for fertilizer, \$1.30 for seed, and 75¢ a week for 22 wk., which wages his mother insists that he shall have for his bank, what are the total expenses for the season's vegetables?

3. Mrs. Brown used to spend \$54 for these same vegetables at the store. What was the saving in having their own garden?

4. Since Frank's mother no longer buys over the telephone, she decides to cut off that expense. This comes to \$2.25 a month. How much does she save in a year by not using the telephone?

5. Frank notices that at certain times there are bargain sales at the stores. By buying at a sale an \$18 suit at 15% off, three \$1.50 shirts at 10% off, a dozen 25-cent handkerchiefs at 16 $\frac{2}{3}$ % off, a pair of \$4.75 shoes at 25% off, and a hat and some collars and ties amounting to \$5.35 at 20% off, how much does he save in all?

6. How much have Frank and his mother together saved on the purchases in Exs. 1-5, as stated above?

7. They find that they can invest this money at 5% interest. What will it amount to in 10 yr.?

8. At the end of a year after the investment mentioned in Ex. 7 was made, they invested an equal amount again at 5% interest, and they did this every year until the end of the 10 yr. How much did they have at the end of that time, including the interest?

9. Frank will then be old enough to go to college. His mother says that he will need \$1500 if he is studious enough to earn a free scholarship. Will they have money enough saved to allow Frank to go to college? If not, how much must Frank's father contribute to help them out?

HOW TOM EARNED HIS SPENDING-MONEY

1. Tom's father gives him his choice of 35¢ a week for spending-money, or a dozen hens with which to earn what he can. How much would 35¢ a week amount to in a year? If the hens laid 100 eggs apiece in a year and eggs averaged 28¢ a dozen, how much more would Tom have for spending-money by keeping hens?

2. Tom figured this out, but forgot to deduct the expense of feeding his hens. Suppose this to be \$1 per year for each hen, and suppose that the hens averaged 125 eggs apiece, how much better would Tom's annual income be than 35¢ a week?

3. Tom chooses the hens and keeps a record of income and expenses. He finds that there are more table scraps than he had counted on, so that he needs each week only 1 pk. of corn costing 60¢ a bushel, $\frac{1}{2}$ pk. of oats costing 36¢ a bushel, and 5 lb. of bran costing \$1.40 per 100 lb. What is the expense of the feed per year?

4. During the year the hens averaged $2\frac{1}{4}$ doz. eggs a week, and these he sold at an average price of 25¢ a dozen. What is his income per year? Deducting his expenses, does this leave him more than or less than 35¢ a week, and how much for the year?

5. Tom wishes to make more money. He learns that by selling eggs for hatching purposes in the late winter and early spring he can get 75¢ for a setting of 15 eggs. If he sells 20 doz. at this price, how much does it add to his yearly income? What does this make his average income per week?

6. He finds that his friend Dick keeps bees, getting 33 lb. of honey a year from each hive. Dick has four hives, and he sells the honey for 18¢ a pound. Is he doing better or worse than Tom, and how much per week?

7. Tom decides to add bees to his source of income. If he can buy two hives of Italian bees for \$4.25 a hive, and needs in addition \$1.80 worth of supplies, how many weeks will he have to save his income, as found in Ex. 5, in order to pay for the bees and supplies?

SHALL IT BE A CAR OR A HORSE?

1. A physician finds that it costs him \$4.50 a week to keep his horse at a livery stable. He pays \$2.50 twice in every three months for shoeing his horse. Veterinary attention averages \$4 a year. What is the total expense per year?

2. The horse and buggy represent an investment of \$275. At 6% interest, how much must be added to the above to obtain the total outlay for a year?

3. But this is not all, for there is the deterioration to be considered. Figuring that in 15 yr., even with good care meanwhile, he could sell both the horse and buggy for only \$50, how much must he add to his annual outgo to allow for deterioration?

4. He decides to see whether it might not be better to buy an automobile. He finds that he can get a serviceable one for \$765. At 6%, what would be his annual loss in interest on his money?

5. He finds that every 3 yr. he can, by paying \$250 additional, exchange his old car for a new one. How does the deterioration per year compare with that on the horse and buggy, as found in Ex. 3?

6. But there is also the expense of tires. Allowing \$12.50 a month, what will this amount to in a year?

7. Then there are the operating expenses. He figures that he will use 15 gal. of gasoline a week at 20¢ a gallon, besides which he will use 80¢ worth of lubricating oil a week and must pay \$3 a week at the garage. What will this amount to in a year?

8. He figures that the automobile would save him 3 hr. a day. If he counts the time at 75¢ an hour, how much will this amount to in a year, supposing he is actually at work 361 da.?

9. He thinks it would be worth \$200 a year as an advertisement, to be going around in a car instead of driving a horse. Now consider the whole question. What are the total expenses of the horse and buggy? What are the total expenses of the car, less his saving in time and his advertisement? Which is the better for him to have?

APPENDIX

229. Efficiency Tests. Teachers occasionally wish more efficiency tests than a textbook supplies. In this exercise an opportunity is offered to select from thousands of examples.

WRITTEN EXERCISE

1. Add each of the 10 numbers in the first column of I to each of the lettered numbers of the second column, thus giving 100 examples. Then do the same for II, making 200 examples.

For example, $(7) + (c) = 10,125 + 27 = 10,152$. Where the work is so simple as to allow the result to be written at sight, this should be done.

2. In the same way, subtract, giving 200 examples.
3. In the same way, multiply, giving 200 examples.
4. In the same way, divide, giving 200 examples.

I		II	
1. 2025.	<i>a.</i> 9.	1. 1176.	<i>a.</i> 4.
2. 8100.	<i>b.</i> 15.	2. 4704.	<i>b.</i> 6.
3. 6075.	<i>c.</i> 27.	3. 2352.	<i>c.</i> 7.
4. 16,200.	<i>d.</i> 45.	4. 5880.	<i>d.</i> 8.
5. 12,150.	<i>e.</i> 25.	5. 3528.	<i>e.</i> 21.
6. 18,225.	<i>f.</i> 75.	6. 9408.	<i>f.</i> 49.
7. 10,125.	<i>g.</i> 81.	7. 8232.	<i>g.</i> 98.
8. 14,175.	<i>h.</i> 225.	8. 10,584.	<i>h.</i> 147.
9. 22,275.	<i>i.</i> 135.	9. 12,936.	<i>i.</i> 196.
10. 20,250.	<i>j.</i> 675.	10. 11,760.	<i>j.</i> 392.

WRITTEN EXERCISE

Add, subtract, multiply, and divide, using each number of the first columns with each lettered number of the second columns:

I		II	
1. 8640.	a. 2.	1. 10,368.	a. 16.
2. 5184.	b. 8.	2. 20,736.	b. 32.
3. 3456.	c. 4.	3. 51,840.	c. 27.
4. 6912.	d. 6.	4. 41,472.	d. 81.
5. 17,280.	e. 9.	5. 62,208.	e. 54.
6. 34,560.	f. 3.	6. 31,104.	f. 36.
7. 55,296.	g. 16.	7. 72,576.	g. 64.
8. 58,752.	h. 18.	8. 93,312.	h. 48.
9. 65,664.	i. 24.	9. 134,784.	i. 96.
10. 86,400.	j. 27.	10. 155,520.	j. 162.

WRITTEN EXERCISE

Add, subtract, multiply, and divide as above:

I		II	
1. 64.68.	a. 0.4.	1. 189.728.	a. 0.4.
2. 582.12.	b. 0.8.	2. 3794.56.	b. 0.8.
3. 12.936.	c. 0.6.	3. 5.69184.	c. 1.6.
4. 258.72.	d. 2.4.	4. 75,891.2.	d. 3.2.
5. 7.7616.	e. 20.	5. 94.864.	e. 1.4.
6. 905.52.	f. 0.07.	6. 113.8368.	f. 2.8.
7. 3880.8.	g. 1.10.	7. 13.28096.	g. 0.56.
8. 194.04.	h. 7.7.	8. 1517.824.	h. 0.11.
9. 106.424.	i. 5.5.	9. 17.07552.	i. 1.21.
10. 84.084.	j. 26.4.	10. 18.9728.	j. 0.77.

WRITTEN EXERCISE

Add, subtract, multiply, and divide as on page 320 :

I

- | | |
|------------------------|----------------------|
| 1. $\frac{5}{8}$. | a. $\frac{1}{2}$. |
| 2. $\frac{7}{8}$. | b. $\frac{1}{3}$. |
| 3. $\frac{5}{6}$. | c. $\frac{1}{4}$. |
| 4. $\frac{3}{4}$. | d. $\frac{1}{5}$. |
| 5. $\frac{3}{5}$. | e. $\frac{1}{6}$. |
| 6. $\frac{4}{7}$. | f. $\frac{1}{8}$. |
| 7. $\frac{5}{7}$. | g. $\frac{2}{5}$. |
| 8. $\frac{6}{7}$. | h. $\frac{3}{8}$. |
| 9. $\frac{5}{9}$. | i. $\frac{1}{9}$. |
| 10. $\frac{7}{9}$. | j. $\frac{2}{9}$. |
| 11. $\frac{8}{9}$. | k. $\frac{4}{9}$. |
| 12. $\frac{7}{10}$. | l. $\frac{1}{12}$. |
| 13. $\frac{9}{10}$. | m. $\frac{5}{12}$. |
| 14. $\frac{6}{11}$. | n. $\frac{1}{16}$. |
| 15. $\frac{7}{12}$. | o. $\frac{3}{16}$. |
| 16. $\frac{7}{16}$. | p. $\frac{5}{16}$. |
| 17. $\frac{9}{16}$. | q. $\frac{7}{16}$. |
| 18. $\frac{13}{16}$. | r. $\frac{1}{24}$. |
| 19. $\frac{15}{16}$. | s. $\frac{5}{24}$. |
| 20. $\frac{17}{32}$. | t. $\frac{7}{24}$. |
| 21. $\frac{23}{24}$. | u. $\frac{11}{24}$. |
| 22. $\frac{19}{24}$. | v. $\frac{1}{32}$. |
| 23. $\frac{17}{48}$. | w. $\frac{3}{32}$. |
| 24. $\frac{39}{48}$. | x. $\frac{7}{32}$. |
| 25. $\frac{51}{100}$. | y. $\frac{5}{64}$. |

II

- | | |
|-----------------------|-----------------------|
| 1. $4\frac{1}{2}$. | a. 2. |
| 2. $3\frac{1}{4}$. | b. 3. |
| 3. $3\frac{3}{4}$. | c. $1\frac{1}{8}$. |
| 4. $4\frac{1}{8}$. | d. $1\frac{3}{8}$. |
| 5. $5\frac{3}{8}$. | e. $2\frac{5}{8}$. |
| 6. $3\frac{5}{8}$. | f. $2\frac{7}{8}$. |
| 7. $4\frac{7}{8}$. | g. $1\frac{1}{5}$. |
| 8. $3\frac{1}{16}$. | h. $2\frac{3}{5}$. |
| 9. $4\frac{3}{16}$. | i. $1\frac{4}{5}$. |
| 10. $3\frac{5}{16}$. | j. $2\frac{1}{6}$. |
| 11. $3\frac{7}{16}$. | k. $1\frac{5}{6}$. |
| 12. $4\frac{9}{16}$. | l. $1\frac{1}{10}$. |
| 13. $3\frac{1}{16}$. | m. $2\frac{3}{10}$. |
| 14. $4\frac{1}{16}$. | n. $1\frac{1}{16}$. |
| 15. $3\frac{1}{24}$. | o. $1\frac{3}{16}$. |
| 16. $3\frac{5}{24}$. | p. $2\frac{5}{16}$. |
| 17. $4\frac{1}{24}$. | q. $1\frac{9}{16}$. |
| 18. $3\frac{1}{32}$. | r. $1\frac{1}{24}$. |
| 19. $3\frac{3}{32}$. | s. $2\frac{5}{24}$. |
| 20. $4\frac{5}{32}$. | t. $2\frac{7}{24}$. |
| 21. $3\frac{9}{32}$. | u. $1\frac{3}{32}$. |
| 22. $3\frac{1}{48}$. | v. $1\frac{9}{32}$. |
| 23. $4\frac{5}{48}$. | w. $2\frac{1}{48}$. |
| 24. $3\frac{7}{48}$. | x. $1\frac{1}{64}$. |
| 25. $3\frac{1}{10}$. | y. $2\frac{3}{100}$. |

WRITTEN EXERCISE

Take, in turn, each per cent in the first column of I and find that per cent of each number in the second column; do the same for II:

I		II	
1. 6%.	a. 2064.	1. 50%.	a. 84.88.
2. $12\frac{1}{2}\%$.	b. 2664.	2. $62\frac{1}{2}\%$.	b. 73.68.
3. 25%.	c. 5064.	3. $87\frac{1}{2}\%$.	c. 6.656.
4. $33\frac{1}{3}\%$.	d. 9564.	4. $66\frac{2}{3}\%$.	d. 5.664.
5. $37\frac{1}{2}\%$.	e. 9648.	5. 75%.	e. 0.1776.

Find, in turn, what per cent each number in the first column of I is of each lettered number in the second column, and then do the same for II:

I		II	
1. 12.	a. 120.	1. 128.	a. 1024.
2. 36.	b. 132.	2. 256.	b. 1536.
3. 48.	c. 144.	3. 512.	c. 3072.
4. 72.	d. 252.	4. 1024.	d. 4608.
5. 96.	e. 496.	5. 3072.	e. 6144.

Find, in turn, the sum of each number in the first column of I and each lettered number in the second column and then find their difference; do the same for II:

I		II	
1. 29 ft. 1 in.	a. 12 ft. 4 in.	1. 50 lb. 1 oz.	a. 29 lb. 5 oz.
2. 37 ft. 2 in.	b. 13 ft. 9 in.	2. 61 lb. 2 oz.	b. 32 lb. 7 oz.
3. 68 ft. 3 in.	c. 16 ft. 10 in.	3. 73 lb. 9 oz.	c. 38 lb. 12 oz.
4. 72 ft. 9 in.	d. 23 ft. 10 in.	4. 82 lb. 6 oz.	d. 39 lb. 14 oz.
5. 90 ft. 4 in.	e. 28 ft. 11 in.	5. 94 lb. 8 oz.	e. 48 lb. 15 oz.
6. 47 ft. 5 in.	f. 24 ft. 6 in.	6. 43 lb. 2 oz.	f. 29 lb. 6 oz.

TABLES FOR REFERENCE

Length

- 12 inches (in.) = 1 foot (ft.)
3 feet = 1 yard (yd.)
 $5\frac{1}{2}$ yards, or $16\frac{1}{2}$ feet = 1 rod (rd.)
320 rods, or 5280 feet = 1 mile (mi.)

Square Measure

- 144 square inches (sq. in.) = 1 square foot (sq. ft.)
9 square feet = 1 square yard (sq. yd.)
 $30\frac{1}{4}$ square yards = 1 square rod (sq. rd.)
160 square rods = 1 acre (A.)
640 acres = 1 square mile (sq. mi.)

Cubic Measure

- 1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.)
27 cubic feet = 1 cubic yard (cu. yd.)
128 cubic feet = 1 cord (cd.)

Weight

- 16 ounces (oz.) = 1 pound (lb.)
2000 pounds = 1 ton (T.)

Liquid Measure

- 4 gills (gi.) = 1 pint (pt.)
2 pints = 1 quart (qt.)
4 quarts = 1 gallon (gal.)

Dry Measure

- 2 pints (pt.) = 1 quart (qt.)
8 quarts = 1 peck (pk.)
4 pecks = 1 bushel (bu.)
2150.42 cu. in. (approximately $1\frac{1}{4}$ cu. ft.) = 1 bushel

Time

60 seconds (sec.) = 1 minute (min.)

60 minutes = 1 hour (hr.)

24 hours = 1 day (da.)

7 days = 1 week (wk.)

12 months (mo.) = 1 year (yr.)

365 days = 1 common year

366 days = 1 leap year

Value

10 mills = 1 cent (¢ or ct.)

10 cents = 1 dime (d.)

10 dimes = 1 dollar (\$)

10 dollars = 1 eagle (E.)

Angles and Arcs

60 seconds (60'') = 1 minute (1')

60 minutes = 1 degree (1°)

Surveyors' Table of Length

7.92 inches = 1 link (li.)

100 links = 4 rods = 1 chain (ch.)

80 chains = 5280 feet = 1 mile

Counting

12 units = 1 dozen (doz.)

12 dozen, or 144 = 1 gross (gr.)

Paper

24 sheets = 1 quire

20 quires, or 480 sheets = 1 ream

For convenience in counting, 500 sheets are now more often called a *ream*; and the word "quire" is used only for folded note paper, other paper being usually sold by the pound.

SUGGESTIONS TO TEACHERS

Suggestions for the Fifth Grade, First Half. The following suggestions relating to the teaching of arithmetic in Grade V, First Half, are taken from the bulletin prepared by the Department of Public Instruction of New Jersey. Other suggestions from the same bulletin appear throughout the text.

The new work of the fifth grade is the teaching of addition, subtraction, multiplication, and division of fractions—common and decimal. This should be divided about equally between the two halves of the year. The first half of the year common fractions (addition and subtraction thoroughly, multiplication and division less thoroughly) should be taught.

The fifth-grade teacher should study carefully the work of the preceding grades, and she may profitably spend two weeks at the beginning of the term in reviewing carefully the fourth-grade work, reteaching those parts upon which the pupils are not clear, and devising new drills for those parts that need emphasis. This review may continue while the new work in fractions is being introduced.

There are two types of problems solved by the process of division :

Type 1. A man sold land for \$15.00 per acre, receiving \$750.00 for it. How many acres did he sell?

Type 2. A man sold 50 acres of land for \$750.00. What was the price per acre?

Formerly it was the practice to think through the problem, determine the "name" of the answer, and perform the division as if it were a pure abstract computation. For instance, in the problem indicated Type 1 the problem calls for "acres." The problem evidently calls for the use of the division process.

The official course of study in arithmetic should be in the hands of every teacher. It can be secured by applying to the State Department of Public Instruction.

Suggestions for the Fifth Grade, Second Half. In the official course of study the State Department of Public Instruction makes certain suggestions for the work of this half year, including the following :

Review carefully and systematically addition, subtraction, multiplication, and division of common fractions. Have much practice with mixed numbers.

Continue reviews of all subjects treated in previous grades, with special attention to training for skill in interpretation of problems, in estimating results, and in economy of calculation.

The work of this year will be much simplified if the numbers are kept small and the problems are kept simple. The emphasis should be placed on familiarizing pupils with the processes.

Instruction in decimals should be introduced by illustrations with United States money. Teach addition, subtraction, multiplication, and division of decimal fractions, using decimals to three places. In reading mixed numbers use the word "and" only at the decimal point. Keep the work simple. In multiplication limit the work to examples having the decimal point in the multiplier only, and in division to examples having the decimal point in the dividend only.

Ability to see readily the relations between numbers and to choose the economical way to use these relations in calculation is gained only by training. Only a few of these economical uses of number relations have been mentioned for purposes of illustration. If pupils are shown the way, if they are trained to keep an open mind (mathematically), and if they are given opportunities for abundant practice, they will develop skill, varying in degree with the individual, without pressure. Again it should be emphasized that no encouragement is here given to train "lightning calculators." Such specialized training as that term implies has no place in school. The purpose is so to teach elementary arithmetic that the pupils may grow in power to understand and to interpret the problems which belong properly to the elementary field of arithmetic, and that they may have such a command of number combinations and of number relations that they may with accuracy, readiness, and economy of labor perform the processes necessary for the solution of such problems. Training will cause the mind instinctively to picture calculation possibilities and will give skill in the selection of the economical plan of procedure.

Suggestions for the Sixth Grade, First Half. The State Department of Public Instruction makes certain suggestions for the work of this half year, including the following:

Teachers of this grade should not expect pupils at the beginning of the term to be proficient in the work of the preceding grade. Two or three weeks, or even a longer time, should be spent in a careful and systematic review of the work in fractions. The sixth-grade teacher should be at home in the work of the fifth-grade teacher.

During the time of this review much practice may be given in interpretation of problems, in estimating results, in becoming skillful in economical calculation, and in choosing short methods of solving problems by taking advantage of number relations, using the aliquot parts of \$1.00 or 100. Pupils should continue to draw to scale during this period.

Often pupils are taught to calculate properly in oral work but are allowed to use long, primitive, laborious methods in written work. Economical methods should be used in both written and oral calculations. Pupils should be encouraged to find and use economical methods of working all problems. However, here as elsewhere, good judgment should control, in order that their calculations may be in reality economical. Pupils should be encouraged to use the pencil as little as possible in the solution of all problems.

Closely allied to training for skill in calculation is training in estimating what the reasonable answers to problems are, as a preliminary to their solution. This training may begin early in school life in estimating lengths, areas, capacities, weights, etc. Much oral and written work of this character with all kinds of problems may profitably be given in all grades until pupils have established the habit of doing their mathematical work with an expectation of arriving at a reasonable result. Good textbooks give frequent suggestions and opportunities for this training.

The habit of checking work as it progresses should be fixed in the early grades. This checking consists in *repeating* one process before the next process is begun. In adding, the calculation should be repeated in the reverse order. In subtracting, the remainder and subtrahend may be added. The repetition should be made after each part of the problem has been solved rather than after the entire problem has been solved. This is the business custom.

Suggestions for the Sixth Grade, Second Half. The State Department of Public Instruction makes certain suggestions for the work of this half year, including the following:

The purpose of the review work in this grade is to establish a strong foundation for future mathematical study by making pupils sure and skillful in the use of the essential processes previously studied.

At the outset the teacher should make sure that every pupil can add and subtract accurately and use the multiplication combinations automatically. All cases of arrested development, such as counting in adding, adding instead of using the multiplication combinations, and writing down numbers to be added to the next higher order in multiplication should be earnestly sought out and corrected. The teacher, particularly of this grade, should feel that, unless this fundamental work is thoroughly accomplished, her work in arithmetic is, to a considerable extent, a failure.

Children in school rely too much on the pencil. What can be done mentally should not be done with the pencil. In mental and oral exercises the pencil may often be used to record data and answers. In written exercises the pencil should not take the place of the mind. All pupils will not be alike in their ability to free themselves from slavery to the pencil, but all may be taught how to do it, and they may be encouraged and rewarded for successful effort in this direction.

Mental and oral work should be given daily, if possible, for about one third of the time of the recitation, usually preceding the other work. In five minutes two or three examples in abstract arithmetic and two or three problems may be solved. The problems may be dictated to the class, and the calculations may be made mentally and the answers recorded on a slip of paper. The problems that are not understood by the pupils should be noted, and the principles involved should be introduced at another time.

At the close of Grade VI every pupil should be perfectly familiar with the fundamental operations with integers, common fractions, decimal fractions, and ordinary compound numbers. In most countries the work in arithmetic closes at this point, except as it is carried along, one day in the week, in connection with simple work in algebra and geometry. In this country we should seek to bring our standards up to the same level. The important point is that the pupils should now be able to compute accurately and quickly.

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